The price of storage

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EMP: shameless advertisement

Allows (GAMS) models to be manipulated to form other problems of interest via a simple EMP info file:

- VI(f, C):

$$0 \in f(x) + N_C(x)$$

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vi f x cons
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generates a variational inequality where C defined by 'cons'

- Either generates the equivalent complementarity (KKT) problem, or provides problem structure for algorithmic exploitation
- QVI can be specified in the same manner

MOPEC

$$\min_{x_i} \theta_i(x_i, x_{-i}, y) \text{ s.t. } g_i(x_i, x_{-i}, y) \leq 0, \forall i$$

and

$$y \text{ solves } \mathrm{VI}(h(x, \cdot), C)$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h y cons
```

is solved in a Nash manner

- Allows multipliers from one problem to be used in another problems
  `dualvar p g(1)`
  where for example the definition of h involves the additional variable p
- Also has extensions for stochastic programming

PJM buy/sell model (2009)

- Storage transfers energy over time (horizon $= T$).
- PJM: given price path p_t, determine charge q_t^+ and discharge q_t^-:

$$
\begin{aligned}
\max_{h_t, q_t^+, q_t^-} \; & \sum_{t=0}^{T} p_t (q_t^- - q_t^+) \\
\text{s.t. } & \partial h_t = e q_t^+ - q_t^- \\
& 0 \le h_t \le \mathcal{S} \\
& 0 \le q_t^+ \le \mathcal{Q} \\
& 0 \le q_t^- \le \mathcal{Q} \\
& h_0, h_T \text{ fixed}
\end{aligned}
$$

- Uses: price shaving, load shifting, transmission line deferral
- what about different storage technologies?

Characterization of storage

$\mathcal{Q}$	power (discharge) capacity	MW
$\mathcal{S}$	energy capacity (size)	MWh
cycles	measure of duration	
c^0	fixed cost	$/h
c^1	variable cost	$/MWh
e	efficiency/energy loss in charging	

- Costs approximate the unit construction and depreciation due to charge and discharge cycles

$$\sum_{t=0}^{T} p_t(q_t^+ - q_t^-) + c^1(q_t^+ + q_t^-) + c^0$$

$c^1(q_t^+ + q_t^-)$ approximates cost of cycles

Stochastic price paths (day ahead market)

$$\min_{x,s,q^+,q^-} c^0(x) + \mathbb{E}_\omega \left[\sum_{t=0}^{T} p_{\omega t}(q^+_{\omega t} - q^-_{\omega t}) + c^1(q^+_{\omega t} + q^-_{\omega t}) \right]$$

$$\text{s.t. } \partial h_{\omega t} = e q^+_{\omega t} - q^-_{\omega t}$$

$$0 \leq h_{\omega t} \leq \mathcal{S} x$$

$$0 \leq q^+_{\omega t}, q^-_{\omega t} \leq \mathcal{Q} x$$

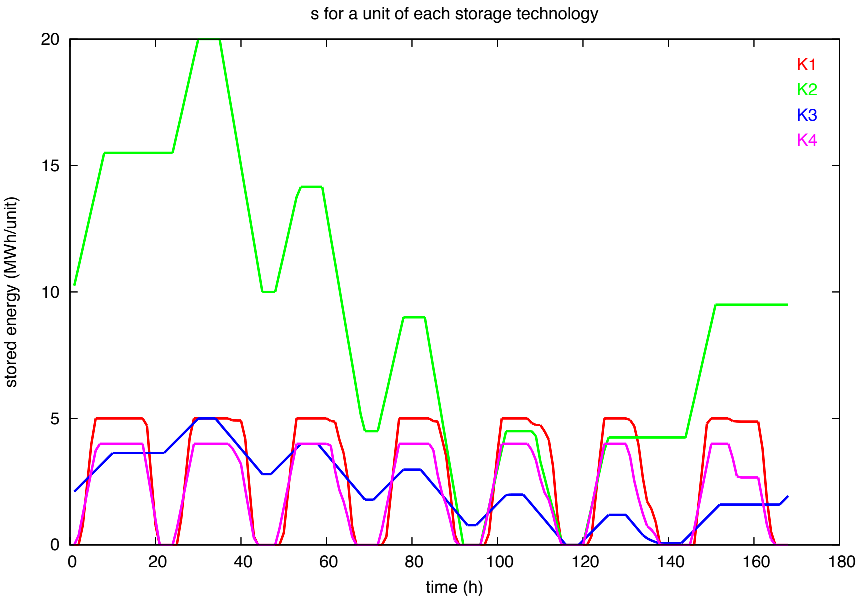
$$h_{\omega 0}, h_{\omega T} \text{ fixed}$$

- First stage decision x: amount of storage to deploy.
- Second stage decision: charging strategy in face of uncertainty

Four technology example

	k_1	k_2	k_3	k_4
$\mathcal{S}$	5	20	5	4
$\mathcal{Q}$	2	1	0.2	1
e	0.8	0.75	0.85	0.84
c^0	0.9	0.7	0.4	0.75
c^1	0.55	0.6	0.45	1.1

- k_1 and k_4 have only a daily cycle of operation
- k_2 and k_3 display a significant weekly cycle in addition to their daily cycle
- Could enforce $q^+_{\omega kt} = q^+_{kt}$, $q^-_{\omega kt} = q^-_{kt}$, deterministic operating plan
- Note ratio of $c_1/\mathcal{Q}$ is relevant


s for a unit of each storage technology
stored energy (MWh/unit)
time (h)
0
5
10
15
20
0
20
40
60
80
100
120
140
160
180
K1
K2
K3
K4

Distribution of (multiple) storage types

Determine storage facilities x_k to build, given distribution of price paths: no entry barriers into market, etc. MOPEC: for all k solve a two stage stochastic program

$$\forall k: \min_{x_k, h_k, q_k^+, q_k^-} c_k^0(x_k) + \mathbb{E}_\omega \left[\sum_{t=0}^{T} p_{\omega t}(q_{\omega kt}^+ - q_{\omega kt}^-) + c_k^1(q_{\omega kt}^+ + q_{\omega kt}^-) \right]$$
$$\text{s.t. } \partial h_{\omega kt} = e q_{\omega kt}^+ - q_{\omega kt}^-$$
$$0 \le h_{\omega kt} \le \mathcal{S} x_k$$
$$0 \le q_{\omega kt}^+, q_{\omega kt}^- \le \mathcal{Q} x_k$$
$$h_{\omega k0}, h_{\omega kT} \text{ fixed}$$

and

$$p_{\omega t} = f\left(\theta, \mathcal{D}_{\omega t} + \sum_k (q_{\omega kt}^+ - q_{\omega kt}^-)\right)$$

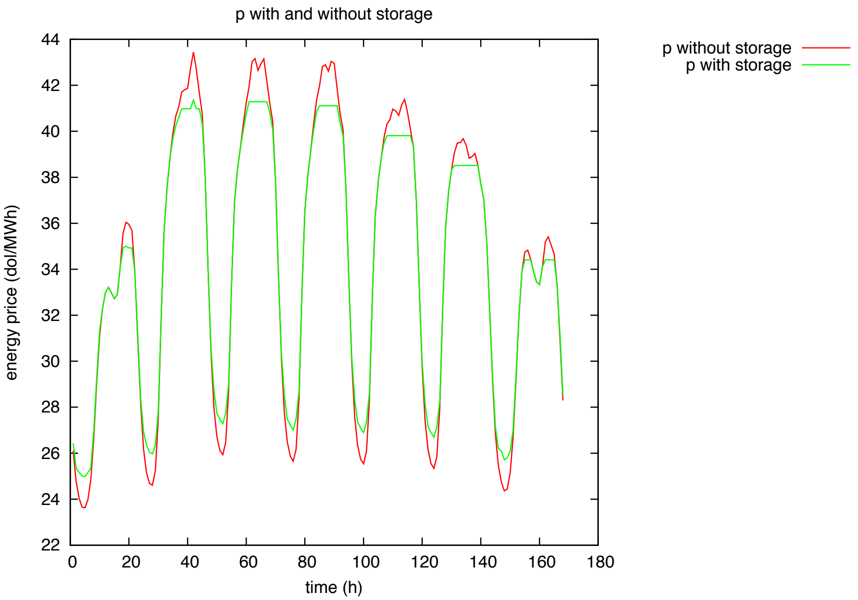
Parametric function (θ) determined by regression. Storage operators react to shift in demand.

Comparison to expected value solution

Interestingly enough, the resulting equilibria in the two models are quite different. Investment variables in the equilibria:

k	x_k, EV soln	x_k, Stochastic soln
k_1	102.063	143.631
k_2	51.606	621.195
k_3	479.859	0.118
k_4	246.806	85.582

Stochastic programming is not kind to k_3 and k_4. A possible explanation for this is that both these technologies have quite high variable costs relative to their charging capacities, meaning that their recourse actions are expensive.


p with and without storage
energy price (dol/MWh)
time (h)
p without storage
p with storage
44
42
40
38
36
34
32
30
28
26
24
22
0
20
40
60
80
100
120
140
160
180

Central Zonal Model (nodes i, time t)

Operationally, dispatch model involves nodes i and transmission:

$$
\begin{aligned}
\min_{z,\theta,g,q^+,q^-,s} \quad & \sum_{i,t} C_i(g_{i,t}) \\
\text{s.t.} \quad & z = B\mathcal{A}\theta, z \in [-\bar{z}, \bar{z}] \\
& g + q^- - q^+ - \mathcal{A}^T z \geq \mathcal{D} \\
& \underline{g}_i \leq g_{i,t} \leq \bar{g}_i, \\
& \partial h_{i,t} = e q^+_{i,t} - q^-_{i,t}, \\
& 0 \leq q^+_{i,t}, q^-_{i,t} \leq \mathcal{Q}_i, \\
& 0 \leq h_{i,t} \leq \mathcal{S}_i
\end{aligned}
$$

$\mathcal{A}$ is the node-arc incidence matrix

Distributed Model

At a bus i, given the hourly clearing prices $p_{i,t}$, the generator solves:

$$\begin{aligned} \max_{g_i} \quad & \sum_t p_{i,t} g_{i,t} - C_i(g_{i,t}) & \\ \text{s.t.} \quad & \underline{g}_i \leq g_{i,t} \leq \bar{g}_i, & \forall i, t \end{aligned}$$

and the storage owner solves:

$$\begin{aligned} \max_{q_i^+, q_i^-, h_i} \quad & \sum_t p_{i,t}(q_{i,t}^- - q_{i,t}^+) \\ \text{s.t.} \quad & \partial h_{i,t} = e q_{i,t}^+ - q_{i,t}^-, \\ & 0 \leq q_{i,t}^+, q_{i,t}^- \leq \mathcal{Q}_i, \\ & 0 \leq h_{i,t} \leq \mathcal{S}_i \end{aligned}$$

Locational pricing of storage

Given the distributed decisions g, q^+, q^-, s, the ISO maintains the transmission constraints and supply-demand balance, and produces the clearing prices, by enforcing the complementarity constraints:

$$
\begin{aligned}
& z - B\mathcal{A}\theta = 0 && \perp \lambda, \\
& \mathcal{D} \le g + q^- - q^+ - \mathcal{A}^T z && \perp p \ge 0, \\
& -\lambda + \mathcal{A}p && \perp z \in [-\bar{z}, \bar{z}], \\
& -\mathcal{A}^T B^T \lambda = 0 && \perp \theta
\end{aligned}
$$

Together these optimization problems form a MOPEC and can be solved directly within GAMS.
These models are equivalent to the central model, but exhibit the behaviors of each player in the market.

Approximating transmission

- Generator maximization (given p)
- Storage operation optimization (given p)
- Transmission and market clearing complementarity (given g, q and s)

- Last piece of model (transmission and market clearing) can be replaced by stochastic price process on p (given g, q and s)

$$p_{it} = f\left(\theta, g_{it} + q_{it}^{-} - q_{it}^{+} - \mathcal{D}_{it}\right)$$

- The stochastic process educated by data will model failures and outages but not detailed transmission: complex tradeoff
- Need to add in investment problem as additional optimization

Conclusions

- Stochastic MOPEC models capture behavioral effects (extended mathematical programming)
- Separate stochastic approximation from optimization
- Tools exist to facilitate easy modeling and solution within GAMS
- Collections of models needed for specific decisions
- Policy implications addressable using Stochastic MOPEC
- Can show certain technologies dominate others, some are not viable at all