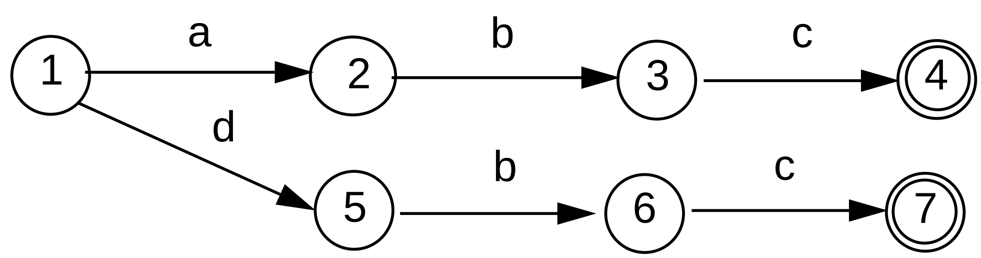
transition to follow for some character, the merged state is *split* into two or more smaller states that do agree.

As an example, assume we start with the following automaton:



Initially we have a merged non-accepting state {1,2,3,5,6} and a merged accepting state {4,7}.

A merger is legal if and only if all constituent states agree on the same successor state for all characters. For example, states 3 and 6 would go to an accepting state given character c; states 1, 2, 5 would not, so a split must occur.

We will add an error state s_E to the original DFA that is the successor state under any illegal character. (Thus reaching s_E becomes equivalent to detecting an illegal token.) s_E is not a real state; rather it allows us to assume every state has a successor under every character. s_E is never merged with any real state.

Algorithm **`Split`**, shown below, splits merged states whose constituents do not agree on a common successor state for all characters. When **`Split`** terminates, we know that the states that remain merged are equivalent in that they always agree on common successors.

```
Split(FASet StateSet) {
 repeat
  for(each merged state S in StateSet) {
    Let S correspond to {s1,...,sn}
    for(each char c in Alphabet){
      Let t1,...,tn be the successor
       states to s1,...,sn under c
      if(t1,...,tn do not all belong to
          the same merged state){
           Split S into two or more new
           states such that si and sj
           remain in the same merged
           state if and only if ti and tj
           are in the same merged state}
      }
 until no more splits are possible
}
```

Returning to our example, we initially have states {1,2,3,5,6} and {4,7}. Invoking `Split`, we first observe that states 3 and 6 have a common successor under c, and states 1, 2, and 5 have no successor under c (equivalently, have the error state s_E as a successor).

This forces a split, yielding {1,2,5}, {3,6} and {4,7}.

Now, for character b, states 2 and 5 would go to the merged state {3,6}, but state 1 would not, so another split occurs.

We now have: {1}, {2,5}, {3,6} and {4,7}.

At this point we are done, as all constituents of merged states agree on the same successor for each input symbol.

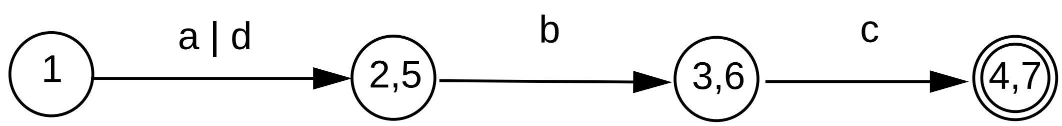
Once `Split` is executed, we are essentially done.

Transitions between merged states are the same as the transitions between states in the original DFA.

Thus, if there was a transition between state s_i and s_j under character c, there is now a transition under c from the merged state containing s_i to the merged state containing s_j. The start state is that merged state containing the original start state.

Accepting states are those merged states containing accepting states (recall that accepting and non-accepting states are never merged).

Returning to our example, the minimum state automaton we obtain is



Properties of Regular Expressions and Finite Automata

- Some token patterns *can't* be defined as regular expressions or finite automata. Consider the set of balanced brackets of the form [[[…]]]. This set is defined formally as
  $\{ [^m]^m \mid m \geq 1 \}$.
  This set is *not* regular.
  No finite automaton that recognizes *exactly* this set can exist.
  Why? Consider the inputs [, [[, [[[, ...
  For two different counts (call them i and j) $[^i$ and $[^j$ must reach the same state of a given FA! (Why?)
  Once that happens, we know that if $[^i]^i$ is accepted (as it should be), the $[^j]^i$ will also be accepted (and that should not happen).

- $\overline{R} = V^* - R$ is regular if R is.
  Why?
  Build a finite automaton for R. Be careful to include transitions to an "error state" s_E for illegal characters. Now invert final and non-final states. What was previously accepted is now rejected, and what was rejected is now accepted. That is, $\overline{R}$ is accepted by the modified automaton.

- Not all subsets of a regular set are themselves regular. The regular expression $[^+]^+$ has a subset that isn't regular. (What is that subset?)

- Let R be a set of strings. Define R^{rev} as all strings in R, in reversed (backward) character order.
  Thus if R = {abc, def}
  then R^{rev} = {cba, fed}.
  If R is regular, then R^{rev} is too.
  Why? Build a finite automaton for R. Make sure the automaton has only one final state. Now *reverse* the direction of all transitions, and interchange the start and final states. What does the modified automation accept?

- If R_1 and R_2 are both regular, then $R_1 \cap R_2$ is also regular. We can show this two different ways:
  1. Build two finite automata, one for R1 and one for R2. Pair together states of the two automata to match R1 and R2 simultaneously. The paired-state automaton accepts only if both R1 and R2 would, so R1 $\cap$ R2 is matched.
  2. We can use the fact that R1 $\cap$ R2 is = $\overline{\overline{R_1} \cup \overline{R_2}}$ We already know union and complementation are regular.

Reading Assignment

- Read Chapter 4 of **Crafting a Compiler, Second Edition**.

Context Free Grammars

A context-free grammar (CFG) is defined as:

- A finite terminal set V_t;
  these are the tokens produced by the scanner.
- A set of intermediate symbols, called non-terminals, V_n.
- A start symbol, a designated non-terminal, that starts all derivations.
- A set of productions (sometimes called rewriting rules) of the form

  $A \rightarrow X_1 \ldots X_m$

  X_1 to X_m may be any combination of terminals and non-terminals.

  If $m = 0$ we have $A \rightarrow \lambda$
  which is a valid production.

Example

Prog $\rightarrow$ { Stmts }

Stmts $\rightarrow$Stmts ; Stmt

Stmts $\rightarrow$Stmt

Stmt $\rightarrow$id = Expr

Expr $\rightarrow$id

Expr $\rightarrow$Expr + id

Often more than one production shares the same left-hand side.

Rather than repeat the left hand side, an “or notation” is used:

Prog → { Stmts }

Stmts →Stmts ; Stmt

| Stmt

Stmt →id = Expr

Expr →id

| Expr + id

Derivations

Starting with the start symbol, non-terminals are rewritten using productions until only terminals remain.

Any terminal sequence that can be generated in this manner is syntactically valid.

If a terminal sequence can't be generated using the productions of the grammar it is invalid (has syntax errors).

The set of strings derivable from the start symbol is the *language* of the grammar (sometimes denoted L(G)).

For example, starting at **Prog** we generate a terminal sequence, by repeatedly applying productions:

Prog

{ Stmts }

{ Stmts ; Stmt }

{ Stmt ; Stmt }

{ id = Expr ; Stmt }

{ id = id ; Stmt }

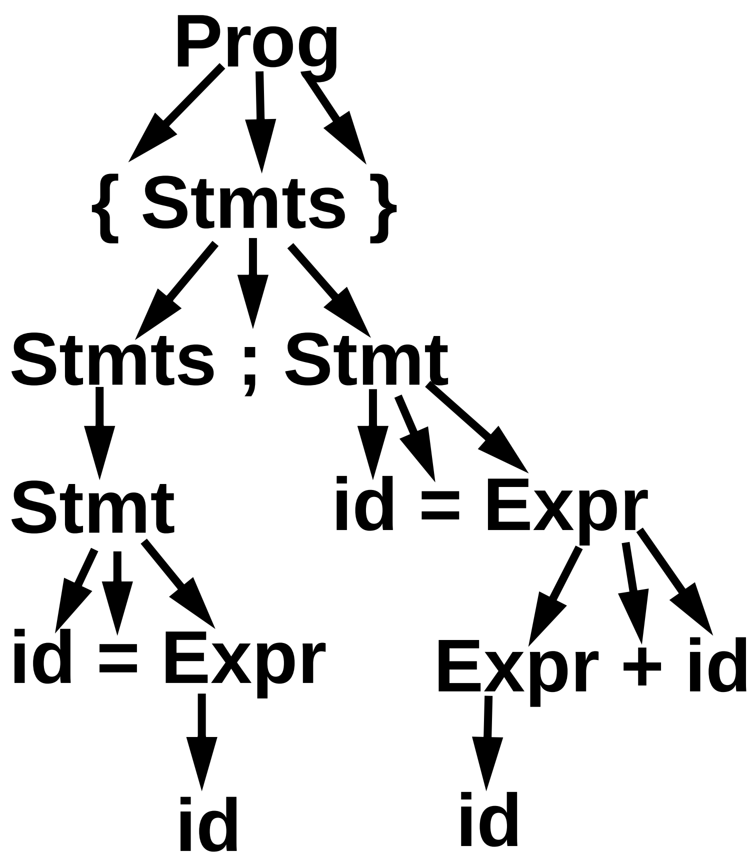
{ id = id ; id = Expr }

{ id = id ; id = Expr + id}

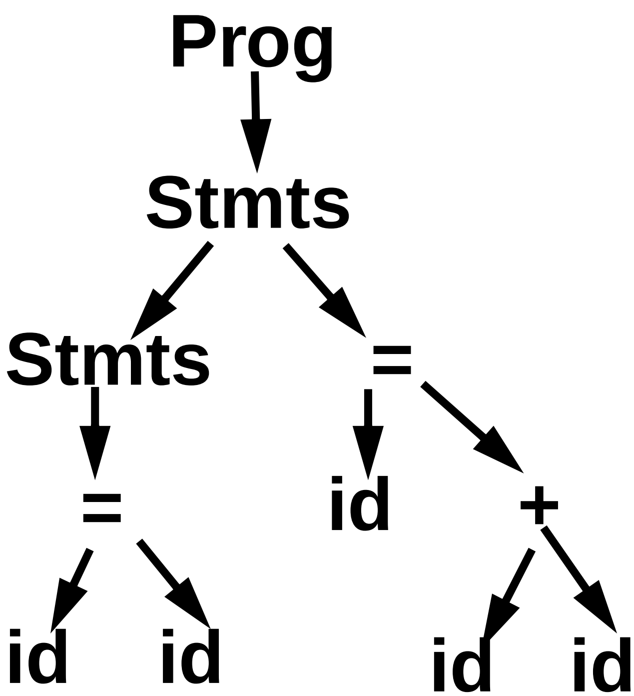
{ id = id ; id = id + id}

Parse Trees

To illustrate a derivation, we can draw a *derivation tree* (also called a *parse tree*):



An *abstract syntax tree* (AST) shows essential structure but eliminates unnecessary delimiters and intermediate symbols:



If $A \rightarrow \gamma$ is a production then
$\alpha A \beta \Rightarrow \alpha \gamma \beta$
where $\Rightarrow$ denotes a one step derivation (using production $A \rightarrow \gamma$).

We extend $\Rightarrow$ to $\Rightarrow^+$ (derives in one or more steps), and $\Rightarrow^*$ (derives in zero or more steps).

We can show our earlier derivation as

Prog $\Rightarrow$

{ Stmts } $\Rightarrow$

{ Stmts ; Stmt } $\Rightarrow$

{ Stmt ; Stmt } $\Rightarrow$

{ id = Expr ; Stmt } $\Rightarrow$

{ id = id ; Stmt } $\Rightarrow$

{ id = id ; id = Expr } $\Rightarrow$

{ id = id ; id = Expr + id} $\Rightarrow$

{ id = id ; id = id + id}

Prog $\Rightarrow^+$ **{ id = id ; id = id + id}**

When deriving a token sequence, if more than one non-terminal is present, we have a choice of which to expand next.

We must specify, at each step, which non-terminal is expanded, and what production is applied.

For simplicity we adopt a convention on what non-terminal is expanded at each step.

We can choose the leftmost possible non-terminal at each step.

A derivation that follows this rule is a *leftmost derivation*.

If we know a derivation is leftmost, we need only specify what productions are used; the choice of non-terminal is always fixed.

To denote derivations that are leftmost,

we use $\Rightarrow_L$, $\Rightarrow^+_L$, and $\Rightarrow^*_L$

The production sequence discovered by a large class of parsers (the top-down parsers) is a leftmost derivation, hence these parsers produce a *leftmost parse*.

Prog $\Rightarrow_L$

{ Stmts } $\Rightarrow_L$

{ Stmts ; Stmt } $\Rightarrow_L$

{ Stmt ; Stmt } $\Rightarrow_L$

{ id = Expr ; Stmt } $\Rightarrow_L$

{ id = id ; Stmt } $\Rightarrow_L$

{ id = id ; id = Expr } $\Rightarrow_L$

{ id = id ; id = Expr + id} $\Rightarrow_L$

{ id = id ; id = id + id}

Prog $\Rightarrow^+_L$ **{ id = id ; id = id + id}**

Rightmost Derivations

A rightmost derivation is an alternative to a leftmost derivation. Now the rightmost non-terminal is always expanded.

This derivation sequence may seem less intuitive given our normal left-to-right bias, but it corresponds to an important class of parsers (the bottom-up parsers, including CUP).

As a bottom-up parser discovers the productions used to derive a token sequence, it discovers a rightmost derivation, but in *reverse order*.

The last production applied in a rightmost derivation is the first that is discovered. The first production used, involving the start symbol, is discovered last.

The sequence of productions recognized by a bottom-up parser is a rightmost parse.

It is the exact reverse of the production sequence that represents a rightmost derivation.

For rightmost derivations, we use the notation $\Rightarrow_R$, $\Rightarrow^+_R$, and $\Rightarrow^*_R$

Prog $\Rightarrow_R$

{ Stmts } $\Rightarrow_R$

{ Stmts ; Stmt } $\Rightarrow_R$

{ Stmts ; id = Expr } $\Rightarrow_R$

{ Stmts ; id = Expr + id } $\Rightarrow_R$

{ Stmts ; id = id + id } $\Rightarrow_R$

{ Stmt ; id = id + id } $\Rightarrow_R$

{ id = Expr ; id = id + id } $\Rightarrow_R$

{ id = id ; id = id + id}

Prog $\Rightarrow^+$ **{ id = id ; id = id + id}**

You can derive the same set of tokens using leftmost and rightmost derivations; the only difference is the order in which productions are used.

Ambiguous Grammars

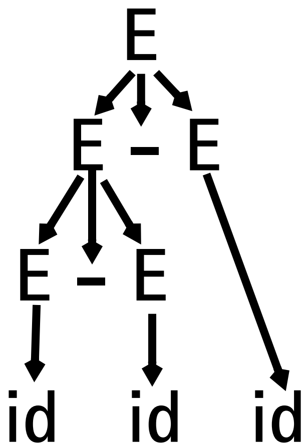
Some grammars allow more than one parse tree for the same token sequence. Such grammars are *ambiguous*. Because compilers use syntactic structure to drive translation, ambiguity is undesirable—it may lead to an unexpected translation.

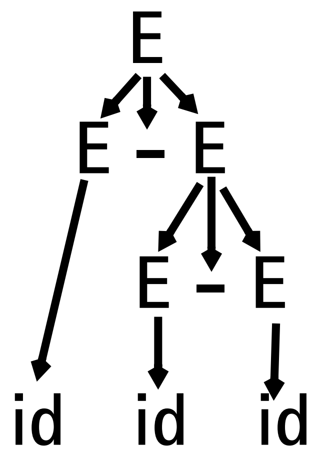
Consider

$$\mathbf{E} \rightarrow \mathbf{E} \text{ - } \mathbf{E}$$

$$\quad | \quad \mathbf{id}$$

When parsing the input a-b-c (where a, b and c are scanned as identifiers) we can build the following two parse trees:





The effect is to parse a-b-c as either (a-b)-c or a-(b-c). These two groupings are certainly not equivalent.

Ambiguous grammars are usually voided in building compilers; the tools we use, like Yacc and CUP, strongly prefer unambiguous grammars.

To correct this ambiguity, we use

$$\mathbf{E} \rightarrow \mathbf{E\ \text{-}\ id}$$
$$\quad | \quad \mathbf{id}$$

Now a-b-c can only be parsed as: