**Currying**

ML chooses the most general (least-restrictive) type possible for user-defined functions. Functions are first-class objects, as in Scheme.

The function definition

```
fun f x y = expression;
```

defines a function `f` (of `x`) that returns a function (of `y`).

Reducing multiple argument functions to a sequence of one argument functions is called currying (after Haskell Curry, a mathematician who popularized the approach).

Thus

```
fun f x y = x :: [y];
val f = fn : 'a -> 'a -> 'a list
```
says that `f` takes a parameter `x`, of type `'a`, and returns a function (of `y`, whose type is `'a`) that returns a list of `'a`.

Contrast this with the more conventional

```
fun g(x,y) = x :: [y];
val g = fn : 'a * 'a -> 'a list
```

Here `g` takes a pair of arguments (each of type `'a`) and returns a value of type `'a` list.

The advantage of currying is that we can bind one argument and leave the remaining argument(s) free.

For example

```
f(1);  
is a legal call. It returns a function of type

fn : int -> int list
```

The function returned is equivalent to

```
fun h b = 1 :: [b];
val h = fn : int -> int list
```

**Map Revisited**

ML supports the `map` function, which can be defined as

```
fun map(f,[]) = []
| map(f,x::y) = (f x) :: map(f,y);
val map =
  fn : ('a -> 'b) * 'a list -> 'b list
```

This type says that `map` takes a pair of arguments. One is a function from type `'a` to type `'b`.

The second argument is a list of type `'a`. The result is a list of type `'b`.

In curried form `map` is defined as

```
fun map f [] = []
| map f (x::y) = (f x) :: map f y;
val map =
  fn : ('a -> 'b) -> 'a list -> 'b list
```
This type says that \texttt{map} takes one argument that is a function from type \texttt{'a} to type \texttt{'b}. It returns a function that takes an argument that is a list of type \texttt{'a} and returns a list of type \texttt{'b}.

The advantage of the curried form of \texttt{map} is that we can now use \texttt{map} to create "specialized" functions in which the function that is mapped is fixed.

For example,
\begin{verbatim}
val neg = map not;
val neg =
  fn : bool list -> bool list
neg [true,false,true];
val it = [false,true,false] : bool list
\end{verbatim}

Now we define \texttt{pow}. We define the powerset of the empty list, [], to be \texttt{[[]]}. That is, the power set of the empty set is set that contains only the empty set.

For a non-empty list, consisting of \texttt{h::t}, we compute the power set of \texttt{t}, which we call \texttt{pset}. Then the power set for \texttt{h::t} is just \texttt{h} distributed through \texttt{pset} appended to \texttt{pset}.

We distribute \texttt{h} through \texttt{pset} very elegantly: we just map the function \texttt{(cons h)} to \texttt{pset}. \texttt{(cons h)} adds \texttt{h} to the head of any list it is given. Thus mapping \texttt{(cons h)} to \texttt{pset} adds \texttt{h} to \textit{all} lists in \texttt{pset}.

\begin{verbatim}
The complete definition is simply
fun pow [] = [[]] |
  pow (h::t) =
  let
    val pset = pow t
    in
      (map (cons h) pset) @ pset
    end;
val pow =
  fn : 'a list -> 'a list list
\end{verbatim}

Let's trace the computation of \texttt{pow [1,2]}.

Here \texttt{h = 1} and \texttt{t = [2]}. We need to compute \texttt{pow [2]}.

We know \texttt{pow [1] = [[]]}, so \texttt{pow [2] = (map (cons 2) [[]])@ [[]] = ([[2]])@ [[]] = [[2],[[]]]}
Therefore $\text{pow} \ [1,2] = (\text{map} \ (\text{cons} \ 1) \ [2],[]) @ [2],[] = [[1,2],[1]] @ [2],[] = [[1,2],[1],[2],[]]$

**Composing Functions**

We can define a composition function that composes two functions into one:

```sml
fun comp \(f,g\)\(x\) = \(f(g(x))\);
```

```sml
val comp = fn :
('a -> 'b) * ('c -> 'a) -> 'c -> 'b
```

In curried form we have

```sml
fun comp \(f\ g\ x\) = \(f(g(x))\);
```

```sml
val comp = fn :
('a -> 'b) -> ('c -> 'a) -> 'c -> 'b
```

For example,

```sml
fun sqr \(x\ : \text{int}\ = x*x;\)
val sqr = fn : \text{int} -> \text{int}
comp sqr sqr;
val it = fn : \text{int} -> \text{int}
comp sqr sqr 3;
val it = 81 : \text{int}
```

In SML \(\circ\) (lower-case O) is the infix composition operator.

Hence

\[ \text{sqr} \circ \text{sqr} \equiv \text{comp} \ \text{sqr} \ \text{sqr} \]

**Lambda Terms**

ML needs a notation to write down unnamed (anonymous) functions, similar to the lambda expressions Scheme uses. That notation is

```sml
\(\text{fn \ arg} \Rightarrow \text{body};\)
```

For example,

```sml
val sqr = \(\text{fn} \ x : \text{int} \Rightarrow x*x;\)
val sqr = fn : \text{int} -> \text{int}
```

In fact the notation used to define functions,

```sml
\(\text{fun \ name \ arg} \Rightarrow \text{body};\)
```

is actually just an abbreviation for the more verbose

```sml
val name = \(\text{fn} \ \text{arg} \Rightarrow \text{body};\)
```
An anonymous function can be used wherever a function value is needed.
For example,
map (fn x => [x]) [1,2,3];
val it = [[1],[2],[3]] : int list list
We can use patterns too:
(fn [] => []
 |(h::t) => h::h::t);
val it = fn : 'a list -> 'a list
(What does this function do?)

Overloading, as in C++ and Java, allows alternative definitions of the same method or operator, with selection based on type.
Thus in Java + may represent integer addition, floating point addition or string concatenation, even though these are really rather different operations.
In ML +, -, *, and = are overloaded.
When = is used (to test equality), ML deduces that an equality type is required. (Most, but not all, types can be compared for equality).
When ML decides an equality type is needed, it uses a type variable that begins with two tics rather than one.
fun eq(x,y) = (x=y);
val eq = fn : 'a * 'a -> bool

Polymorphism vs. Overloading
ML supports polymorphism.
A function may accept a polypye (a set of types) rather than a single fixed type.
In all cases, the same function definition is used. Details of the supplied type are irrelevant and may be ignored.
For example,
fun id x = x;
val id = fn : 'a -> 'a
fun toList x = [x];
val toList = fn : 'a -> 'a list
(What does this function do?)

Defining New Types in ML
We can create new names for existing types (type abbreviations) using
type id = def;
For example,
type triple = int*real*string;
type triple = int * real * string
type rec1 =
 {a:int,b:real,c:string};
type rec1 =
 {a:int, b:real, c:string}
type 'a triple3 = 'a*'a*'a;
type 'a triple3 = 'a * 'a * 'a
type intTriple = int triple3;
type intTriple = int triple3
These type definitions are essentially macro-like name substitutions.
The Datatype Mechanism

The **datatype** mechanism specifies new data types using **value constructors**. For example,

```sml
datatype color = red | blue | green;
```

Pattern matching works too using the type's constructors:

```sml
fun translate red = "rot"
  | translate blue = "blau"
  | translate green = "gruen";
val translate = fn : color -> string
```

```sml
fun jumble red = blue
  | jumble blue = green
  | jumble green = red;
val jumble = fn : color -> color
```

```sml
translate (jumble green);
```

SML Examples

Source code for most of the SML examples presented here may be found in

```
~cs538-1/public/sml/class.sml
```

Parameterized Constructors

The constructors used to define data types may be parameterized:

```sml
datatype money = none
  | coin of int
  | bill of int
  | iou of real * string;
```

```sml
datatype money =
  bill of int | coin of int
  | iou of real * string | none
```

Now expressions like `coin(25)` or `bill(5)` or `iou(10.25,"Lisa")` represent valid values of type `money`.

```sml
val dime = coin(10);
val dime = coin 10 : money
val deadbeat = iou(25.00,"Homer Simpson");
val deadbeat = iou (25.0,"Homer Simpson") : money
fun amount(none) = 0.0
  | amount(coin(cents)) = real(cents)/100.0
  | amount(bill(dollars)) = real(dollars)
  | amount(iou(amt,__)) = 0.5*amt;
val amount = fn : money -> real
```
Polymorphic Datatypes

A user-defined data type may be polymorphic. An excellent example is:

```ml
datatype 'a option = none | some of 'a
val zilch = none;
val mucho = some(10e10);
```

```ml
type studentInfo = {name:string, ssNumber:int option};
type studentInfo = {name:string, ssNumber:int option}
val newStudent = {name="Mystery Man", ssNumber=none} : studentInfo;
```

Datatypes may be Recursive

Recursive datatypes allow linked structures without explicit pointers.

```ml
datatype binTree = null | leaf | node of binTree * binTree;
fun size(null) = 0 | size(leaf) = 1 | size(node(t1,t2)) = size(t1)+size(t2) + 1
val size = fn : binTree -> int
```

Recursive Datatypes may be Polymorphic

```ml
datatype 'a binTree = null | leaf of 'a | node of 'a binTree * 'a binTree
datatype 'a binTree = null | leaf of 'a | node of 'a binTree * 'a binTree
fun frontier(null) = [] | frontier(leaf(v)) = [v] | frontier(node(t1,t2)) = frontier(t1) @ frontier(t2)
val frontier = fn : 'a binTree -> 'a list
```
We can model n-ary trees by using lists of subtrees:

```ml
datatype 'a Tree =
  null
| leaf of 'a
| node of 'a Tree list;

datatype 'a Tree = leaf of 'a |
  node of 'a Tree list | null

fun frontier(null) = []
| frontier(leaf(v)) = [v]
| frontier(node(h::t)) =
  frontier(h) @
  frontier(node(t))
| frontier(node([])) = []

val frontier = fn :
  'a Tree -> 'a list
```