Redefinition of an identifier is OK, but this is redefinition *not* assignment;

Thus

```ml
val x = 100;
val x = (x=100);
```
is fine; there is no type error even though the first `x` is an integer and then it is a boolean.

```ml
val x = 100 : int
val x = true : bool
```
Examples

val x = 1;
val x = 1 : int
val z = (x,x,x);
val z = (1,1,1) : int * int * int
val L = [z,z];
val L = [(1,1,1),(1,1,1)] :
  (int * int * int) list
val r = {a=L};
val r = {a=[(1,1,1),(1,1,1)]} :
  {a:(int * int * int) list}

After rebinding, the “nearest” (most recent) binding is used.

The and symbol (not boolean and) is used for simultaneous binding:

val x = 10;
val x = 10 : int
val x = true and y = x;
val x = true : bool
val y = 10 : int
Local definitions are temporary value definitions:

```ml
local
    val x = 10
in
    val u = x*x;
end;
val u = 100 : int
```

Let bindings are used in expressions:

```ml
let
    val x = 10
in
    5*x
end;
val it = 50 : int
```
Patterns

Scheme (and most other languages) use access or decomposition functions to access the components of a structured object.

Thus we might write

(let ((h (car L) (t (cdr L))))
  body)

Here car and cdr are used as access functions to locate the parts of L we want to access.

In ML we can access components of lists (or tuples, or records) directly by using patterns. The context in which the identifier appears tells us the part of the structure it references.
val x = (1,2);
val x = (1,2) : int * int
val (h,t) = x;
val h = 1 : int
val t = 2 : int
val L = [1,2,3];
val L = [1,2,3] : int list
val [v1,v2,v3] = L;
val v1 = 1 : int
val v2 = 2 : int
val v3 = 3 : int
val [1,x,3] = L;
val x = 2 : int
val [1,rest] = L;
(* This is illegal. Why? *)
val yy::rest = L;
val yy = 1 : int
val rest = [2,3] : int list
Wildcards

An underscore (\_) may be used as a “wildcard” or “don’t care” symbol. It matches part of a structure without defining a new binding.

```ml
val zz::_ = L;
val zz = 1 : int
```

Pattern matching works in records too.

```ml
val r = {a=1,b=2};
val r = {a=1,b=2} : {a:int, b:int}
val {a=va,b=vb} = r;
val va = 1 : int
val vb = 2 : int
val {a=wa,b=_}=r;
val wa = 1 : int
val {a=za, ...}=r;
val za = 1 : int
```
Patterns can be nested too.

```ocaml
val x = ((1,3.0),5);
val x = ((1,3.0),5) : (int * real) * int
val ((1,y),_)=x;
val y = 3.0 : real
```
Functions

Functions take a single argument (which can be a tuple). Function calls are of the form

```
function_name argument;
```

For example

```
size "xyz";
cos 3.14159;
```

The more conventional form

```
size("xyz"); Or cos(3.14159);
```

is OK (the parentheses around the argument are allowed, but unnecessary).

The form `(size "xyz")` or `(cos 3.14159)`

is OK too.
Note that the call 
\texttt{plus(1,2);}
passes \textit{one} argument, the tuple \((1,2)\)
to \texttt{plus}.

The call \texttt{dummy();}
passes \textit{one} argument, the unit value, to \texttt{dummy}.

All parameters are passed by value.
Function Types

The type of a function in ML is denoted as $\textbf{T}_1 \to \textbf{T}_2$. This says that a parameter of type $\textbf{T}_1$ is mapped to a result of type $\textbf{T}_2$.

The symbol \texttt{fn} denotes a value that is a function.

Thus

\begin{verbatim}
size;
val it = fn : string -> int
not;
val it = fn : bool -> bool
Math.cos;
val it = fn : real -> real
\end{verbatim}

(Math is an ML structure—an external library member that contains separately compiled definitions).
User-Defined Functions

The general form is

\[
\text{fun name arg = expression;}
\]

ML answers back with the name defined, the fact that it is a function (the \texttt{fn} symbol) and its inferred type.

For example,

\[
\text{fun twice \hspace{1em} x = \hspace{1em} 2*x;}
\]

\[
\text{val twice = fn : int -> int}
\]

\[
\text{fun twotimes(x) = 2*x;}
\]

\[
\text{val twotimes = fn : int -> int}
\]

\[
\text{fun fact n =}
\]

\[
\text{if n=0}
\]

\[
\text{then 1}
\]

\[
\text{else n*fact(n-1);}
\]

\[
\text{val fact = fn : int -> int}
\]
fun plus(x,y):int = x+y;
val plus = fn : int * int -> int

The :int suffix is a type constraint.

It is needed to help ML decide that + is integer plus rather than real plus.
The following defines a predicate that tests whether a list, \( \text{L} \) is null (the predefined \texttt{null} function already does this).

\[
\text{fun isNull L =}
\begin{array}{ll}
\text{if } L = ][] \text{ then true } & \text{else false;}
\end{array}
\]

\[
\text{val isNull = fn : 'a list -> bool}
\]

However, we can decompose the definition using \textit{patterns} to get a simpler and more elegant definition:

\[
\text{fun isNull [] = true}
\begin{array}{ll}
\text{| isNull(_::_) = false;}
\end{array}
\]

\[
\text{val isNull = fn : 'a list -> bool}
\]
The “|” divides the function definition into different argument patterns; no explicit conditional logic is needed. The definition that matches a particular actual parameter is automatically selected.

```plaintext
fun fact(1) = 1
    | fact(n) = n*fact(n-1);
val fact = fn : int -> int
```

If patterns that cover all possible arguments aren’t specified, you may get a run-time Match exception.

If patterns overlap you may get a warning from the compiler.
fun append([],L) = L
  | append(hd::tl,L) = hd::append(tl,L);
val append = fn :
  'a list * 'a list -> 'a list

If we add the pattern
append(L,[]) = L
we get a redundant pattern
warning (Why?)

fun append ([],L) = L
  | append(hd::tl,L) = hd::append(tl,L)
  | append(L,[]) = L;

stdIn:151.1-153.20 Error: match redundant
  (nil,L) => ...
  (hd :: tl,L) => ...
  --> (L,nil) => ...

But a more precise decomposition is fine:

```plaintext
fun append ([], L) = L
    | append(hd::tl, hd2::tl2) = hd::append(tl, hd2::tl2)
    | append(hd::tl, []) = hd::tl;

val append = fn : 'a list * 'a list -> 'a list
```
**Function Types Can be Polytypes**

Recall that 'a, 'b, ... represent type variables. That is, any valid type may be substituted for them when checking type correctness.

ML said the type of `append` is

```ml
val append = fn :
    'a list * 'a list -> 'a list
```

Why does 'a appear three times?

We can define `eitherNull`, a predicate that determines whether either of two lists is null as

```ml
fun eitherNull(L1,L2) = 
    null(L1) orelse null(L2);
val eitherNull =
    fn : 'a list * 'b list -> bool
```

Why are both 'a and 'b used in `eitherNull`'s type?
Currying

ML chooses the most general (least-restrictive) type possible for user-defined functions. Functions are first-class objects, as in Scheme. The function definition

\[ \text{fun } f \ x \ y = \text{expression}; \]
defines a function \( f \) (of \( x \)) that returns a function (of \( y \)). Reducing multiple argument functions to a sequence of one argument functions is called currying (after Haskell Curry, a mathematician who popularized the approach).
Thus

```haskell
fun f x y = x :: [y];
val f = fn : 'a -> 'a -> 'a list
```
says that \( f \) takes a parameter \( x \), of type \( 'a \), and returns a function (of \( y \), whose type is \( 'a \)) that returns a list of \( 'a \).

Contrast this with the more conventional

```haskell
fun g(x,y) = x :: [y];
val g = fn : 'a * 'a -> 'a list
```

Here \( g \) takes a pair of arguments (each of type \( 'a \)) and returns a value of type \( 'a \) list.

The advantage of currying is that we can bind one argument and leave the remaining argument(s) free.
For example

```
fn (1);
```

is a legal call. It returns a function of type

```
fn : int -> int list
```

The function returned is equivalent to

```
fun h b = 1 :: [b];
val h = fn : int -> int list
```
Map Revisited

ML supports the \texttt{map} function, which can be defined as

\begin{verbatim}
fun map(f,[]) = []
   | map(f,x::y) =
       (f x) :: map(f,y);
\end{verbatim}

val map =
  fn : ('a -> 'b) * 'a list -> 'b list

This type says that \texttt{map} takes a pair of arguments. One is a function from type \texttt{'a} to type \texttt{'b}. The second argument is a list of type \texttt{'a}. The result is a list of type \texttt{'b}.

In curried form \texttt{map} is defined as

\begin{verbatim}
fun map f [] = []
   | map f (x::y) =
       (f x) :: map f y;
\end{verbatim}

val map =
  fn : ('a -> 'b) ->
       'a list -> 'b list
This type says that `map` takes one argument that is a function from type `'a` to type `'b`. It returns a function that takes an argument that is a list of type `'a` and returns a list of type `'b`.

The advantage of the curried form of `map` is that we can now use `map` to create “specialized” functions in which the function that is mapped is fixed.

For example,

```plaintext
val neg = map not;
val neg = 
  fn : bool list -> bool list
neg [true,false,true];
val it = [false,true,false] : bool list
```
Power Sets Revisited

Let’s compute power sets in ML. We want a function `pow` that takes a list of values, viewed as a set, and which returns a list of lists. Each sublist will be one of the possible subsets of the original argument.

For example,

```
pow [1,2] = [[1,2],[1],[2],[[]]]
```

We first define a version of `cons` in curried form:

```
fun cons h t = h::t;
val cons = fn :
  'a -> 'a list -> 'a list
```
Now we define \texttt{pow}. We define the powerset of the empty list, \([\texttt{[]}\)], to be \([\texttt{[[]]}\)]. That is, the power set of the empty set is set that contains only the empty set.

For a non-empty list, consisting of \texttt{h::t}, we compute the power set of \texttt{t}, which we call \texttt{pset}. Then the power set for \texttt{h::t} is just \texttt{h} distributed through \texttt{pset} appended to \texttt{pset}.

We distribute \texttt{h} through \texttt{pset} very elegantly: we just map the function \texttt{(cons h)} to \texttt{pset}. \texttt{(cons h)} adds \texttt{h} to the head of any list it is given. Thus mapping \texttt{(cons h)} to \texttt{pset} adds \texttt{h} to all lists in \texttt{pset}. 
The complete definition is simply

```plaintext
fun pow [] = [[]]
|  pow (h::t) =
  let
    val pset = pow t
  in
    (map (cons h) pset) @ pset
  end;
val pow =
  fn : 'a list -> 'a list list

Let's trace the computation of pow [1,2].
Here h = 1 and t = [2]. We need to compute pow [2].
Now h = 2 and t = [].
We know pow [] = [[]],
so pow [2] =
(map (cons 2) [[]])@[] =
([[2]])@[] = [[]2],[]]
```
Therefore `pow [1,2] = (^ (map (cons 1) [[2],[[]]]))`  
@[[2],[[]]] =  
[[1,2],[1]]@[[2],[[]]] =  
[[1,2],[1],[2],[[]]]