MOPEC Models for Energy Problems

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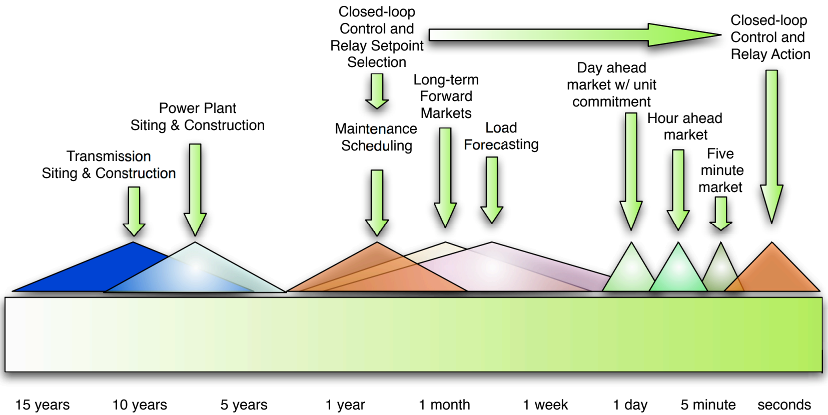
The premise

- The next generation electric grid will be more dynamic, flexible, constrained, and more complicated.
- Decision processes (in this environment) are predominantly hierarchical.
- Models to support such decision processes must also be layered or hierachical.
- Optimization and computation facilitate adaptivity, control, treatment of uncertainties and understanding of interaction effects.
- Coupling of smaller models with well defined interfaces allows validation, understanding, and enhanced solution techniques.

Representative decision-making timescales in electric power systems



A monster model is difficult to validate, inflexible, prone to errors.

Transmission Line Expansion Model (1)

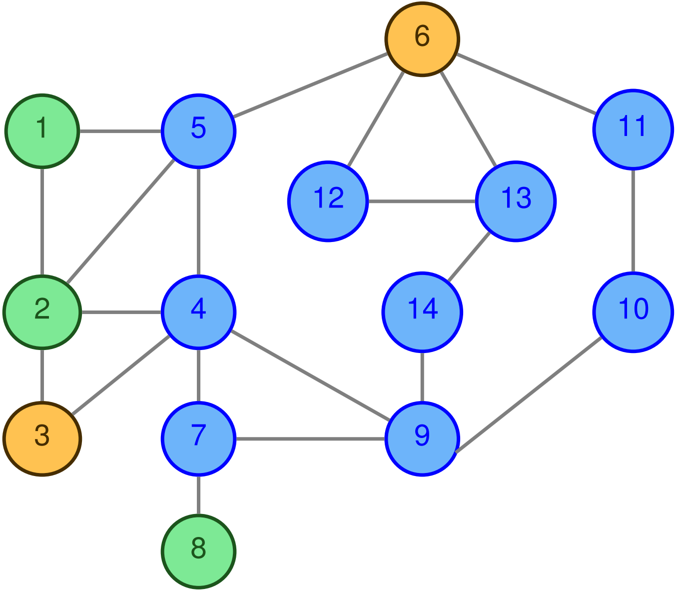
$$\min_{x \in X} \quad \sum_{\omega} \pi_\omega \sum_{i \in N} d_i^\omega p_i^\omega(x)$$

$$s.t. \quad Ax \le b \qquad \text{(RTO budget (and other) constraints)}$$

N:	The set of all nodes
X:	The set of all line expansions being considered
x:	Amount of investment in line $x \in X$
ω:	Demand scenarios
π_ω:	Probability of scenario ω occuring
d_i^ω:	Demand of load node i in in scenario ω
$p_i^\omega(x)$:	Price (LMP) at load node i in scenario ω as a function of x

Solution approach

- Use deriviative free method for the upper level problem (1)
- Requires $p_i^\omega(x)$
- Construct these as multipliers on demand equation (per scenario) in an Economic Dispatch (market clearing) model
- But transmission line capacity expansion typically leads to generator expansion, which interacts with market clearing
- How to combine these models?



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Generator Expansion (2)

$$\forall f \in F: \quad \min_{y_f} \sum_{\omega} \pi_\omega \sum_{j \in G_f} C_j(y_j, q_j^\omega) - r(h_f - \sum_{j \in G_f} y_j)$$

$$s.t. \quad \sum_{j \in G_f} y_j \le h_f \qquad \text{(budget cons)}$$

$$y_f \ge 0$$

F: The set of firms

G_f: The set of all generators belonging to firm f

ω: Demand scenarios

y_j: Amount of investment in generator j

q_j^ω: Real power generated at bus j in scenario ω

C_j: Cost function of generator j as a function of y_j and q_j

r: Interest Rate

Market Clearing Model (3)

$$\forall \omega: \quad \min_{z,\theta,q^\omega} \sum_f \sum_{j \in G_f} C_j(y_j, q_j^\omega)$$

$$\begin{aligned}
\text{s.t.} \quad & q_j^\omega - d_j^\omega = \sum_{i \in I(j)} z_{ij} && \forall j \in N (\perp p_j^\omega) && \text{(flow balance)} \\
& z_{ij} = \Omega_{ij}(\theta_i - \theta_j) && \forall (i,j) \in A && \text{(line data)} \\
& -b_{ij}(x) \le z_{ij} \le b_{ij}(x) && \forall (i,j) \in A && \text{(line capacity)} \\
& \underline{u}_j(y_j) \le q_j^\omega \le \overline{u}_j(y_j) && && \text{(gen capacity)}
\end{aligned}$$

z_{ij}:	Real power flowing along the i-j arc
q_j^ω:	Real power generated at bus j in scenario ω
θ_i:	Voltage phase angle at bus i
Ω_{ij}:	Susceptance of line i-j
$b_{ij}(x)$:	Line capacity as a function of x
$\underline{u}_j(y)$, $\overline{u}_j(y)$:	Generator j limits as a function of y

How to combine: Nash Games

- Non-cooperative game: collection of players $a \in \mathcal{A}$ whose individual objectives depend not only on the selection of their own strategy $x_a \in C_a = \mathrm{dom} f_a(\cdot, x_{-a})$ but also on the strategies selected by the other players $x_{-a} = \{x_a : o \in \mathcal{A} \setminus \{a\}\}$.
- Nash Equilibrium Point:

$$\bar{x}_{\mathcal{A}} = (\bar{x}_a, a \in \mathcal{A}) : \forall a \in \mathcal{A} : \bar{x}_a \in \mathrm{argmin}_{x_a \in C_a} f_a(x_a, \bar{x}_{-a}).$$

1. for all $x \in \mathcal{A}$, $f_a(\cdot, x_{-a})$ is convex
2. $C = \prod_{a \in \mathcal{A}} C_a$ and for all $a \in \mathcal{A}$, C_a is closed convex.

VI reformulation

Define

$$G : \mathbf{R}^N \mapsto \mathbf{R}^N \text{ by } G_a(x_{\mathcal{A}}) = \partial_a f_a(x_a, x_{-a}), a \in \mathcal{A}$$

where ∂_a denotes the subgradient with respect to x_a. Generally, the mapping G is set-valued.

Theorem

Suppose the objectives satisfy (1) and (2), then every solution of the variational inequality

$$x_{\mathcal{A}} \in C \text{ such that } -G(x_{\mathcal{A}}) \in N_C(x_{\mathcal{A}})$$

is a Nash equilibrium point for the game.
Moreover, if C is compact and G is continuous, then the variational inequality has at least one solution that is then also a Nash equilibrium point.

Example

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

empinfo: equilibrium

forall f: min expcost(f) y(f) budget(f)

forall ω: min scencost(ω) q(ω) ...

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	2.86	4.60	4.00	4.12	3.38
ω_2		4.70	4.09	4.24	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	167.83	565.31			266.86
f_2			292.11	207.89	

Flow of information

$$\min_{x \in X} \quad \sum_{\omega} \pi_\omega \sum_{i \in N} d_i^\omega p_i^\omega(x)$$

$$s.t. \min_{y_f \in Y} \sum_{\omega} \pi_\omega \sum_{j \in G_f} C_j(y_j, q_j^\omega) - r(h_f - \sum_{j \in G_f} y_j) \qquad \forall f \in F$$

$$\min_{z,\theta,q^\omega} \sum_{f} \sum_{j \in G_f} C_j(y_j, q_j^\omega) \qquad \forall \omega$$

$$s.t. \quad q_j^\omega - d_j^\omega = \sum_{i \in I(j)} z_{ij} \qquad \forall j \in N (\perp p_j^\omega(x))$$

$$z_{ij} = \Omega_{ij}(\theta_i - \theta_j) \qquad \forall (i,j) \in A$$

$$-b_{ij}(x) \le z_{ij} \le b_{ij}(x) \qquad \forall (i,j) \in A$$

$$\underline{u}_j(y_j) \le q_j^\omega \le \overline{u}_j(y_j)$$

Feasibility

$$\text{KKT of } \min_{y_f \in Y} \sum_{\omega} \pi_\omega \sum_{j \in G_f} C_j(y_j, q_j^\omega) - r(h_f - \sum_{j \in G_f} y_j) \quad \forall f \in F \quad (2)$$

$$\text{KKT of } \min_{(z,\theta,q^\omega) \in Z(x,y)} \sum_f \sum_{j \in G_f} C_j(y_j, q_j^\omega) \quad \forall \omega \quad (3)$$

- Models (2) and (3) form an MCP/VI (via EMP)
- Solve (3) as NLP using global solver (actual $C_j(y_j, q_j^\omega)$ are not convex), per scenario (SNLP) this provides starting point for MCP
- Solve (KKT(2) + KKT(3)) using EMP and PATH, then repeat
- Identifies MCP solution whose components solve the scenario NLP's (3) to global optimality

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	3.05	4.25	3.93	4.34	3.39
ω_2		4.41	4.07	4.55	

EMP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	2.86	4.60	4.00	4.12	3.38
ω_2		4.70	4.09	4.24	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	167.83	565.31			266.86
f_2			292.11	207.89	

Scenario	ω_1	ω_2
Probability	0.5	0.5
Demand Multiplier	8	5.5

SNLP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.35	4.66	5.04	3.91
ω_2		4.70	4.09	4.24	

EMP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1	0.00	5.34	4.62	5.01	3.99
ω_2		4.71	4.07	4.25	

Firm	y_1	y_2	y_3	y_6	y_8
f_1	0.00	622.02			377.98
f_2			283.22	216.79	

Scenario	ω_1	ω_2	ω_3	ω_4	ω_5
Probability	0.2	0.2	0.25	0.1	0.25
Demand Multiplier	4	6.5	9.5	8	8.9

EMP (1):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1		3.40	3.31	2.77	
ω_2		4.35	3.83	3.88	3.35
ω_3	3.53	5.30	4.66	5.04	3.99
ω_4	2.89	4.55	4.00	4.12	3.41
ω_5	3.27	5.00	4.41	4.68	3.73

Firm	y_1	y_2	y_3	y_6	y_8
f_1	194.39	469.99			335.61
f_2			292.89	207.11	

Scenario	ω_1	ω_2	ω_3	ω_4	ω_5
Probability	0.2	0.2	0.25	0.1	0.25
Demand Multiplier	4	6.5	9.5	8	8.9

EMP (2):

Scenario	q_1	q_2	q_3	q_6	q_8
ω_1		5.04	4.45	0.00	
ω_2		4.37	3.86	3.83	3.35
ω_3	3.46	5.33	4.71	5.00	4.01
ω_4	0.00	5.31	4.67	4.97	3.99
ω_5	3.22	5.04	4.45	4.64	3.75

Firm	y_1	y_2	y_3	y_6	y_8
f_1	145.45	507.45			347.05
f_2			320.54	179.46	

Observations

- But this is simply one function evaluation for the outer "transmission capacity expansion" problem
- Number of critical arcs typically very small
- But in this case, p_j^ω are very volatile
- Outer problem is small scale, objectives are open to debate, possibly ill conditioned
- Economic dispatch should use AC power flow model
- Structure of market open to debate
- Types of "generator expansion" also subject to debate
- Suite of tools is very effective in such situations

- Coupling collections of (sub)-models with well defined (information sharing) interfaces facilitates:
  - appropriate detail and consistency of sub-model formulation (each of which may be very large scale, of different types (mixed integer, semidefinite, nonlinear, variational, etc) with different properties (linear, convex, discrete, smooth, etc))
  - ability for individual subproblem solution verification and engagement of decision makers
  - ability to treat uncertainty by stochastic and robust optimization at submodel level and with evolving resolution
  - ability to solve submodels to global optimality (by exploiting size, structure and model format specificity)

(A monster model that mixes several modeling formats loses its ability to exploit the underlying structure and provide guarantees on solution quality)

Extended Mathematical Programs

- Optimization models improve understanding of underlying systems and facilitate operational/strategic improvements under resource constraints
- Problem format is old/traditional

$$\min_x f(x) \text{ s.t. } g(x) \leq 0, h(x) = 0$$

- Extended Mathematical Programs allow annotations of constraint functions to augment this format.
- Developing interfaces and exploiting hierarchical structure using computationally tractable algorithms will provide overall solution speed, understanding of localized effects, and value for the coupling of the system.

Stochastic competing agent models (with Wets)

- Competing agents (consumers, or generators in energy market)
- Each agent maximizes objective independently (utility)
- Market prices are function of all agents activities
- Additional twist: model must "hedge" against uncertainty
- Facilitated by allowing contracts bought now, for goods delivered later
- Conceptually allows to transfer goods from one period to another (provides wealth retention or pricing of ancilliary services in energy market)
- Can investigate new instruments to move to system optimal solutions from equilibrium (or market) solutions

The model details: c.f. Brown, Demarzo, Eaves

Each agent maximizes:

$$u_a = -\sum_s \pi_s \left(\kappa - \prod_i q_{a,s,i}^{\alpha_{a,i}} \right)$$

Time 0:

$$\sum_i p_{0,i} q_{a,0,i} + \sum_j v_j y_{a,j} \leq \sum_i p_{0,i} e_{a,0,i}$$

Time 1:

$$\sum_i p_{s,i} q_{a,s,i} \leq \sum_i p_{s,i} \sum_j D_{s,i,j} y_{a,j} + \sum_i p_{s,i} e_{a,s,i}$$

Additional constraints (complementarity) outside of control of agents:

$$0 \leq -\sum_a y_{a,j} \perp v_j \geq 0$$

$$0 \leq -\sum_a d_{a,s,i} \perp p_{s,i} \geq 0$$

Conclusions

- Modern optimization within applications requires multiple model formats, computational tools and sophisticated solvers
- EMP model type is clear and extensible, additional structure available to solver
- Extended Mathematical Programming available within the GAMS modeling system
- Able to pass additional (structure) information to solvers
- Embedded optimization models automatically reformulated for appropriate solution engine
- Exploit structure in solvers
- Extend application usage further