To see the usefulness of futures, consider the usual definition of Scheme’s map function:

(define (map f L)
  (if (null? L)
      ()
      (cons (f (car L))
            (map f (cdr L))))
)

If we have a call

(map slow-function long-list)

where slow-function executes slowly and long-list is a large data structure, we can expect to wait quite a while for computation of the result list to complete.
Now consider *fastmap*, a version of *map* that uses futures:

\[
\text{(define (fastmap f L)}
\text{  (if (null? L)}
\text{    ()}
\text{    (cons}
\text{      (future (f (car L)))}
\text{      (fastmap f (cdr L))}
\text{    )}
\text{  )})
\]

Now look at the call

\[
\text{(fastmap slow-function long-list)}
\]

We will exploit a useful aspect of futures— they can be cons’ed together without delay, **even if the computation isn’t completed yet.**

Why? Well a *cons* just stores a pair of pointers, and it really doesn’t matter what the pointers reference (a future or an actual result value).
The call to \texttt{fastmap} can actually return before any of the call to \texttt{slow-function} have completed:

Eventually all the futures automatically transform themselves into data values:
Note that `pcall` can be implemented using futures.

That is, instead of

\[(\text{pcall } F \ X \ Y \ Z)\]

we can use

\[(((\text{future } F)) \ (\text{future } X) \ (\text{future } Y) \ (\text{future } Z))\]

In fact the latter version is actually more parallel—execution of \(F\) can begin even if all the parameters aren’t completely evaluated.
Another Example of Futures

The following function, \texttt{partition}, will take a list and a data value (called \texttt{pivot}). \texttt{partition} will partition the list into two sublists:

(a) Those elements $\leq \texttt{pivot}$

(b) Those elements $> \texttt{pivot}$

\begin{verbatim}
(define (partition pivot L)
  (if (null? L)
      (cons () () )
      (let ((tail-part
               (partition pivot (cdr L))))
        (if (<= (car L) pivot)
            (cons
             (cons (car L) (car tail-part))
             (cdr tail-part))
            (cons
             (car tail-part))
            (cons (car L) (cdr tail-part))))
    )
  )
\end{verbatim}
We want to add futures to partition, but where?

It makes sense to use a future when a computation may be lengthy and we may not need to use the value computed immediately.

What computation fits that pattern? The computation of \texttt{tail-part}. We’ll mark it in a blue box to show we plan to evaluate it using a future:
(define (partition pivot L)
  (if (null? L)
      (cons () () )
      (let ((tail-part
              (partition pivot (cdr L))))
        (if (<= (car L) pivot)
            (cons
              (cons (car L) (car tail-part))
              (cdr tail-part))
            (cons
              (car tail-part))
            (cons (car L) (cdr tail-part))))
      ) ) )

But this one change isn’t enough! We soon access the car and cdr of tail-part, which forces us to wait for its computation to complete. To avoid this delay, we can place the four references to car or cdr of tail-part into futures too:
(define (partition pivot L)
  (if (null? L)
      (cons () () )
      (let ((tail-part
             (partition pivot (cdr L))))
        (if (<= (car L) pivot)
            (cons
             (cons (car L) (car tail-part))
             (cdr tail-part))
            (cons
             (car tail-part))
            (cons (car L) (cdr tail-part))))))
) ) ) )
Now we can build the initial part of the partitioned list (that involving pivot and (car L) independently of the recursive call of partition, which completes the rest of the list.

For example,

(partition 17 '(5 3 8 ...))

creates a future (call it future1) to compute

(partition 17 '(3 8 ...))

It also creates future2 to compute (car tail-part) and future3 to compute (cdr tail-part). The call builds

![Diagram](image-url)
Reading Assignment

- Roosta: Section 13.3
- Introduction to Standard ML (linked from class web page)
- Webber: Chapters 5, 7, 9, 11
ML — Meta Language

SML is Standard ML, a popular ML variant.

ML is a functional language that is designed to be efficient and type-safe. It demonstrates that a functional language need not use Scheme’s odd syntax and need not bear the overhead of dynamic typing.

SML’s features and innovations include:

1. Strong, compile-time typing.
2. Automatic type inference rather than user-supplied type declarations.
3. Polymorphism, including “type variables.”
4. Pattern-directed Programming
   
   ```ml
   fun len([]) = 0
   |   len(a::b) = 1+len(b);
   ```

5. Exceptions

6. First-class functions

7. Abstract Data Types
   
   ```ml
   coin of int |
   bill of int |
   check of string*real;
   val dime = coin(10);
   ```

A good ML reference is

“Elements of ML Programming,”

by Jeffrey Ullman
(Prentice Hall, 1998)
SML is Interactive

You enter a definition or expression, and SML returns a result with an inferred type.

The command

    use "file name";

loads a set of ML definitions from a file.

For example (SML responses are in blue):

```ml
21;
val it = 21 : int
(2 div 3);
val it = 0 : int
true;
val it = true : bool
"xyz";
val it = "xyz" : string
```
Basic SML Predefined Types

• Unit
  
  Its only value is (). Type unit is similar to void in C; it is used where a type is needed, but no “real” type is appropriate. For example, a call to a write function may return unit as its result.

• Integer

  Constants are sequences of digits. Negative values are prefixed with a ~ rather than a – (– is a binary subtraction operator). For example, ~123 is negative 123.

  Standard operators include
  
i  +  –  *  div  mod
  <  >  <=  >=  =  <>
• Real

Both fractional (123.456) and exponent forms (10e7) are allowed. Negative signs and exponents use ~ rather than – (~10.0e~12).

Standard operators include
+  -  *   /
<  >  <=  >=

Note that = and <> aren’t allowed! (Why?)

Conversion routines include
real(int) to convert an int to a real,
floor(real) to take the floor of a real,
ceil(real) to take the ceiling of a real.

round(real) to round a real,
trunc(real) to truncate a real.
For example, \texttt{real(3)} returns \texttt{3.0}, \\
\texttt{floor(3.1)} returns \texttt{3}, \\
\texttt{ceiling(3.3)} returns \texttt{4}, \\
\texttt{round(~3.6)} returns \texttt{~4}, \\
\texttt{trunc(3.9)} returns \texttt{3}.

Mixed mode expressions, like \\
\texttt{1 + 2.5} aren’t allowed; you must do \\
explicit conversion, like \\
\texttt{real(1) + 2.5}

• Strings

Strings are delimited by double 
quotes. Newlines are \texttt{\n}, tabs are \texttt{\t}, 
and \texttt{\"} and \texttt{\\} escape double quotes 
and backslashes. E.g. \texttt{"Bye now\n} 
The ^ operator is concatenation. 
\texttt{"abc" ^ "def" = "abcdef"}

The usual relational operators are provided: \texttt{< > <= >= = <>}
• Characters

Single characters are delimited by double quotes and prefixed by a #. For example, "a" or \\t. A character is not a string of length one. The str function may be used to convert a character into a string. Thus str("a") = "a"

• Boolean

Constants are true and false. Operators include andalso (short-circuit and), orelse (short-circuit or), not, = and <>.

A conditional expression, (if boolval v₁ else v₂) is available.
Tuples

A tuple type, composed of two or more values of any type is available. Tuples are delimited by parentheses, and values are separated by commas. Examples include:

(1,2);
val it = (1,2) : int * int
("xyz",1=2);
val it = ("xyz",false) : string * bool
(1,3.0,false);
val it = (1,3.0,false) : int * real * bool
(1,2,(3,4));
val it = (1,2,(3,4)) : int * int * (int * int)
Equality is checked componentwise:

\[(1, 2) = (0+1, 1+1)\];
val it = true : bool

\[(1, 2, 3) = (1, 2)\] causes a compile-time type error (tuples must be of the same length and have corresponding types to be compared).

\[\#i\] selects the \(i\)-th component of a tuple (counting from 1). Hence

\[\#2(1, 2, 3)\];
val it = 2 : int
Lists

Lists are required to have a single element type for all their elements; their length is unbounded.
Lists are delimited by [ and ] and elements are separated by commas.
Thus \([1, 2, 3]\) is an integer list. The empty (or null) list is \([]\) or \(\text{nil}\).
The cons operator is ::
Hence \([1, 2, 3]\) \(\equiv 1 :: 2 :: 3 :: []\)
Lists are automatically typed by ML:
\([1, 2]\);
val it = [1,2] : int list
Cons

Cons is an infix operator represented as ::
The left operand of :: is any value of type T.
The right operand of :: is any list of type T list.
The result of :: is a list of type T list.
Hence :: is polymorphic.

[] is the empty list. It has a type 'a list. The symbol 'a, read as “alpha” or “tic a” is a type variable.
Thus [] is a polymorphic constant.
List Equality

Two lists may be compared for equality if they are of the same type. Lists \( L_1 \) and \( L_2 \) are considered equal if:

1. They have the same number of elements
2. Corresponding members of the two lists are equal.

List Operators

\( \text{hd} \equiv \text{head of list operator} \approx \text{car} \)
\( \text{tl} \equiv \text{tail of list operator} \approx \text{cdr} \)
\( \text{null} \equiv \text{null list predicate} \approx \text{null?} \)
\( @ \equiv \text{infix list append operator} \approx \text{append} \)