Functions are First-class Objects

Functions may be passed as parameters, returned as the value of a function call, stored in data objects, etc.

This is a consequence of the fact that

```
(lambda (args) (body))
```

evaluates to a function just as

```
(+ 1 1)
```

evaluates to an integer.

Scoping

In Scheme scoping is static (lexical). This means that non-local identifiers are bound to containing lambda parameters, or let values, or globally defined values. For example,

```
(define (f x)
        (lambda (y) (+ x y)))
```

Function **f** takes one parameter, **x**. It returns a function (of **y**), with **x** in the returned function bound to the value of **x** used when **f** was called.

Thus

```
(f 10) ≡ (lambda (y) (+ 10 y))
((f 10)  12) ⇒ 22
```


Unbound symbols are assumed to be globals; there is a run-time error if an unbound global is referenced. For example,

```
(define (p y) (+ x y))
(p 20) ; error -- x is unbound
(define x 10)
(p 20) ⇒ 30
```

We can use let bindings to create private local variables for functions:

```
(define F
        (let  ( (X  1) )
            (lambda ()  X)
        )
)
```

F is a function (of no arguments).

(F) calls **F**.

```
(define X 22)
(F) ⇒ 1;X used in F is private
```


We can *encapsulate* internal state with a function by using private, let-bound variables:

```
(define  cnt
   (let  (  (I  0) )
     (lambda ()
         (set! I (+ I 1))  I)
   )
)
```

Now,

```
(cnt) ⇒ 1
(cnt) ⇒ 2
(cnt) ⇒ 3
```

etc.

Let Bindings can be Subtle

You must check to see if the let-bound value is created when the function is *created* or when it is *called*.

Compare

```
(define  cnt
   (let  (  (I  0) )
     (lambda ()
             (set! I (+ I 1))  I)
   )
 )
```

vs.

```
 (define  reset
   (lambda ()
     (let  (  (I  0) )
       (set! I (+ I 1))  I)
   )
 )
```

(reset) ⇒ **1, (reset)** ⇒ **1, etc.**

Simulating Class Objects

Using association lists and private bound values, we can *encapsulate* data and functions. This gives us the effect of class objects.

```
(define (point x y)
  (list
   (list 'rect
         (lambda () (list x y)))
   (list 'polar
         (lambda ()
          (list
           (sqrt (+ (* x x) (* y y)))
           (atan (/ x y))
          )
         )
   )
  )
)
```

A call **(point 1 1)** creates an association list of the form

((rect funct) (polar funct))

We can use **structure** to access components:

(define p (point 1 1))

((structure 'rect p)) ⇒ **(1 1)**

((structure 'polar p)) ⇒

($\sqrt{2}$ $\frac{\pi}{4}$ **)**

We can add new functionality by just adding new **(id function)** pairs to the association list.

```
(define (point x y)
  (list
   (list 'rect
         (lambda () (list x y)))
   (list 'polar
         (lambda ()
          (list
           (sqrt (+ (* x x) (* y y)))
           (atan (/ x y))
   )))
   (list 'set-rect!
         (lambda (newx newy)
                 (set! x newx)
                 (set! y newy)
                 (list x y)
   ))
   (list 'set-polar!
         (lambda (r theta)
           (set! x (* r (sin theta)))
           (set! y (* r (cos theta)))
           (list r theta)
   ))
))
```

Now we have

```
(define  p  (point 1 1) )
( (structure 'rect p) ) ⇒ (1 1)
( (structure 'polar p) ) ⇒
```

$(\sqrt{2}\ \frac{\pi}{4})$

```
((structure 'set-polar! p) 1 π/4)
 ⇒ (1 π/4)
 ( (structure 'rect p) ) ⇒
```

$(\frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}})$

Limiting Access to Internal Structure

We can improve upon our association list approach by returning a single function (similar to a C++ or Java object) rather than an explicit list of (id function) pairs.

The function will take the name of the desired operation as one of its arguments.

First, let's differentiate between

```
(define  def1
  (let  (  (I 0)  )
    (lambda () (set! I (+ I 1)) I)
  )
)
```

and

```
(define  (def2)
  (let  (  (I 0)  )
    (lambda () (set! I (+ I 1)) I)
  )
)
```

`def1` is a zero argument function that increments a local variable and returns its updated value.

`def2` is a a zero argument function that *generates* a function of zero arguments (that increments a local variable and returns its updated value). Each call to **`def2`** creates a *different* function.

Stack Implemented as a Function

```
(define  ( stack )
  (let  (  (s  () ) )
    (lambda (op . args) ; var # args
     (cond
      ((equal? op 'push!)
       (set!  s (cons (car args) s))
        (car s))
      ((equal?  op  'pop!)
        (if (null? s)
            #f
            (let  ( (top  (car s)) )
                (set!  s  (cdr s))
                top )))
      ((equal? op 'empty?)
       (null? s))
      (else   #f)
      )
     )
  )
)
```

```
(define stk (stack));new empty stack
(stk 'push! 1) ⇒ 1 ;s = (1)
(stk 'push! 3) ⇒ 3 ;s = (3 1)
(stk 'push! 'x) ⇒ x ;s = (x 3 1)
(stk 'pop!) ⇒ x ;s = (3 1)
(stk 'empty?) ⇒ #f ;s = (3 1)
(stk 'dump) ⇒ #f ;s = (3 1)
```


Higher-Order Functions

A higher-order function is a function that takes a function as a parameter or one that returns a function as its result.

A very important (and useful) higher-order function is **map**, which applies a function to a list of values and produces a list or results:

```
(define (map f L)
  (if (null? L)
     ()
     (cons (f (car L))
           (map f (cdr L)))
  )
)
```

Note: In Scheme's built-in implementation of **map**, the order of function application is unspecified.


```
(map sqrt '(1 2 3 4 5)) ⇒
  (1 1.414 1.732 2 2.236)
(map (lambda(x) (* x x))
         '(1 2 3 4 5)) ⇒
  (1 4 9 16 25)
```

Map may also be used with multiple argument functions by supplying more than one list of arguments:

```
(map + '(1 2 3) '(4 5 6)) ⇒
  (5 7 9)
```


The Reduce Function

Another useful higher-order function is **reduce**, which reduces a list of values to a single value by repeatedly applying a binary function to the list values.

This function takes a binary function, a list of data values, and an identity value for the binary function:

```
(define
  (reduce f L id)
  (if (null? L)
     id
     (f (car L)
        (reduce f (cdr L) id))
  )
)
(reduce + '(1 2 3 4 5) 0) ⇒   15
(reduce * '(1 2 4 6 8 10) 1) ⇒
3840
```

```
(reduce append
 '((1 2 3) (4 5 6) (7 8)) ())
⇒ (1 2 3 4 5 6 7 8)
(reduce expt '(2 2 2 2) 1) ⇒
```

$2^{2^{2^{2}}}$ `= 65536`

```
(reduce expt '(2 2 2 2 2) 1)
```

⇒ 2^{65536}

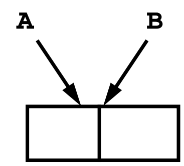
```
(string-length
 (number->string
  (reduce expt '(2 2 2 2 2) 1)))
```

⇒ `19729 ; digits in` 2^{65536}

Sharing vs. Copying

In languages without side-effects an object can be copied by copying a pointer (reference) to the object; a complete new copy of the object isn't needed.

Hence in Scheme **`(define A B)`** normally means

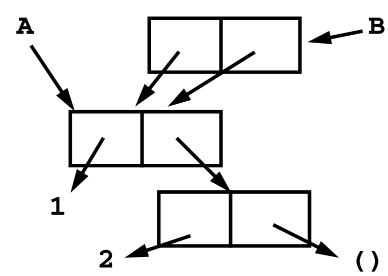


But, if side-effects are possible we may need to force a physical copy of an object or structure:

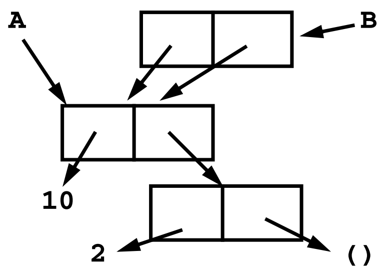
```
(define (copy obj)
  (if (pair? obj)
    (cons (copy (car obj))
          (copy (cdr obj)))
    obj
  )
)
```


For example,

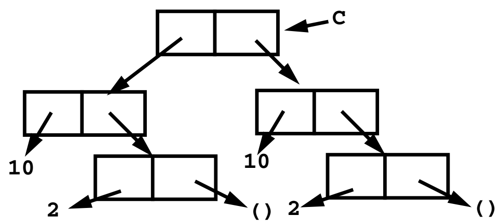
```
(define A '(1 2))
(define B (cons A A))
B = ( (1 2) 1 2)
```



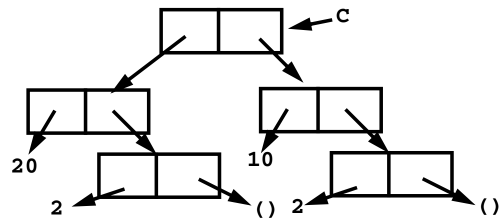
```
(set-car! (car B)  10)
B = ( (10 2) 10 2)
```




```
(define C (cons (copy A) (copy A)))
```




```
(set-car! (car C) 20)
C = ((20 2) 10 2)
```



Similar concerns apply to strings and vectors, because their internal structure can be changed.

Shallow & Deep Copying

A copy operation that copies a pointer (or reference) rather than the object itself is a *shallow copy*.

For example, In Java,

```
Object O1 = new Object();
Object O2 = new Object();
O1 = O2; // shallow copy
```

If the structure within an object is physically copied, the operation is a *deep copy*.

In Java, for objects that support the clone operation,

```
O1 = O2.clone(); // deep copy
```

Even in Java's deep copy (via the **clone()** operation), objects referenced from within an object are shallow copied. Thus given


```
class List {
  int  value;
  List next;
}
List L,M;
M = L.clone();
```

L.value and **M.value** are independent, but **L.next** and **M.next** refer to the same **List** object.

A complete deep copy, that copies all objects linked directly or indirectly, is expensive and tricky to implement.

(Consider a complete copy of a circular linked list).