The price of storage

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EMP: shameless advertisement

Allows (GAMS) models to be manipulated to form other problems of interest via a simple EMP info file:

- $\text{VI}(f, C)$:
  
  \[
  0 \in f(x) + N_C(x)
  \]

  $\text{vi f x cons}$

  generates a variational inequality where $C$ defined by 'cons'

- Either generates the equivalent complementarity (KKT) problem, or provides problem structure for algorithmic exploitation

- QVI can be specified in the same manner
MOPEC

\[
\min_{x_i} \theta_i(x_i, x_{-i}, y) \text{ s.t. } g_i(x_i, x_{-i}, y) \leq 0, \forall i
\]

and

\[y \text{ solves } \text{VI}(h(x, \cdot), C)\]

equilibrium
\[
\min \ \text{theta}(1) \ x(1) \ g(1)
\]

\[
\ldots
\]

\[
\min \ \text{theta}(m) \ x(m) \ g(m)
\]

vi h y cons

is solved in a Nash manner

- Allows multipliers from one problem to be used in another problems
  
  dualvar \( p \) \( g(1) \)

  where for example the definition of \( h \) involves the additional variable \( p \)

- Also has extensions for stochastic programming
PJM buy/sell model (2009)

- Storage transfers energy over time (horizon = \( T \)).
- PJM: given price path \( p_t \), determine charge \( q_t^+ \) and discharge \( q_t^- \):

\[
\max_{h_t, q_t^+, q_t^-} \sum_{t=0}^{T} p_t (q_t^- - q_t^+) \\
\text{s.t. } \partial h_t = e q_t^+ - q_t^- \\
0 \leq h_t \leq S \\
0 \leq q_t^+ \leq Q \\
0 \leq q_t^- \leq Q \\
h_0, h_T \text{ fixed}
\]

- Uses: price shaving, load shifting, transmission line deferral
- what about different storage technologies?
Characterization of storage

\( Q \)  power (discharge) capacity  MW
\( S \)  energy capacity (size)  MWh

cycles measure of duration
\( c^0 \)  fixed cost  $/h
\( c^1 \)  variable cost  $/MWh

\( e \)  efficiency/energy loss in charging

- Costs approximate the unit construction and depreciation due to charge and discharge cycles

\[
\sum_{t=0}^{T} p_t(q_t^+ - q_t^-) + c^1(q_t^+ + q_t^-) + c^0
\]

\( c^1(q_t^+ + q_t^-) \) approximates cost of cycles
Stochastic price paths (day ahead market)

\[
\min_{x,s,q^+,q^-} c^0(x) + \mathbb{E}_\omega \left[ \sum_{t=0}^{T} p_{\omega t}(q^+_{\omega t} - q^-_{\omega t}) + c^1(q^+_{\omega t} + q^-_{\omega t}) \right]
\]

s.t. \( \partial h_{\omega t} = e q^+_{\omega t} - q^-_{\omega t} \)
\( 0 \leq h_{\omega t} \leq S x \)
\( 0 \leq q^+_{\omega t}, q^-_{\omega t} \leq Q x \)
\( h_{\omega 0}, h_{\omega T} \) fixed

- First stage decision \( x \): amount of storage to deploy.
- Second stage decision: charging strategy in face of uncertainty
Four technology example

<table>
<thead>
<tr>
<th></th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$k_3$</th>
<th>$k_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>5</td>
<td>20</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>$Q$</td>
<td>2</td>
<td>1</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>$e$</td>
<td>0.8</td>
<td>0.75</td>
<td>0.85</td>
<td>0.84</td>
</tr>
<tr>
<td>$c^0$</td>
<td>0.9</td>
<td>0.7</td>
<td>0.4</td>
<td>0.75</td>
</tr>
<tr>
<td>$c^1$</td>
<td>0.55</td>
<td>0.6</td>
<td>0.45</td>
<td>1.1</td>
</tr>
</tbody>
</table>

- $k_1$ and $k_4$ have only a daily cycle of operation
- $k_2$ and $k_3$ display a significant weekly cycle in addition to their daily cycle
- Could enforce $q_{\omega kt}^+ = q_{kt}^+$, $q_{\omega kt}^- = q_{kt}^-$, deterministic operating plan
- Note ratio of $c_1/Q$ is relevant
s for a unit of each storage technology

Ferris and Holzer (Wisconsin)

Price of storage

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Distribution of (multiple) storage types

Determine storage facilities $x_k$ to build, given distribution of price paths: no entry barriers into market, etc. MOPEC: for all $k$ solve a two stage stochastic program

$$\forall k : \min_{x_k, h_k, q_k^+, q_k^-} c^0_k(x_k) + \mathbb{E}_\omega \left[ \sum_{t=0}^{T} p_{\omega t} (q^+_{\omega kt} - q^-_{\omega kt}) + c^1_k(q^+_{\omega kt} + q^-_{\omega kt}) \right]$$

s.t.  \[ \partial h_{\omega kt} = eq^+_{\omega kt} - q^-_{\omega kt} \]

\[ 0 \leq h_{\omega kt} \leq Sx_k \]

\[ 0 \leq q^+_{\omega kt}, q^-_{\omega kt} \leq Qx_k \]

\[ h_{\omega k0}, h_{\omega kT} \text{ fixed} \]

and

$$p_{\omega t} = f \left( \theta, D_{\omega t} + \sum_k (q^+_{\omega kt} - q^-_{\omega kt}) \right)$$

Parametric function ($\theta$) determined by regression. Storage operators react to shift in demand.
Comparison to expected value solution

Interestingly enough, the resulting equilibria in the two models are quite different. Investment variables in the equilibria:

<table>
<thead>
<tr>
<th>$k$</th>
<th>$x_k$, EV soln</th>
<th>$x_k$, Stochastic soln</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>102.063</td>
<td>143.631</td>
</tr>
<tr>
<td>$k_2$</td>
<td>51.606</td>
<td>621.195</td>
</tr>
<tr>
<td>$k_3$</td>
<td>479.859</td>
<td>0.118</td>
</tr>
<tr>
<td>$k_4$</td>
<td>246.806</td>
<td>85.582</td>
</tr>
</tbody>
</table>

Stochastic programming is not kind to $k_3$ and $k_4$. A possible explanation for this is that both these technologies have quite high variable costs relative to their charging capacities, meaning that their recourse actions are expensive.
Central Zonal Model (nodes $i$, time $t$)

Operationally, **dispatch** model involves nodes $i$ and transmission:

\[
\begin{align*}
\min_{z, \theta, g, q^+, q^-, s} & \sum_{i,t} C_i(g_i, t) \\
\text{s.t.} & z = B A \theta, z \in [-\bar{z}, \bar{z}] \\
& g + q^- - q^+ - A^T z \geq D \\
& g_i \leq g_{i,t} \leq \bar{g}_i, \\
& \partial h_{i,t} = e q^+_{i,t} - q^-_{i,t}, \\
& 0 \leq q^+_{i,t}, q^-_{i,t} \leq Q_i, \\
& 0 \leq h_{i,t} \leq S_i
\end{align*}
\]

$A$ is the node-arc incidence matrix
Distributed Model

At a bus $i$, given the hourly clearing prices $p_{i,t}$, the generator solves:

$$\max_{g_i} \sum_t p_{i,t} g_{i,t} - C_i(g_{i,t})$$

s.t.

$$g_i \leq g_{i,t} \leq \bar{g}_i, \quad \forall i, t$$

and the storage owner solves:

$$\max_{q_i^+,q_i^-,h_i} \sum_t p_{i,t} (q_{i,t}^- - q_{i,t}^+)$$

s.t.

$$\partial h_{i,t} = eq_{i,t}^+ - q_{i,t}^-, \quad 0 \leq q_{i,t}^+, q_{i,t}^- \leq Q_i,$$

$$0 \leq h_{i,t} \leq S_i$$
Locational pricing of storage

Given the distributed decisions $g, q^+, q^-, s$, the ISO maintains the transmission constraints and supply-demand balance, and produces the clearing prices, by enforcing the complementarity constraints:

\[
\begin{align*}
z - BA\theta &= 0 & \perp \lambda, \\
D &\leq g + q^- - q^+ - ATz & \perp p \geq 0, \\
- \lambda + Ap &\quad \perp z \in [-\bar{z}, \bar{z}], \\
- ATB^T\lambda &= 0 & \perp \theta
\end{align*}
\]

Together these optimization problems form a MOPEC and can be solved directly within GAMS. These models are equivalent to the central model, but exhibit the behaviors of each player in the market.
Approximating transmission

- Generator maximization (given $p$)
- Storage operation optimization (given $p$)
- Transmission and market clearing complementarity (given $g$, $q$ and $s$)

Last piece of model (transmission and market clearing) can be replaced by stochastic price process on $p$ (given $g$, $q$ and $s$)

$$p_{it} = f(\theta, g_{it} + q_{it}^− - q_{it}^+ - D_{it})$$

- The stochastic process educated by data will model failures and outages but not detailed transmission: complex tradeoff
- Need to add in investment problem as additional optimization
Conclusions

- Stochastic MOPEC models capture behavioral effects (extended mathematical programming)
- Separate stochastic approximation from optimization
- Tools exist to facilitate easy modeling and solution within GAMS
- Collections of models needed for specific decisions
- Policy implications addressable using Stochastic MOPEC
- Can show certain technologies dominate others, some are not viable at all