

Modeling and Computation of Security-constrained Economic Dispatch with Multi-stage Rescheduling

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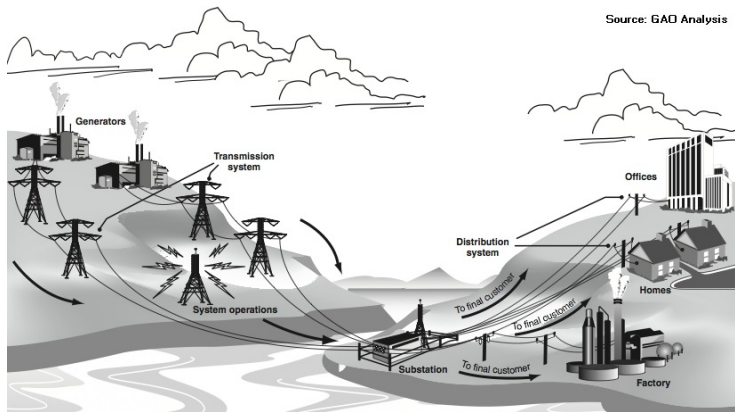
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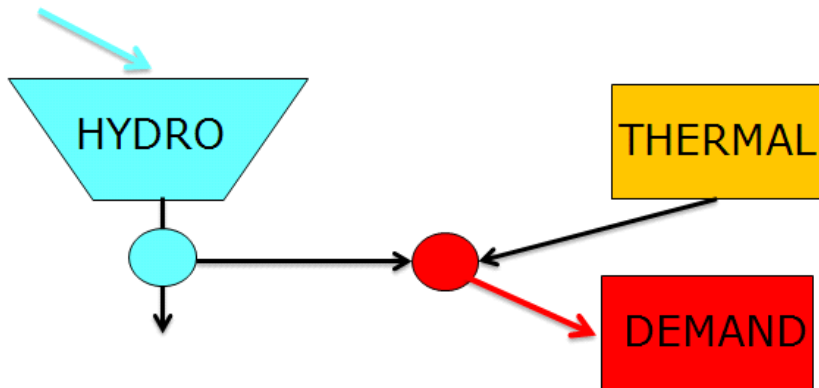
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Power generation, transmission and distribution



- Determine generators' output to reliably meet the load
 - ▶ $\sum \text{Gen MW} = \sum \text{Load MW}$, at all times.
 - ▶ Power flows cannot exceed lines' transfer capacity.

Hydro-Thermal System (Philpott/F./Wets)



Simple electricity “system optimization” problem

$$\begin{aligned} \text{SO: } \max_{d_k, u_i, v_j, x_i \geq 0} \quad & \sum_{k \in \mathcal{K}} W_k(d_k) - \sum_{j \in \mathcal{T}} C_j(v_j) + \sum_{i \in \mathcal{H}} V_i(x_i) \\ \text{s.t.} \quad & \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k, \\ & x_i = x_i^0 - u_i + h_i^1, \quad i \in \mathcal{H} \end{aligned}$$

- u_i water release of hydro reservoir $i \in \mathcal{H}$
- v_j thermal generation of plant $j \in \mathcal{T}$
- x_i water level in reservoir $i \in \mathcal{H}$
- prod fn U_i (strictly concave) converts water release to energy
- $C_j(v_j)$ denote the cost of generation by thermal plant
- $V_i(x_i)$ future value of terminating with storage x (assumed separable)
- $W_k(d_k)$ utility of consumption d_k

SO equivalent to CE

Consumers $k \in \mathcal{K}$ solve CP(k): $\max_{d_k \geq 0} W_k(d_k) - p^T d_k$

Thermal plants $j \in \mathcal{T}$ solve TP(j): $\max_{v_j \geq 0} p^T v_j - C_j(v_j)$

Hydro plants $i \in \mathcal{H}$ solve HP(i): $\max_{u_i, x_i \geq 0} p^T U_i(u_i) + V_i(x_i)$
s.t. $x_i = x_i^0 - u_i + h_i^1$

Perfectly competitive (Walrasian) equilibrium is a MOPEC

CE: $d_k \in \arg \max CP(k),$ $k \in \mathcal{K},$

$v_j \in \arg \max TP(j),$ $j \in \mathcal{T},$

$u_i, x_i \in \arg \max HP(i),$ $i \in \mathcal{H},$

$$0 \leq p \perp \sum_{i \in \mathcal{H}} U_i(u_i) + \sum_{j \in \mathcal{T}} v_j \geq \sum_{k \in \mathcal{K}} d_k.$$

Nash Equilibria (as a MOPEC)

- Nash Games: x^* is a Nash Equilibrium if

$$x_i^* \in \arg \min_{x_i \in X_i} \ell_i(x_i, x_{-i}^*, p), \forall i \in \mathcal{I}$$

x_{-i} are the decisions of other players.

- Prices p given exogenously, or via complementarity:

$$0 \leq H(x, p) \perp p \geq 0$$

- **empinfo: equilibrium**
min loss(i) x(i) cons(i)
vi H p
- Applications: Discrete-Time Finite-State Stochastic Games. Specifically, the Ericson & Pakes (1995) model of dynamic competition in an oligopolistic industry.

Key point: models generated correctly solve quickly

Here S is mesh spacing parameter

| S | Var | rows | non-zero | dense(%) | Steps | RT (m:s) |
|-----|--------|--------|----------|----------|-------|----------|
| 20 | 2400 | 2568 | 31536 | 0.48 | 5 | 0 : 03 |
| 50 | 15000 | 15408 | 195816 | 0.08 | 5 | 0 : 19 |
| 100 | 60000 | 60808 | 781616 | 0.02 | 5 | 1 : 16 |
| 200 | 240000 | 241608 | 3123216 | 0.01 | 5 | 5 : 12 |

Convergence for $S = 200$ (with new basis extensions in PATH)

| Iteration | Residual |
|-----------|-----------|
| 0 | 1.56(+4) |
| 1 | 1.06(+1) |
| 2 | 1.34 |
| 3 | 2.04(-2) |
| 4 | 1.74(-5) |
| 5 | 2.97(-11) |

Agents have stochastic recourse?

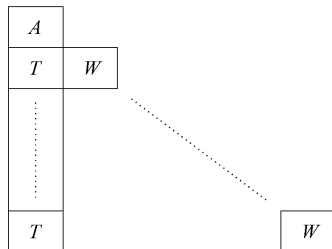
- Two stage stochastic programming, x^1 is here-and-now decision, recourse decisions x^2 depend on realization of a random variable
- ρ is a risk measure (e.g. expectation, CVaR)

$$\text{SP: max } c^T x^1 + \rho[q^T x^2]$$

$$\text{s.t. } Ax^1 = b, \quad x^1 \geq 0,$$

$$T(\omega)x^1 + W(\omega)x^2(\omega) \geq d(\omega),$$

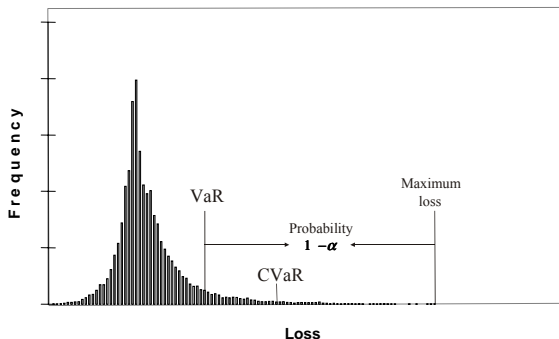
$$x^2(\omega) \geq 0, \forall \omega \in \Omega.$$



EMP/SP extensions to facilitate these models

Risk Measures

- Modern approach to modeling risk aversion uses concept of risk measures
- \overline{CVaR}_α : mean of upper tail beyond α -quantile (e.g. $\alpha = 0.95$)



- mean-risk, mean deviations from quantiles, VaR, CVaR
- Much more in mathematical economics and finance literature
- Optimization approaches still valid, different objectives, varying convex/non-convex difficulty

Stochastic unit commitment: different risk measures

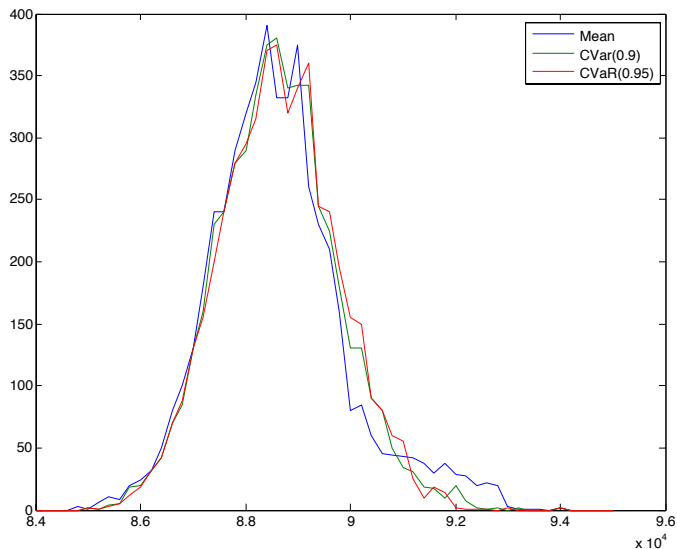


Figure : Frequency plot for cost for 5000 (out-of-sample) simulations

Equilibrium or optimization?

- Each agent has its own risk measure
- Is there a system risk measure?
- Is there a system optimization problem?

$$\min \sum_i C(x_i^1) + \rho_i (C(x_i^2(\omega)))????$$

- Can we modify (complete) system to have a social optimum by trading risk?
- How do we design these instruments? How many are needed? What is cost of deficiency?
- Can we solve efficiently / distributively?

Contracts in MOPEC (F./Wets)

- Competing agents (consumers, or generators in energy market)
- Each agent minimizes objective independently (cost)
- Market prices are function of all agents activities
- Additional twist: model must “hedge” against uncertainty
- Facilitated by allowing contracts bought now, for goods delivered later (e.g. Arrow-Debreu Securities)
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancillary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Example as MOPEC: agents solve a Stochastic Program

Buy y_i contracts in period 1, to deliver $D(\omega)y_i$ in period 2, scenario ω
Each agent i :

$$\begin{aligned} \min \quad & C(x_i^1) + \rho_i (C(x_i^2(\omega))) \\ \text{s.t.} \quad & p^1 x_i^1 + v y_i \leq p^1 e_i^1 && \text{(budget time 1)} \\ & p^2(\omega) x_i^2(\omega) \leq p^2(\omega) (D(\omega) y_i + e_i^2(\omega)) && \text{(budget time 2)} \end{aligned}$$

$$0 \leq v \perp - \sum_i y_i \geq 0 \quad \text{(contract)}$$

$$0 \leq p^1 \perp \sum_i (e_i^1 - x_i^1) \geq 0 \quad \text{(walras 1)}$$

$$0 \leq p^2(\omega) \perp \sum_i (D(\omega) y_i + e_i^2(\omega) - x_i^2(\omega)) \geq 0 \quad \text{(walras 2)}$$

Theory and Observations

- agent problems are multistage stochastic optimization models
- perfectly **competitive partial equilibrium** still corresponds to a **social optimum** when all agents are **risk neutral** and share common knowledge of the probability distribution governing future inflows
- **situation complicated when agents are risk averse**
 - ▶ utilize stochastic process over scenario tree
 - ▶ under mild conditions a social optimum corresponds to a competitive market equilibrium if agents have time-consistent dynamic coherent risk measures and there are **enough traded market instruments (over tree)** to hedge inflow uncertainty
- Otherwise, must solve the stochastic equilibrium problem
- Solution possible via disaggregation only seems possible in special cases
 - ▶ When problem is block diagonally dominant (Wathen/F./Rutherford)
 - ▶ When overall (complementarity) problem is monotone
 - ▶ (Pang): when problem is a potential game
- **Research challenge: develop reliable algorithms for large scale decomposition approaches to MOPEC**

Security-constrained Economic Dispatch

- Base-case network topology g_0 and line flow x_0 .
- If the k -th line fails, line flow jumps to x_k in new topology g_k .
- Ensure that x_k is within limit, for all k .
- SCED model:

$$\min_{u, x_0, \dots, x_k} c^T u + \rho(u)$$

$$\text{s.t.} \quad 0 \leq u \leq \bar{u}$$

$$g_0(x_0, u) = 0$$

$$-\bar{x} \leq x_0 \leq \bar{x}$$

$$g_k(x_k, u) = 0, \quad k = 1, \dots, K$$

$$-\bar{x} \leq x_k \leq \bar{x}, \quad k = 1, \dots, K$$

▷ Total cost

▷ GEN capacity const.

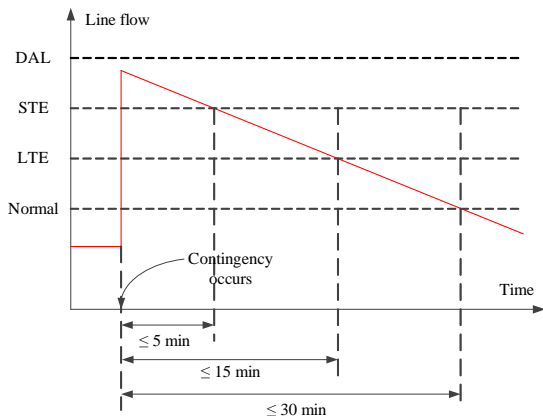
▷ Base-case network eqn.

▷ Base-case flow limit

▷ Ctgcy network eqn.

▷ Ctgcy flow limit

Reality offers a sweeter deal...



Operating procedure (ISO-NE) requires post-contingency line loadings be:

- \leq STE (short time emergency) rating in 5 minutes;
- \leq LTE (long time emergency) rating in 15 minutes;
- \leq Normal rating in 30 minutes.

What we will contribute

Research issues:

- Corrective actions are not modeled in ISO's dispatch software.
- Because it was “insolvable” due to its large size ($\geq 10\text{GB LP}$).
 - ▶ “We looked into SCED with corrective actions before, and were hindered by the computational challenge.” – Feng Zhao, senior analyst at ISO-NE, via private correspondence.

Our contributions:

- We **model** the *multi-period* corrective rescheduling in SCED; solutions much better quality
- **Enhance** the Benders' **algorithm** to solve the problem faster
- **Achieve** about $50\times$ **speedup** compared to traditional approaches

Our model (K contingencies, T periods)

$$\begin{aligned} \min_{x_0, \dots, x_k, u_0, \dots, u_k} \quad & c^T u_0 \\ \text{s.t.} \quad & g_0(x_0, u_0) = 0 \\ & h_0(x_0, u_0) \leq 0 \\ & g_k(x_k^t, u_k^t) = 0 \quad k = 1, \dots, K, t = 0, \dots, T \\ & h_k(x_k^t, u_k^t) \leq 0 \quad k = 1, \dots, K, t = 0, \dots, T \\ & |u_k^t - u_k^{t-1}| \leq \Delta_t \quad k = 1, \dots, K, t = 1, \dots, T \\ & u_k^0 - u_0 = 0 \quad k = 1, \dots, K \end{aligned}$$

- Subscript 0 indicates a quantity in the base-case network topology.
- This is a large-scale linear program.
- What special structure does it have?

Model structure

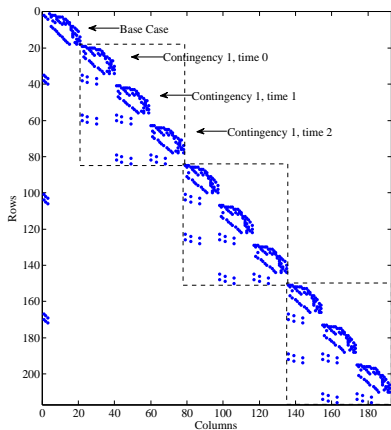


Figure : Sparsity structure of the Jacobian matrix of a 6-bus case, considering 3 contingencies and 3 post-contingency checkpoints.

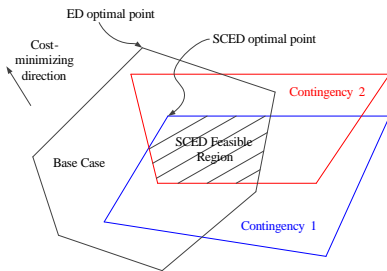


Figure : On the u_0 plane, the feasible region of a SCED is the intersection of $K+1$ polyhedra.

Current state of the art (unsatisfactory)

Table : CPLEX v.s. Vanilla Benders Algorithm

| Case | Ctgcy | Big LP (time) | | Vanilla Benders | | |
|----------|-------|---------------|----------------------|-----------------|------|--------|
| | | Simplex | Barrier ¹ | Iter | LPs | Time |
| 118-bus | 183 | 207.8 | 13.8 | 8 | 1464 | 123.5 |
| 2383-bus | 20 | 175.0 | 205.5 | 52 | 1040 | 1281.2 |
| 2383-bus | 50 | 1403.2 | 123.1 | 49 | 2450 | 2799.3 |
| 2383-bus | 100 | 3621.8 | 240.6 | 32 | 3200 | 3688.6 |
| 2383-bus | 400 | - | 2354.5 | - | - | - |

- Three time-periods: 5-min STE, 15-min LTE and 30-min Normal.
- Vanilla Benders' algorithm is inferior to the big LP formulation.
- Big LP cannot handle large instances.

¹Barrier method without crossover. Crossover may take even more time.

How we enhanced the Benders' algorithm ...

- 1 Reduce the number of LPs
- 2 Solve subproblem LPs faster
- 3 Parallel computing
- 4 Add difficult contingencies to master model

| Case | Ctgcy | Big LP (time) | | Enhanced Benders | | |
|----------|-------|---------------|---------|------------------|------|-------|
| | | Simplex | Barrier | Iter | LPs | Time |
| 118-bus | 183 | 207.8 | 13.8 | 12 | 755 | 13.5 |
| 2383-bus | 20 | 175.0 | 205.5 | 11 | 60 | 41.5 |
| 2383-bus | 50 | 1403 | 123.1 | 11 | 135 | 46.5 |
| 2383-bus | 100 | 3621 | 240.6 | 12 | 245 | 79.4 |
| 2383-bus | 400 | - | 2354.5 | 13 | 879 | 197.8 |
| 2383 wp | 2349 | | | 21 | 9529 | 515.7 |
| 2736 sp | 2749 | | | 4 | 5500 | 220.9 |
| 2737 sop | 2753 | | | 1 | 2753 | 100.5 |
| 2746 wop | 2794 | | | 1 | 2794 | 118.5 |
| 2746 wp | 2719 | | | 14 | 5558 | 333.5 |

Illustration

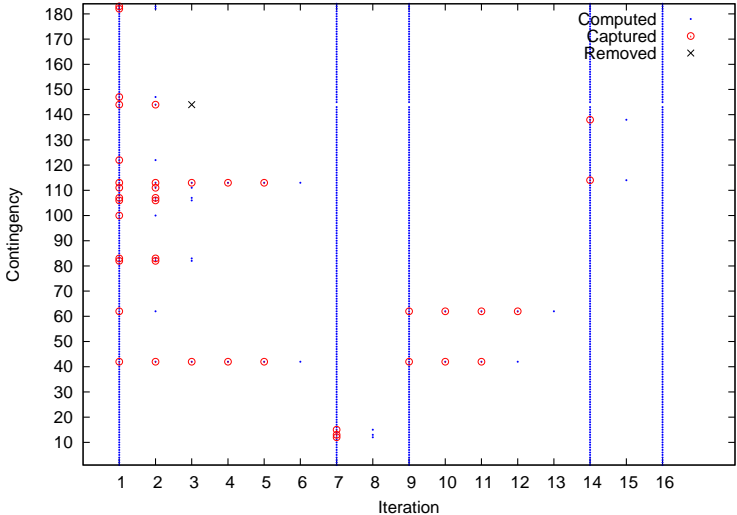


Figure : Benders' algorithm with reduced number of subproblem LPs, 118-bus case

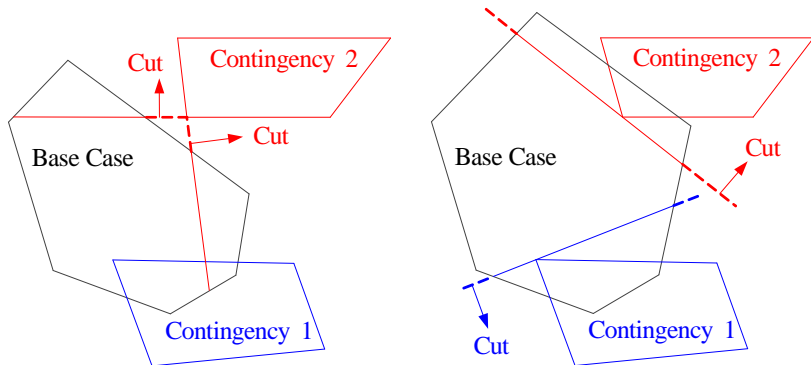
Computational Results

| Case | Ctgcy | RedLP+Opt | | | Paraguss (8) | | | Fatmaster (5) | | |
|---------|-------|-----------|-----|-------|--------------|-----|-------|---------------|-----|-------|
| | | Iter | LPs | Time | Iter | LPs | Time | Iter | LPs | Time |
| 118-bus | 183 | 10 | 764 | 72.6 | 14 | 776 | 15.1 | 12 | 755 | 13.5 |
| 2383 wp | 20 | 46 | 115 | 99.8 | 48 | 117 | 95.4 | 11 | 60 | 41.5 |
| 2383 wp | 50 | 48 | 193 | 160.3 | 48 | 193 | 101.7 | 11 | 135 | 46.5 |
| 2383 wp | 100 | 33 | 289 | 226.0 | 32 | 288 | 96.3 | 12 | 245 | 79.4 |
| 2383 wp | 400 | 35 | 953 | 913.3 | 38 | 956 | 218.0 | 13 | 879 | 197.8 |

| Case | Ctgcy | RedLP+Opt | | | Paraguss (40) | | | Fatmaster (5) | | |
|---------------|-------------|-----------|-------|-------|---------------|------|------|---------------|------|------|
| | | Iter | LPs | Time | Iter | LPs | Time | Iter | LPs | Time |
| 2383wp | 2349 | 106 | 12123 | 12165 | 104 | 9788 | 770 | 21 | 9529 | 516 |
| 2736sp | 2749 | 45 | 5543 | 5836 | 44 | 5542 | 366 | 4 | 5500 | 221 |
| 2737sop | 2753 | 1 | 2753 | 2801 | 1 | 2753 | 100 | 1 | 2753 | 101 |
| 2746wop | 2794 | 1 | 2794 | 3046 | 1 | 2794 | 118 | 1 | 2794 | 119 |
| 2746wp | 2719 | 262 | 8646 | 9738 | 278 | 8622 | 1428 | 14 | 5558 | 334 |

- Big LP for **2383-bus 2349-contingency** case generates a 18GB LP. CPLEX could not solve it in 3 hours.
- Computer used for the lower table: Dell R710 (opt-a006) 2 3.46G Chips 12 Cores, 288G Memory.

Dealing with Infeasibility



(a) Contingency 2 is intrinsically infeasible. Either the corresponding subproblem is infeasible or its Benders' cuts will render the master problem infeasible.

(b) Each individual contingency is feasible, but they are not simultaneously feasible. Their Benders' cuts will render the master problem infeasible.

Figure : Two cases of infeasibility.

Identifying infeasible contingencies in Benders' algorithm

- If a subproblem is infeasible (in the first iteration), the corresponding contingency is intrinsically infeasible. Remove (tabu) it.
 - ▶ Typically line failure results in an islanded load node or sub-network.
- Master problem infeasible: solve a modified master model to find the “minimal” set of problematic contingencies using sparse optimization.

$$\begin{aligned} \min_{x_0, u_0} \quad & f_0(x_0, u_0) + \sum_{k \in K} M v_k \\ \text{s.t.} \quad & g_0(x_0, u_0) = 0, h_0(x_0, u_0) \leq 0 \\ & \bar{w}_k^i + \bar{\lambda}_k^i (u_0 - \bar{u}_0^i) - v_k \leq 0, v_k \geq 0 \quad \forall (k, i) \in \text{CUT} \end{aligned}$$

- ▶ Solution of this model indicates the violated cuts.
 - ▶ Tabu the contingency that has contributed one or more violated cuts.
- Start a pre-screening daemon in parallel when the Active List size is smaller than L^{fc} .
 - ▶ Tabu infeasible ones, and add feasible ones to the master problem.

Computational Results

Table : Solution for big cases on opt-a006, 80 threads, $L^{fc} = 5$

| Case | Ctgcy | Iter | LPs | Time | To Master | Tabu |
|----------|-------|------|------|-------|-----------|------|
| 2383 wp | 2896 | 15 | 7694 | 522.1 | 6 | 547 |
| 2736 sp | 3269 | 4 | 6020 | 252.9 | 1 | 520 |
| 2737 sop | 3269 | 4 | 6023 | 242.2 | 0 | 516 |
| 2746 wop | 3307 | 4 | 6102 | 280.2 | 0 | 513 |
| 2746 wp | 3279 | 8 | 6053 | 334.3 | 4 | 560 |
| 2383 wp | 2353 | 16 | 7156 | 460.6 | 6 | 4 |
| 2736 sp | 2749 | 4 | 5498 | 245.9 | 1 | 0 |
| 2737 sop | 2753 | 1 | 2753 | 110.8 | 0 | 0 |
| 2746 wop | 2794 | 1 | 2794 | 131.7 | 0 | 0 |
| 2746 wp | 2719 | 14 | 5558 | 354.4 | 4 | 0 |

- Upper: all lines are in the Contingency List (N-1 security).
- Lower: all pre-screened lines are in the Contingency List.

SCED with SDP subproblems

- Economic dispatch is a short-term planning problem, so a “DC” model is OK.
- Contingency response is an operational problem, and should be studied on full AC network representation.
- But AC power flow gives a nonconvex problem, which cannot generate valid cuts from a Benders’ subproblem.

Idea

Relaxing the AC feasibility problem using semi-definite programming (SDP) to obtain a convex subproblem.

Goal

Producing a base-case dispatch solution such that contingencies are “really” controllable in the AC context.

SDP relaxation of AC feasibility problem

Model ACF-SDP:

$$\begin{aligned} \min_{W \succeq 0} \quad & A_0 \bullet W \\ \text{s.t.} \quad & \sum_{g \in \mathcal{G}_i} \underline{G}_g^{\text{real}} - D_i^{\text{real}} \leq A_{1i} \bullet W \leq \sum_{g \in \mathcal{G}_i} \bar{G}_g^{\text{real}} - D_i^{\text{real}} & \forall i \in \text{BUS} \\ & \sum_{g \in \mathcal{G}_i} \underline{G}_g^{\text{imag}} - D_i^{\text{imag}} \leq A_{2i} \bullet W \leq \sum_{g \in \mathcal{G}_i} \bar{G}_g^{\text{imag}} - D_i^{\text{imag}} & \forall i \in \text{BUS} \\ & -\bar{F}_{i,j} \leq A_{3ij} \bullet W \leq \bar{F}_{i,j} & \forall (i,j) \in \text{LINE} \\ & (\underline{V}_i)^2 \leq A_{4i} \bullet W \leq (\bar{V}_i)^2 & \forall i \in \text{BUS} \\ & \sum_{g \in \mathcal{G}_i} (G_g^0 - \Delta_g) \leq A_{5i} \bullet W \leq \sum_{g \in \mathcal{G}_i} (G_g^0 + \Delta_g) & \forall i \in \text{BUS} \end{aligned}$$

- It is a convex optimization problem and weak (strong) duality holds.
- It is a relaxation because the requirement that W has rank 1 is dropped.

Experiments

| Case | Cont | Solution | | | | Performance | | |
|------|------|----------|------|----------|---------|-------------|----|----|
| | | Model | Tabu | Cost | Time | IF | FS | FT |
| 14 | 20 | LP | 0 | 13253.3 | 4.2 | 12 | 12 | 0 |
| | | SDP | 6 | 16065.8 | 18.4 | 6 | 0 | 0 |
| | | SDP0 | 6 | 16003.4 | 11.9 | 6 | 0 | 0 |
| 30 | 40 | LP | 0 | 582.0 | 4.0 | 1 | 1 | 0 |
| | | SDP | 1 | 585.0 | 20.1 | 1 | 0 | 0 |
| | | SDP0 | 1 | 600.5 | 22.1 | 1 | 0 | 0 |
| 57 | 20 | LP | 0 | 12508.0 | 1.9 | 1 | 1 | 0 |
| | | SDP | 1 | 12508.0 | 13.2 | 1 | 0 | 0 |
| | | SDP0 | 1 | 12560.0 | 50.9 | 1 | 0 | 0 |
| 118 | 15 | LP | 0 | 139716.8 | 54.0 | 16 | 16 | 0 |
| | | SDP | 0 | 141372.2 | 2414.3 | 1 | 1 | 0 |
| | | SDP0 | 0 | 144220.1 | 11951.1 | 0 | 0 | 0 |

- SDP subproblem is “exact” in contingency response, no False Secure, no False Tabu.
- It takes longer time to solve (with room for improvement).

Summary

- 1 SCED is a million-dollar problem for system operators.
- 2 SCED with corrective actions can save money, but is hard to solve.
 - ▶ Too big for CPLEX
 - ▶ Original Benders' decomposition algorithm is slow.
- 3 Our algorithmic enhancements yield significant speedup.
- 4 Potential for practical deployment.
- 5 SDP extension allows for more accurate operational modeling.

Extension

1. Decomposition approach is useful in many applications.
2. Currently in collaboration with ISO-NE to deploy our algorithm.

Conclusions

- Optimization critical for understanding of power system markets
- Different behaviors are present in practice and modeled here
- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- Policy implications addressable using MOPEC
- Stochastic MOPEC models capture behavioral effects (as an EMP)
- Extended Mathematical Programming available within the GAMS modeling system
- Modeling, optimization, statistics and computation embedded within the application domain is critical

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS