Stochastic Multiple Optimization Problems with Equilibrium Constraints

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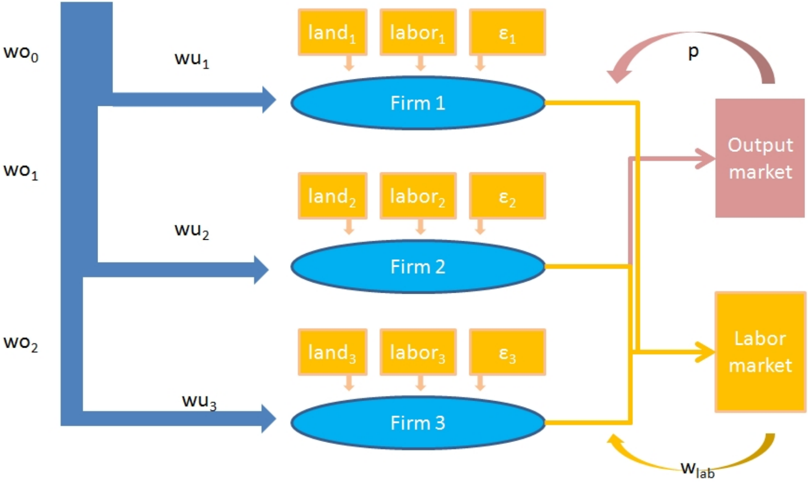
Water rights pricing (Britz/F./Kuhn)



The model AO

$$\max_{q_i, x_i, wo_i \geq 0} \sum_i \left(q_i \cdot p - \sum_{f \in \{int, lab\}} x_{i,f} \cdot w_f \right)$$

$$\text{s.t.} \quad q_i \leq \prod_f \left(x_{i,f} + e_{i,f} \right)^{\epsilon_{i,f}}$$

$$x_{i,land} \leq e_{i,land}$$

$$wo_{i-1} = x_{i,wat} + wo_i$$

The model AO

$$\max_{q_i, x_i, wo_i \geq 0} \sum_i \left(q_i \cdot p - \sum_{f \in \{int, lab\}} x_{i,f} \cdot w_f \right)$$

$$\begin{aligned} \text{s.t.} \quad & q_i \leq \prod_f (x_{i,f} + e_{i,f})^{\epsilon_{i,f}} \\ & x_{i,land} \leq e_{i,land} \\ & wo_{i-1} = x_{i,wat} + wo_i \end{aligned}$$

$$\begin{aligned} 0 \leq & \quad \textstyle\sum_i q_i - d(p) & \perp \quad & p \geq 0 \\ 0 \leq & \sum_i e_{i,lab} - \sum_i x_{i,lab} & \perp \quad & w_{lab} \geq 0 \end{aligned}$$

(M)OPEC

$$\max_{x} \theta(x, p) \text{ s.t. } g(x, p) \leq 0,$$

and

$$0 \leq h(x, p) \perp p \geq 0$$

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equilibrium
max theta x g
vi h p
```

is solved concurrently (in a Nash manner)

(M)OPEC

$$\max_{x} \theta(x, p) \text{ s.t. } g(x, p) \leq 0,$$

and

$$h(x, p) = 0$$

```
equilibrium
max theta x g
vi h p
```

is solved concurrently (in a Nash manner)

MOPEC

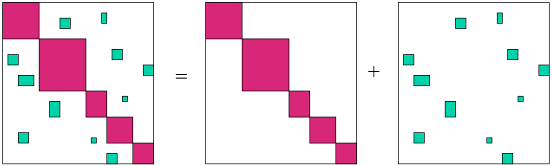
$$\min_{x_i} \theta_i(x_i, x_{-i}, p) \text{ s.t. } g_i(x_i, x_{-i}, p) \leq 0, \forall i$$

and

$$p \text{ solves } \mathrm{VI}(h(x, \cdot), C)$$

```
equilibrium
min theta(1) x(1) g(1)
...
min theta(m) x(m) g(m)
vi h p cons
```

- Reformulate optimization problem as first order conditions (complementarity)
- Use nonsmooth Newton methods to solve complementarity problem
- Precondition using "individual optimization" with fixed externalities



The model IO

$$
\begin{aligned}
\max_{q_i, x_i, wo_i \geq 0} \quad & \left(q_i \cdot p - \sum_f x_{i,f} \cdot w_f \right) \\
\text{s.t.} \quad & q_i \leq \prod_f \left(x_{i,f} + e_{i,f} \right)^{\epsilon_{i,f}} \\
& x_{i,land} \leq e_{i,land} \\
& wo_{i-1} = x_{i,wat} + wo_i
\end{aligned}
$$

$$
\begin{aligned}
0 \leq & \quad \textstyle\sum_i q_i - d(p) \quad \perp \quad p \geq 0 \\
0 \leq & \quad \sum_i e_{i,lab} - \sum_i x_{i,lab} \quad \perp \quad w_{lab} \geq 0
\end{aligned}
$$

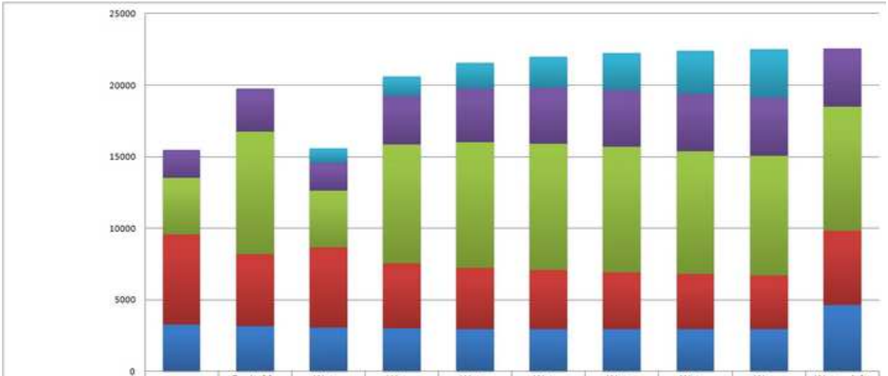
The model IO

$$\max_{q_i, x_i, wo_i, wr_i^b, wr_i^s \geq 0} \left(q_i \cdot p - \sum_f x_{i,f} \cdot w_f - wr_i^b \cdot (w_{wr} + \tau) + wr_i^s \cdot w_{wr} \right)$$

$$\begin{aligned}
\text{s.t.} \quad & q_i \leq \prod_f (x_{i,f} + e_{i,f})^{\epsilon_{i,f}} \\
& x_{i,land} \leq e_{i,land} \\
& wo_{i-1} = x_{i,wat} + wo_i \\
& wr_i + wr_i^b \geq x_{i,wat} + wr_i^s
\end{aligned}$$

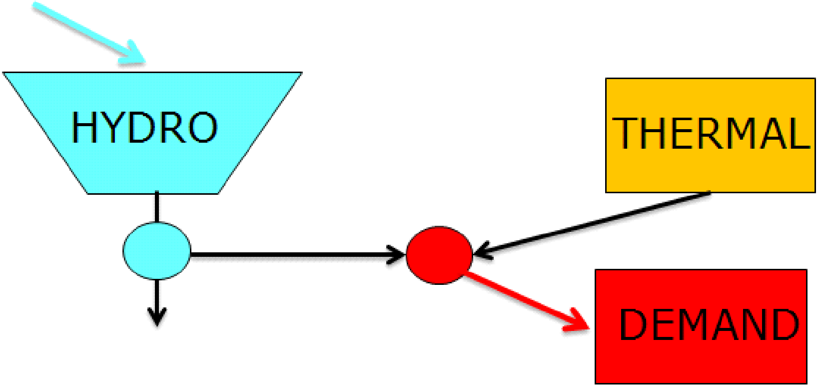
$$\begin{aligned}
0 \leq & \sum_i q_i - d(p) & \perp \quad & p \geq 0 \\
0 \leq & \sum_i e_{i,lab} - \sum_i x_{i,lab} & \perp \quad & w_{lab} \geq 0 \\
0 \leq & \sum_i wr_i^s - \sum_i wr_i^b & \perp \quad & w_{wr} \geq 0
\end{aligned}$$

Different Management Strategies



	No institution	Equitable rights	Water charges 10	Water charges 14	Water charges 18	Water charges 22	Water charges 26	Water charges 30	Water charges 34	Water right trade
Water charge revenue	0	0	1000	1400	1800	2200	2600	3000	3400	0
Consumer surplus	1943	3040	1969	3398	3705	3865	3959	4018	4054	4099
Profit Firm 3	3983	8513	3979	8279	8776	8869	8784	8601	8356	8642
Profit Firm 2	6306	5033	5580	4557	4281	4097	3960	3853	3766	5217
Profit Firm 1	3255	3176	3068	2995	2972	2957	2947	2940	2936	4617

Hydro-Thermal System (Philpott/F./Wets)



Simple electricity system optimization problem

$$\text{SSP:} \quad \min \quad \sum_{j \in \mathcal{T}} C_j(v(j)) - \sum_{i \in \mathcal{H}} V_i(x(i))$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{i \in \mathcal{H}} U_i(u(i)) + \sum_{j \in \mathcal{T}} v(j) \geq d, \\ & x(i) = x_0(i) - u(i), \quad i \in \mathcal{H} \\ & u(i), v(j), x(i) \geq 0. \end{aligned}$$

- $u(i)$ water release of hydro reservoir $i \in \mathcal{H}$
- $v(j)$ thermal generation of plant $j \in \mathcal{T}$
- production function U_i (strictly concave) converts water release to energy
- water level reservoir $i \in \mathcal{H}$ is denoted $x(i)$
- $C_j(v(j))$ denote the cost of generation by thermal plant
- $V_i(x(i))$ to be the future value of terminating the period with storage x (assumed separable)

SSP equivalent to CE

Thermal plants solve

$$\text{TP}(j): \quad \max \quad p^T v(j) - C_j(v(j))$$

$$\text{s.t.} \qquad v(j) \geq 0.$$

The hydro plants $i \in \mathcal{H}$ solve

$$\text{HP}(i): \quad \max \quad p^T U_i\left(u(i)\right) + V_i(x(i))$$

$$\begin{aligned} \text{s.t.} \quad & x(i) = x_0(i) - u(i) \\ & u(i), x(i) \geq 0. \end{aligned}$$

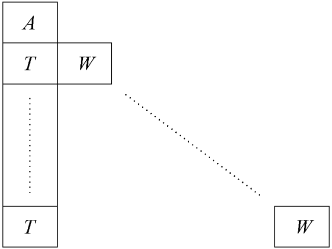
Perfectly competitive (Walrasian) equilibrium is a MOPEC

$$\begin{aligned} \text{CE:} \quad & u(i), x(i) \in \arg\max \text{HP}(i), & i \in \mathcal{H}, \\ & v(j) \in \arg\max \text{TP}(j), & j \in \mathcal{T}, \\ & 0 \leq \left(\textstyle\sum_{i \in \mathcal{H}} U_i\left(u(i)\right) + \sum_{j \in \mathcal{T}} v(j)\right) - d \perp p \geq 0. & \end{aligned}$$

Agents have stochastic recourse?

- Two stage stochastic programming, x is here-and-now decision, recourse decisions y depend on realization of a random variable
- $\mathbb{R}$ is a risk measure (e.g. expectation, CVaR)

$$
\begin{aligned}
\text{SP:} \quad \min \quad & c^\top x + \mathbb{R}[q^\top y] \\
\text{s.t.} \quad & Ax = b, \quad x \geq 0, \\
\forall \omega \in \Omega : \quad & T(\omega)x + W(\omega)y(\omega) \leq d(\omega), \\
& y(\omega) \geq 0.
\end{aligned}
$$



EMP/SP extensions to facilitate these models

Two stage problems

$$\text{TP}(j): \quad \max \quad p_1 v_1(j) - C_j(v_1(j)) + R_\omega[p_2(\omega) v_2(j,\omega) - C_j(v_2(j,\omega))]$$

$$\text{s.t.} \quad v_1(j) \geq 0, \quad v_2(j,\omega) \geq 0, \qquad \text{for all } \omega \in \Omega.$$

$$\text{HP}(i): \quad \max \quad p_1 U_i(u_1(i)) + R_\omega[p_2(\omega) U_i(u_2(i,\omega)) + V_i(x_2(i,\omega))]$$

$$\begin{aligned} \text{s.t.} \quad & x_1(i) = x_0(i) - u_1(i) + h_1(i), & \\ & x_2(i,\omega) = x_1(i) - u_2(i,\omega) + h_2(i,\omega), & \text{for all } \omega \in \Omega, \\ & u_1(i), x_1(i) \geq 0, \quad u_2(i,\omega), x_2(i,\omega) \geq 0, & \text{for all } \omega \in \Omega. \end{aligned}$$

Results

- Suppose every agent is risk neutral and has knowledge of all deterministic data, as well as sharing the same probability distribution for inflows. SP solution is same as CE solution
- Using coherent risk measure (weighted sum of expected value and conditional variance at risk), 10 scenarios for rain
  1. High initial storage: risk-averse central plan (RSP) and the risk-averse competitive equilibrium (RCE) have same solution (but different to risk neutral case)
  2. Low initial storage: RSP and RCE are very different. Since the hydro generator and the system do not agree on a worst-case outcome, the probability distributions that correspond to an equivalent risk neutral decision will not be common.
  3. Extension: Construct MOPEC models for trading risk

Stochastic competing agent models (F./Wets)

- Competing agents (consumers, or generators in energy market)
- Each agent minimizes objective independently (cost)
- Market prices are function of all agents activities
- Additional twist: model must “hedge” against uncertainty
- Facilitated by allowing contracts bought now, for goods delivered later
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to mitigate risk, or move to system optimal solutions from equilibrium (or market) solutions

Example as MOPEC: agents solve a Stochastic Program

Each agent minimizes:

$$u_a = (\kappa - f(q_{a,0}))^2 + \sum_s \pi_s (\kappa - f(q_{a,s}))^2$$

Budget time 0: $p_0^T q_{a,0} + v^T y_a \leq p_0^T e_{a,0}$

Budget time 1: $p_s^T q_{a,s} \leq p_s^T (D_s y_a + e_{a,s})$

Additional constraints (complementarity) outside of control of agents:

$$\text{(contract) } 0 \leq -\sum_a y_a \perp v \geq 0$$

$$\text{(walras) } 0 \leq \sum_a (D_s y_a + e_{a,s} - q_{a,s}) \perp p_s \geq 0$$

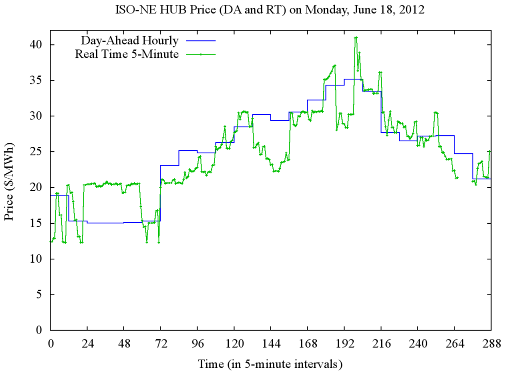
Observations

- Examples from literature solved using homotopy continuation seem incorrect - need transaction costs to guarantee solution
- Solution possible via disaggregation only seems possible in special cases
  - When problem is block diagonally dominant
  - When overall (complementarity) problem is monotone
  - (Pang): when problem is a potential game
- Progressive hedging possible to decompose in these settings by agent and scenario

PJM buy/sell model (2009)

- Storage transfers energy over time (horizon $= T$).
- PJM: given price path p_t, determine charge q_t^+ and discharge q_t^-:

$$\max_{h_t, q_t^+, q_t^-} \sum_{t=0}^{T} p_t (q_t^- - q_t^+)$$
$$\text{s.t. } \partial h_t = e q_t^+ - q_t^-$$
$$0 \le h_t \le \mathcal{S}$$
$$0 \le q_t^+ \le \mathcal{Q}$$
$$0 \le q_t^- \le \mathcal{Q}$$
$$h_0, h_T \text{ fixed}$$



- Uses: price shaving, load shifting, transmission line deferral
- what about different storage technologies?

Stochastic price paths (day ahead market)

$$\min_{x,s,q^+,q^-} c^0(x) + \mathbb{E}_\omega \left[\sum_{t=0}^{T} p_{\omega t}(q^+_{\omega t} - q^-_{\omega t}) + c^1(q^+_{\omega t} + q^-_{\omega t}) \right]$$
$$\text{s.t. } \partial h_{\omega t} = e q^+_{\omega t} - q^-_{\omega t}$$
$$0 \leq h_{\omega t} \leq \mathcal{S} x$$
$$0 \leq q^+_{\omega t}, q^-_{\omega t} \leq \mathcal{Q} x$$
$$h_{\omega 0}, h_{\omega T} \text{ fixed}$$

- First stage decision x: amount of storage to deploy.
- Second stage decision: charging strategy in face of uncertainty

Distribution of (multiple) storage types

Determine storage facilities x_k to build, given distribution of price paths: no entry barriers into market, etc. MOPEC: for all k solve a two stage stochastic program

$$\forall k : \min_{x_k, h_k, q_k^+, q_k^-} c_k^0(x_k) + \mathbb{E}_\omega \left[\sum_{t=0}^{T} p_{\omega t}(q_{\omega kt}^+ - q_{\omega kt}^-) + c_k^1(q_{\omega kt}^+ + q_{\omega kt}^-) \right]$$

$$\begin{aligned} \text{s.t. } & \partial h_{\omega kt} = e q_{\omega kt}^+ - q_{\omega kt}^- \\ & 0 \le h_{\omega kt} \le \mathcal{S} x_k \\ & 0 \le q_{\omega kt}^+, q_{\omega kt}^- \le \mathcal{Q} x_k \\ & h_{\omega k0}, h_{\omega kT} \text{ fixed} \end{aligned}$$

and

$$p_{\omega t} = f\left(\theta, \mathcal{D}_{\omega t} + \sum_k (q_{\omega kt}^+ - q_{\omega kt}^-)\right)$$

Parametric function (θ) determined by regression. Storage operators react to shift in demand.

What is EMP?

Annotates existing equations/variables/models for modeler to provide/define additional structure

- equilibrium
- vi (agents can solve min/max/vi)
- bilevel (reformulate as MPEC, or as SOCP)
- disjunction (or other constraint logic primitives)
- randvar
- dualvar (use multipliers from one agent as variables for another)
- extended nonlinear programs (library of plq functions)

Currently available within GAMS

Conclusions

- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- EMP model type is clear and extensible, additional structure available to solver
- Stochastic MOPEC models capture behavioral effects (as an EMP)
- Policy implications addressable using Stochastic MOPEC
- Extended Mathematical Programming available within the GAMS modeling system