Recall that compilers prefer an unambiguous grammar because a unique parse tree structure can be guaranteed for all inputs.

Hence a unique translation, guided by the parse tree structure, will be obtained.

We would like an algorithm that checks if a grammar is ambiguous.

Unfortunately, it is undecidable whether a given CFG is ambiguous, so such an algorithm is impossible to create.

Fortunately for certain grammar classes, including those for which we can generate parsers, we can prove included grammars are unambiguous.

Potentially, the most serious flaw that a grammar might have is that it generates the “wrong language."

This is a subtle point as a grammar serves as the *definition* of a language.

For established languages (like C or Java) there is usually a suite of programs created to test and validate new compilers. An incorrect grammar will almost certainly lead to incorrect compilations of test programs, which can be automatically recognized.

For new languages, initial implementors must thoroughly test the parser to verify that inputs are scanned and parsed as expected.

Parsers and Recognizers

Given a sequence of tokens, we can ask:

"Is this input syntactically valid?"

(Is it generable from the grammar?).

A program that answers this question is a *recognizer*.

Alternatively, we can ask:

"Is this input valid and, if it is, what is its structure (parse tree)?"

A program that answers this more general question is termed a *parser*.

We plan to use language structure to drive compilers, so we will be especially interested in parsers.

Two general approaches to parsing exist.

The first approach is *top- down*.

A parser is top- down if it "discovers" the parse tree corresponding to a token sequence by starting at the top of the tree (the start symbol), and then expanding the tree (via predictions) in a depth- first manner.

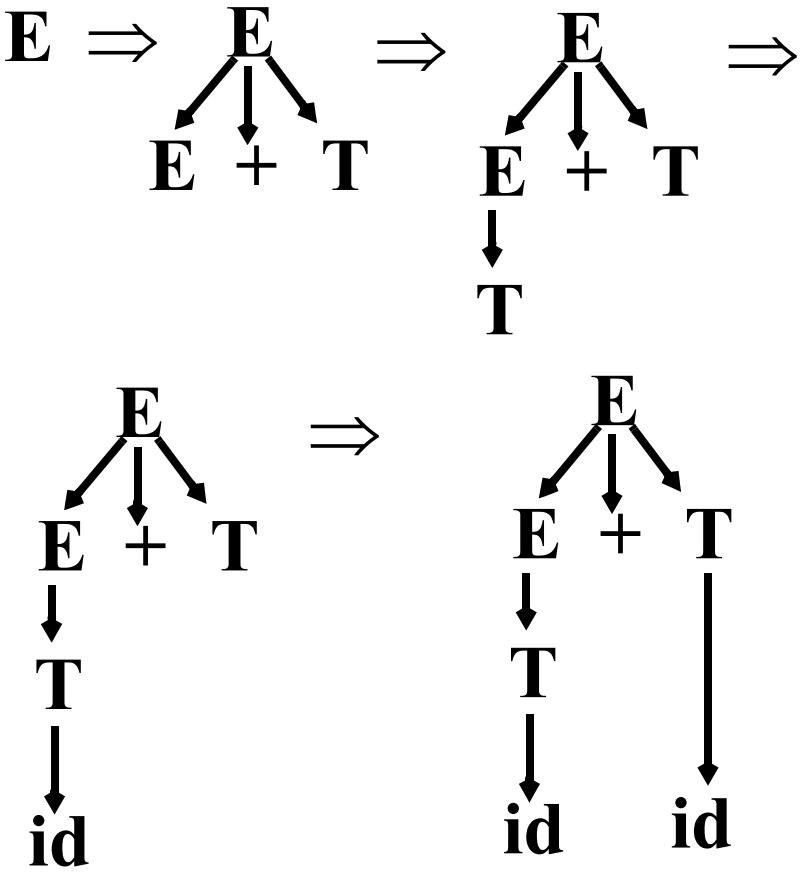
Top- down parsing techniques are *predictive* in nature because they always predict the production that is to be matched before matching actually begins.

Consider

$\mathbf{E \rightarrow E + T \mid T}$

$\mathbf{T \rightarrow T * id \mid id}$

To parse **id + id** in a top- down manner, a parser build a parse tree in the following steps:



A wide variety of parsing techniques take a different approach.

They belong to the class of *bottom- up* parsers.

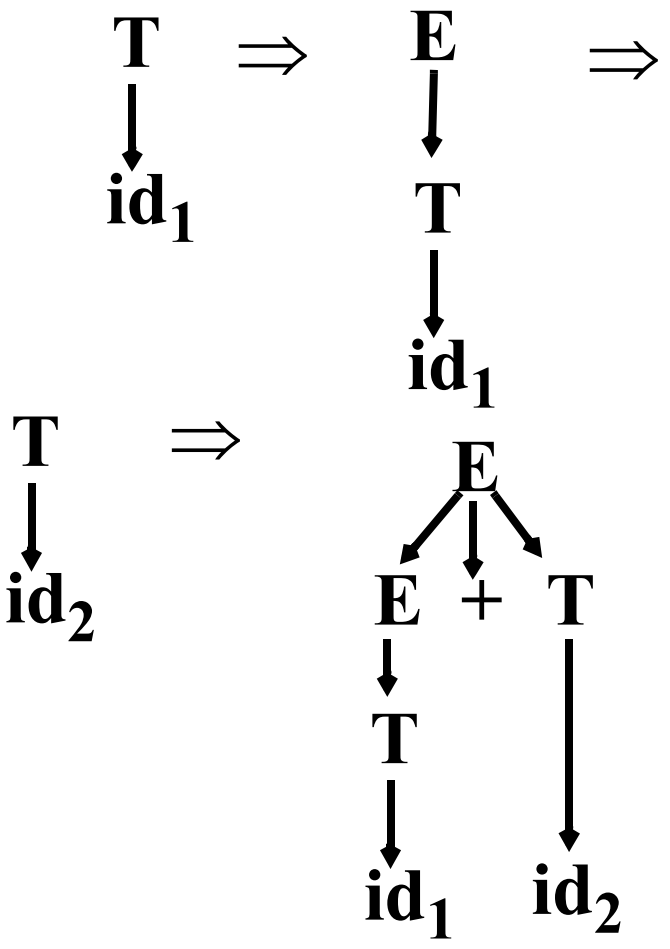
As the name suggests, bottom- up parsers discover the structure of a parse tree by beginning at its bottom (at the leaves of the tree which are terminal symbols) and determining the productions used to generate the leaves.

Then the productions used to generate the immediate parents of the leaves are discovered.

The parser continues until it reaches the production used to expand the start symbol.

At this point the entire parse tree has been determined.

A bottom- up parse of $id_1 + id_2$ would follow the following steps:



A Simple Top-Down Parser

We'll build a rudimentary top-down parser that simply tries each possible expansion of a non-terminal, in order of production definition.

If an expansion leads to a token sequence that doesn't match the current token being parsed, we *backup* and try the next possible production choice.

We stop when all the input tokens are correctly matched or when all possible production choices have been tried.

Example

Given the productions

S → **a**

| **(S)**

we try **a**, then **(a)**, then **((a))**, etc.

Let's next try an additional alternative:

S → **a**

| **(S)**

| **(S]**

Let's try to parse **a**, then **(a]**, then **((a]]**, etc.

We'll count the number of productions we try for each input.

- For input = **a**
  We try **S** → **a** which works.
  Matches needed = 1
- For input = **(a]**
  We try **S** → **a** which fails.
  We next try **S** → **(S)**.
  We expand the inner **S** three different ways; all fail.
  Finally, we try **S** → **(S]**.
  The inner **S** expands to **a**, which works.
  Total matches tried =
  1 + (1+3)+(1+1)= 7.
- For input = **((a]]**
  We try **S** → **a** which fails.
  We next try **S** → **(S)**.
  We match the inner **S** to **(a]** using 7 steps, then fail to match the last **]**.
  Finally, we try **S** → **(S]**.
  We match the inner **S** to **(a]** using 7

steps, then match the last].
Total matches tried =
1 + (1+7)+(1+7)= 17.

- For input = **(((a]]]**
  We try **S** → **a** which fails.
  We next try **S** → **(S)**.
  We match the inner **S** to **((a]]** using 17 steps, then fail to match the last].
  Finally, we try **S** → **(S]**.
  We match the inner **S** to **((a]]** using 17 steps, then match the last].
  Total matches tried =
  1 + (1+17) + (1+17) = 37.

Adding one extra **(...]** pair *doubles* the number of matches we need to do the parse.

In fact to parse $(^i\mathbf{a}]^i$ takes $5*2^i - 3$ matches. This is *exponential* growth!

With a more effective *dynamic programming* approach, in which results of intermediate parsing steps are cached, we can reduce the number of matches needed to $\mathbf{n}^3$ for an input with **n** tokens.

Is this acceptable?

No!

Typical source programs have at least 1000 tokens, and $1000^3 = 10^9$ is a lot of steps, even for a fast modern computer.

The solution?

—Smarter selection in the choice of productions we try.

Reading Assignment

Read Chapter 5 of

Crafting a Compiler, Second Edition.

Prediction

We want to avoid trying productions that can't possibly work.

For example, if the current token to be parsed is an identifier, it is useless to try a production that begins with an integer literal.

Before we try a production, we'll consider the set of terminals it might initially produce. If the current token is in this set, we'll try the production.

If it isn't, there is no way the production being considered could be part of the parse, so we'll ignore it.

A *predict function* tells us the set of tokens that might be initially generated from any production.

For $A \rightarrow X_1 \ldots X_n$, Predict($A \rightarrow X_1 \ldots X_n$) = Set of all initial (first) tokens derivable from $A \rightarrow X_1 \ldots X_n$

$= \{a \text{ in } V_t \mid A \rightarrow X_1 \ldots X_n \Rightarrow^* a \ldots\}$

For example, given

Stmt → Label id = Expr ;

| Label if Expr then Stmt ;

| Label read (IdList) ;

| Label id (Args) ;

Label → intlit :

| λ

Production	Predict Set
Stmt → Label id = Expr ;	**{id, intlit}**
Stmt → Label if Expr then Stmt ;	**{if, intlit}**
Stmt → Label read (IdList) ;	**{read, intlit}**
Stmt → Label id (Args) ;	**{id, intlit}**

We now will match a production p only if the next unmatched token is in p's predict set. We'll avoid trying productions that clearly won't work, so parsing will be faster.

But what is the predict set of a λ- production?

It can't be what's generated by λ (which is nothing!), so we'll define it as the tokens that can *follow* the use of a λ- production.

That is, Predict($A \rightarrow \lambda$) = Follow(A)

where (by definition)

Follow(A) = {a in V_t | $S \Rightarrow^+$ ...Aa...}

In our example,
Follow(Label $\rightarrow \lambda$) ={ id, if, read }

(since these terminals can immediately follow uses of Label in the given productions).

Now let's parse

id (intlit) ;

Our start symbol is **Stmt** and the initial token is **id**.

id can predict

Stmt $\rightarrow$ **Label id = Expr ;**

id then predicts **Label** $\rightarrow \lambda$

The **id** is matched, but "**(**" doesn't match "**=**" so we backup and try a different production for **Stmt**.

id also predicts

Stmt $\rightarrow$ **Label id (Args) ;**

Again, **Label** $\rightarrow \lambda$ is predicted and used, and the input tokens can match the rest of the remaining production.

We had only one misprediction, which is better than before.

Now we'll rewrite the productions a bit to make predictions easier.

We remove the **Label** prefix from all the statement productions (now **intlit** won't predict all four productions).

We now have

Stmt $\rightarrow$ **Label BasicStmt**

BasicStmt $\rightarrow$ **id = Expr ;**

| **if Expr then Stmt ;**

| **read (IdList) ;**

| **id (Args) ;**

Label $\rightarrow$ **intlit :**

| λ

Now **id** predicts two different **BasicStmt** productions. If we rewrite these two productions into

BasicStmt $\rightarrow$ **id StmtSuffix**

StmtSuffix $\rightarrow$ **= Expr ;**

| **(Args) ;**

we no longer have any doubt over which production id predicts.

We now have

Production	Predict Set
Stmt → Label BasicStmt	**Not needed!**
BasicStmt → id StmtSuffix	**{id}**
BasicStmt → if Expr then Stmt ;	**{if}**
BasicStmt → read (IdList) ;	**{read}**
StmtSuffix → (Args) ;	**{ (}**
StmtSuffix → = Expr ;	**{ = }**
Label → intlit :	**{intlit}**
Label → λ	**{if, id, read}**

This grammar generates the same statements as our original grammar did, but now prediction never fails!

Whenever we must decide what production to use, the predict sets for productions with the same lefthand side are always disjoint.

Any input token will predict a unique production or no production at all (indicating a syntax error).

If we never mispredict a production, we never backup, so parsing will be fast and absolutely accurate!

LL(1) Grammars

A context- free grammar whose Predict sets are always disjoint (for the same non- terminal) is said to be *LL(1)*.

LL(1) grammars are ideally suited for top- down parsing because it is always possible to correctly predict the expansion of any non-terminal. No backup is ever needed.

Formally, let

First($X_1 ... X_n$) =

$\{a \text{ in } V_t \mid A \rightarrow X_1 ... X_n \Rightarrow^* a...\}$

Follow(A) = $\{a \text{ in } V_t \mid S \Rightarrow^+ ...Aa...\}$

Predict($A \rightarrow X_1 ... X_n$) =

If $X_1 ... X_n \Rightarrow^* \lambda$

Then First($X_1 ... X_n$) U Follow(A)

Else First($X_1 ... X_n$)

If some CFG, G, has the property that for all pairs of distinct productions with the same lefthand side,
$A \rightarrow X_1 ... X_n$ and $A \rightarrow Y_1 ... Y_m$
it is the case that

Predict($A \rightarrow X_1 ... X_n$) $\cap$

Predict($A \rightarrow Y_1 ... Y_m$) = ϕ

then G is LL(1).

LL(1) grammars are easy to parse in a top- down manner since predictions are always correct.

Example

Production	Predict Set
$S \rightarrow A\ a$	{b,d,a}
$A \rightarrow B\ D$	{b, d, a}
$B \rightarrow b$	{ b }
$B \rightarrow \lambda$	{d, a}
$D \rightarrow d$	{ d }
$D \rightarrow \lambda$	{ a }

Since the predict sets of both B productions and both D productions are disjoint, this grammar is LL(1).

Recursive Descent Parsers

An early implementation of top-down (LL(1)) parsing was recursive descent.

A parser was organized as a set of *parsing procedures*, one for each non- terminal. Each parsing procedure was responsible for parsing a sequence of tokens derivable from its non- terminal.

For example, a parsing procedure, A, when called, would call the scanner and match a token sequence derivable from A.

Starting with the start symbol's parsing procedure, we would then match the entire input, which must be derivable from the start symbol.

This approach is called recursive descent because the parsing procedures were typically *recursive*, and they *descended* down the input's parse tree (as top- down parsers always do).

Building A Recursive Descent Parser

We start with a procedure **Match**, that matches the current input token against a predicted token:

```
void Match(Terminal a) {
  if (a == currentToken)
     currentToken = Scanner();
  else SyntaxErrror();}
```

To build a parsing procedure for a non- terminal A, we look at all productions with A on the lefthand side:

$A \rightarrow X_1 ... X_n \mid A \rightarrow Y_1 ... Y_m \mid ...$

We use predict sets to decide which production to match (LL(1) grammars always have disjoint predict sets).

We match a production's righthand side by calling **Match** to

match terminals, and calling parsing procedures to match non- terminals.

The general form of a parsing procedure for

$A \rightarrow X_1 \ldots X_n \mid A \rightarrow Y_1 \ldots Y_m \mid \ldots$ is

```
void A() {
 if (currentToken in Predict(A→X1…Xn))
   for(i=1;i<=n;i++)
     if (X[i] is a terminal)
         Match(X[i]);
     else X[i]();
 else
 if (currentToken in Predict(A→Y1…Ym))
   for(i=1;i<=m;i++)
     if (Y[i] is a terminal)
         Match(Y[i]);
     else Y[i]();
 else
      // Handle other A →… productions
 else // No production predicted
     SyntaxError();
}
```

Usually this general form isn't used.

Instead, each production is "macro- expanded" into a sequence of **`Match`** and parsing procedure calls.

Example: CSX-Lite

Production	Predict Set
Prog → { Stmts } Eof	{
Stmts → Stmt Stmts	id if
Stmts → λ	}
Stmt → id = Expr ;	id
Stmt → if (Expr) Stmt	if
Expr → id Etail	id
Etail → + Expr	+
Etail → - Expr	-
Etail → λ	) ;

CSX-Lite Parsing Procedures

```
void Prog() {
  Match("{");
  Stmts();
  Match("}");
  Match(Eof);
}

void Stmts() {
 if (currentToken == id ||
     currentToken == if){
     Stmt();
     Stmts();
 } else {
     /* null */
}}

void Stmt() {
 if (currentToken == id){
     Match(id);
     Match("=");
     Expr();
     Match(";");
 } else {
     Match(if);
     Match("(");
     Expr();
     Match(")");
     Stmt();
}}
```

```
void Expr() {
  Match(id);
  Etail();
}

void Etail() {
 if (currentToken == "+") {
     Match("+");
     Expr();
 } else if (currentToken == "-"){
     Match("-");
     Expr();
 } else {
     /* null */
}}
```

Let's use recursive descent to parse

`{ a = b + c; } Eof`

We start by calling `Prog()` since this represents the start symbol.

Calls Pending	Remaining Input
`Prog()`	`{ a = b + c; } Eof`

`Match("{");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`{ a = b + c; } Eof`

`Stmts();`
`Match("}");`
`Match(Eof);`
	`a = b + c; } Eof`

`Stmt();`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`a = b + c; } Eof`

`Match(id);`
`Match("=");`
`Expr();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`a = b + c; } Eof`

Calls Pending	Remaining Input

`Match("=");`
`Expr();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`= b + c; } Eof`

`Expr();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`b + c; } Eof`

`Match(id);`
`Etail();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`b + c; } Eof`

`Etail();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`+ c; } Eof`

Calls Pending	Remaining Input

`Match("+");`
`Expr();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`+ c; } Eof`

`Expr();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`c; } Eof`

`Match(id);`
`Etail();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`c; } Eof`

`Etail();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`; } Eof`

`/* null */`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`; } Eof`

Calls Pending	Remaining Input

`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`; } Eof`

`Stmts();`
`Match("}");`
`Match(Eof);`
	`} Eof`

`/* null */`
`Match("}");`
`Match(Eof);`
	`} Eof`

`Match("}");` `Match(Eof);`	`} Eof`
`Match(Eof);`	`Eof`
Done!	All input matched

Syntax Errors in Recursive Descent Parsing

In recursive descent parsing, syntax errors are automatically detected. In fact, they are detected *as soon as possible* (as soon as the first illegal token is seen).

How? When an illegal token is seen by the parser, either it fails to predict any valid production or it fails to match an expected token in a call to **`Match`**.

Let's see how the following illegal CSX- lite program is parsed:

```
{ b + c = a; } Eof
```

(Where should the first syntax error be detected?)

Calls Pending	Remaining Input
`Prog()`	`{ b + c = a; } Eof`

`Match("{");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`{ b + c = a; } Eof`

`Stmts();`
`Match("}");`
`Match(Eof);`
	`b + c = a; } Eof`

`Stmt();`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`b + c = a; } Eof`

`Match(id);`
`Match("=");`
`Expr();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`b + c = a; } Eof`

Calls Pending	Remaining Input

`Match("=");`
`Expr();`
`Match(";");`
`Stmts();`
`Match("}");`
`Match(Eof);`
	`+ c = a; } Eof`

Call to Match fails!	`+ c = a; } Eof`

Table-Driven Top-Down Parsers

Recursive descent parsers have many attractive features. They are actual pieces of code that can be read by programmers and extended.

This makes it fairly easy to understand how parsing is done.

Parsing procedures are also convenient places to add code to build ASTs, or to do type-checking, or to generate code.

A major drawback of recursive descent is that it is quite inconvenient to change the grammar being parsed. Any change, even a minor one, may force parsing procedures to be

reprogrammed, as productions and predict sets are modified.

To a less extent, recursive descent parsing is less efficient than it might be, since subprograms are called just to match a single token or to recognize a righthand side.

An alternative to parsing procedures is to encode all prediction in a parsing table. A pre- programed driver program can use a parse table (and list of productions) to parse any LL(1) grammar.

If a grammar is changed, the parse table and list of productions will change, but the driver need not be changed.

LL(1) Parse Tables

An LL(1) parse table, T, is a two-dimensional array. Entries in T are production numbers or blank (error) entries.

T is indexed by:

- A, a non- terminal. A is the non-terminal we want to expand.
- CT, the current token that is to be matched.
- T[A][CT] = $A \rightarrow X_1 \ldots X_n$
  if CT is in Predict($A \rightarrow X_1 \ldots X_n$)
  T[A][CT] = error
  if CT predicts no production with A as its lefthand side

CSX-lite Example

	Production	Predict Set
1	Prog → { Stmts } Eof	{
2	Stmts → Stmt Stmts	id if
3	Stmts → λ	}
4	Stmt → id = Expr ;	id
5	Stmt → if (Expr) Stmt	if
6	Expr → id Etail	id
7	Etail → + Expr	+
8	Etail → - Expr	-
9	Etail → λ	) ;

	{	}	if	(	)	id	=	+	-	;	eof
Prog	1										
Stmts		3	2			2					
Stmt			5			4					
Expr						6					
Etail					9			7	8	9	

LL(1) Parser Driver

Here is the driver we'll use with the LL(1) parse table. We'll also use a *parse stack* that remembers symbols we have yet to match.

```
void LLDriver(){
  Push(StartSymbol);
  while(! stackEmpty()){
  //Let X=Top symbol on parse stack
  //Let CT = current token to match
    if (isTerminal(X)) {
      match(X); //CT is updated
      pop();    //X is updated
    } else if (T[X][CT] != Error){
      //Let T[X][CT] = X→Y1...Ym
      Replace X with
         Y1...Ym on parse stack
    } else SyntaxError(CT);
  }
}
```

Example of LL(1) Parsing

We'll again parse

```
{ a = b + c; } Eof
```

We start by placing Prog (the start symbol) on the parse stack.

Parse Stack	Remaining Input
`Prog`	`{ a = b + c; } Eof`

`{`
`Stmts`
`}`
`Eof`
	`{ a = b + c; } Eof`

`Stmts`
`}`
`Eof`
	`a = b + c; } Eof`

`Stmt`
`Stmts`
`}`
`Eof`
	`a = b + c; } Eof`

Parse Stack	Remaining Input
id = Expr ; Stmts } Eof	a = b + c; } Eof
= Expr ; Stmts } Eof	= b + c; } Eof
Expr ; Stmts } Eof	b + c; } Eof
id Etail ; Stmts } Eof	b + c; } Eof

Parse Stack	Remaining Input
Etail ; Stmts } Eof	+ c; } Eof
+ Expr ; Stmts } Eof	+ c; } Eof
Expr ; Stmts } Eof	c; } Eof
id Etail ; Stmts } Eof	c; } Eof

Parse Stack	Remaining Input
Etail ; Stmts } Eof	; } Eof
; Stmts } Eof	; } Eof
Stmts } Eof	} Eof
} Eof	} Eof
Eof	Eof
Done!	All input matched

Syntax Errors in LL(1) Parsing

In LL(1) parsing, syntax errors are automatically detected as soon as the first illegal token is seen.

How? When an illegal token is seen by the parser, either it fetches an error entry from the LL(1) parse table *or* it fails to match an expected token.

Let's see how the following illegal CSX- lite program is parsed:

```
{ b + c = a; } Eof
```

(Where should the first syntax error be detected?)

Parse Stack	Remaining Input
`Prog`	{ b + c = a; } Eof

`{`
`Stmts`
`}`
`Eof`
	{ b + c = a; } Eof

`Stmts`
`}`
`Eof`
	b + c = a; } Eof

`Stmt`
`Stmts`
`}`
`Eof`
	b + c = a; } Eof

`id`
`=`
`Expr`
`;`
`Stmts`
`}`
`Eof`
	b + c = a; } Eof

Parse Stack	Remaining Input
= Expr ; Stmts } Eof	+ c = a; } Eof
Current token (+) fails to match expected token (=)!	+ c = a; } Eof

How do LL(1) Parsers Build Syntax Trees?

So far our LL(1) parser has acted like a recognizer. It verifies that input token are syntactically correct, but it produces no output.

Building complete (concrete) parse trees automatically is fairly easy.

As tokens and non- terminals are matched, they are pushed onto a second stack, the *semantic stack*.

At the end of each production, an action routine pops off n items from the semantic stack (where n is the length of the production's righthand side). It then builds a syntax tree whose root is the

lefthand side, and whose children are the n items just popped off.

For example, for production

Stmt → id = Expr ;

the parser would include an action symbol after the ";" whose actions are:

```
P4 = pop(); // Semicolon token
P3 = pop(): // Syntax tree for Expr
P2 = pop(); // Assignment token
P1 = pop(); // Identifier token
Push(new StmtNode(P1,P2,P3,P4));
```

Creating Abstract Syntax Trees

Recall that we prefer that parsers generate abstract syntax trees, since they are simpler and more concise.

Since a parser generator can't know what tree structure we want to keep, we must allow the user to define "custom" action code, just as Java CUP does.

We allow users to include "code snippets" in Java or C. We also allow labels on symbols so that we can refer to the tokens and tress we wish to access. Our production and action code will now look like this:

Stmt → id:i = Expr:e ;

```
{: RESULT = new StmtNode(i,e); :}
```