String Operations

ML provides a wide variety of string manipulation routines. Included are:
- The string concatenation operator,
  `"abc" ^ "def" = "abcdef"
- The standard 6 relational operators:
  `<` `>` `<=` `=>` `=` `<>
- The string size operator:
  `val size : string -> int`
  `size ("abcd");`
  `val it = 4 : int`
- The string subscripting operator
  (indexing from 0):
  `val sub = fn : string * int -> char`
  `sub ("abcde", 2);`
  `val it = #"c" : char`

- The substring function
  `val substring : string * int * int -> string`
  This function is called as
  `substring (string, start, len)`
  `start` is the starting position, counting from 0.
  `len` is the length of the desired substring. For example,
  `substring ("abcdefghij", 3, 4)`
  `val it = "defg" : string`
- Concatenation of a list of strings
  into a single string:
  `val concat : string list -> string`
  For example,
  `concat ["What’s", " up", "] ;`
  `val it = "What’s up?" : string`
- Convert a character into a string:
  `val str : char -> string`
  For example,
  `str (#"x");`
  `val it = "x" : string`
- "Expode" a string into a list of characters:
  `val explode : string -> char list`
  For example,
  `explode ("abcde");`
  `val it = [ #"a", #"b", #"c", #"d", #"e"] : char list`
- "Implode" a list of characters into a string.
  `val implode : char list -> string`
  For example,
  `implode [ #"a", #"b", #"c", #"d", #"e"] ;`
  `val it = "abcde" : string`

Structures and Signatures

In C++ and Java you can group variable and function definitions into classes. In Java
you can also group classes into packages.
In ML you can group value, exception and function definitions into structures.
You can then import selected definitions from the structure
(using the notation `structure.name`) or you can open
the structure, thereby importing all the definitions
within the structure.
(Examples used in this section may be found at
`~cs538-1/public/sml/struct.sml`)
The general form of a structure definition is
```
structure name =
struct
  val, exception and
  fun definitions
end
```
For example,
```
structure Mapping =
struct
  exception NotFound;
  val create = [];
  fun lookup(key,[]) = raise NotFound
  | lookup(key, (key1,value1)::rest) = if key = key1
    then value1
    else lookup(key,rest);
end;
```
We can access members of this structure as `Mapping.name`. Thus
```
Mapping.insert(538,"languages",[]);
val it = [(538,"languages")]: (int * string) list
```

**Signatures**

Each structure has a *signature*, which is its type.
For example, `Mapping`'s signature is
```
structure Mapping :
  sig
    exception NotFound
    val create : 'a list
    val insert : 'a * 'b * ('a * 'b) list -> ('a * 'b) list
    val lookup : 'a * ('a * 'b) list -> 'b
  end
```

You can define a signature as
```
signature name = sig
  type definitions for values, functions and exceptions
end
```
For example,
```
signature Str2IntMapping =
sig
  exception NotFound;
  val lookup: string * (string*int) list -> int;
end;
```
Signatures can be used to
\begin{itemize}
\item Restrict the type of a value or function in a structure.
\item Hide selected definitions that appear in a structure
\end{itemize}
For example
\begin{verbatim}
structure Str2IntMap :
    Str2IntMapping = Mapping;
\end{verbatim}
defines a new structure, \textit{Str2IntMap}, created by restricting \textit{Mapping} to the \textit{Str2IntMapping} signature. When we do this we get
\begin{verbatim}
open Str2IntMap;
exception NotFound
val lookup : string *
    (string * int) list -> int
\end{verbatim}
Only \textit{lookup} and \textit{NotFound} are created, and \textit{lookup} is limited to keys that are strings.

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### Extending ML's Polymorphism

In languages like C++ and Java we must use types like \texttt{void*} or \texttt{Object} to simulate the polymorphism that ML provides. In ML whenever possible a general type (a polytype) is used rather than a fixed type. Thus in
\begin{verbatim}
fun len([]) = 0
    | len(a::b) = 1 + len(b);
\end{verbatim}
we get a type of
\begin{verbatim}
'a list -> int
\end{verbatim}
because this is the most general type possible that is consistent with \texttt{len}'s definition.
Is this form of polymorphism general enough to capture the general idea of making program definitions as type-independent as possible?
It isn't, and to see why consider the following ML definition of a merge sort. A merge sort operates by first splitting a list into two equal length sublists. The following function does this:
\begin{verbatim}
fun split [] = ([],[])
    | split [a] = ([a],[])
    | split (a::b::rest) = 
        let val (left,right) = 
            split(rest) in
        (a::left, b::right) end;
\end{verbatim}
After the input list is split into two halves, each half is recursively sorted, then the sorted halves are merged together into a single list.

The following ML function merges two sorted lists into one:

```ml
fun merge([],[]) = []
| merge([],hd::tl) = hd::tl
| merge(hd::tl,[]) = hd::tl
| merge(hd::tl,h::t) =
  if hd <= h
  then hd::merge(tl,h::t)
  else h::merge(hd::tl,t)
```

With these two subroutines, a definition of a sort is easy:

```ml
fun sort [] = []
| sort([a]) = [a]
| sort(a::b::rest) =
  let val (left,right) = split(a::b::rest) in
    merge(sort(left),sort(right))
  end;
```

This definition looks very general—it should work for a list of any type.

Unfortunately, when ML types the functions we get a surprise:

```ml
val split = fn : 'a list ->
  'a list * 'a list
val merge = fn : int list *
  int list -> int list
val sort = fn :
  int list -> int list
split is polymorphic, but merge and sort are limited to integer lists!
Where did this restriction come from?
```

The problem is that we did a comparison in `merge` using the `<=` operator, and ML typed this as an integer comparison.

We can make our definition of sort more general by adding a comparison function, `le(a,b)` as a parameter to `merge` and `sort`. If we curry this parameter we may be able to hide it from end users. Our updated definitions are:

```ml
fun merge(le,[],[]) = []
| merge(le,[],hd::tl) = hd::tl
| merge(le,hd::tl,[]) = hd::tl
| merge(le,hd::tl,h::t) =
  if le(hd,h)
  then hd::merge(le,tl,h::t)
  else h::merge(le,hd::tl,t)
```
fun sort le [] = []
| sort le [a] = [a]
| sort le (a::b::rest) = let val (left,right) = split(a::b::rest) in
  merge(le, sort le left, sort le right)
end;

Now the types of \texttt{merge} and \texttt{sort} are:
\[
\begin{align*}
\text{val merge} & = \text{fn : (} 'a \times 'a \rightarrow \text{bool} \text{)} \times \text{'}a\text{ list} \times \text{'}a\text{ list} \rightarrow \text{'}a\text{ list} \\
\text{val sort} & = \text{fn : (} 'a \times 'a \rightarrow \text{bool} \text{)} \rightarrow \text{'}a\text{ list} \rightarrow \text{'}a\text{ list}
\end{align*}
\]

We can now “customize” \texttt{sort} by choosing a particular definition for the \texttt{le} parameter:
\[
\begin{align*}
\text{fun le(a,b)} & = a \leq b; \\
\text{val le} & = \text{fn : int} \times \text{int} \rightarrow \text{bool}
\end{align*}
\]

The structure mechanism we studied earlier can help. We can put a single definition of \texttt{le} in the structure, and share it:
\[
\begin{align*}
\text{structure Sorting = struct} \\
\text{  fun le(a,b)} & = a \leq b; \\
\text{  val le} & = \text{fn : int} \times \text{int} \rightarrow \text{bool}
\end{align*}
\]

Making the comparison relation an explicit parameter works, but it is a bit ugly and inefficient. Moreover, if we have several functions that depend on the comparison relation, we need to ensure that they all use the same relation. Thus if we wish to define a predicate \texttt{inOrder} that tests if a list is already sorted, we can use:
\[
\begin{align*}
\text{fun inOrder le [] = true} \\
| \text{inOrder le [a] = true} \\
| \text{inOrder le (a::b::rest) = le(a,b) andalso inOrder le (b::rest);} \\
\text{val inOrder} & = \text{fn : (} 'a \times 'a \rightarrow \text{bool} \text{)} \rightarrow \text{'}a\text{ list} \rightarrow \text{bool}
\end{align*}
\]

Now \texttt{sort} and \texttt{inOrder} need to use the same definition of \texttt{le}. But how can we enforce this?
fun sort [] = []
| sort([a]) = [a]
| sort(a::b::rest) = 
  let val (left,right) = 
    split(a::b::rest) in 
    merge(sort(left),
          sort(right))
  end;
fun inOrder [] = true
| inOrder [a] = true
| inOrder (a::b::rest) = 
  le(a,b) andalso 
  inOrder (b::rest);
end;
structure Sorting : 
  sig
    val inOrder : int list -> bool
    val le : int * int -> bool
    val merge : int list * 
              int list -> int list
    val sort : 
              int list -> int list
    val split : 'a list -> 
               'a list * 'a list
  end

To sort a type other than integers, we replace the definition of le in the structure. But rather than actually edit that definition, ML gives us a powerful mechanism to parameterize a structure. This is the functor, which allows us to use one or more structures as parameters in the definition of a structure.

We first define
signature Order =
  sig
    type elem
    val le : elem*elem -> bool
  end;
This defines the type of a structure that defines a le predicate defined on a pair of types called elem.
An example of such a structure is
structure IntOrder:Order = 
  struct
    type elem = int;
    fun le(a,b) = a <= b;
  end;

**Functors**

The general form of a functor is

functor name
  (structName:signature) =
  structure definition;

This functor will create a specific version of the structure definition using the structure parameter passed to it.
For our purposes this is ideal—we pass in a structure defining an ordering relation (the le function). This then creates a custom version of all the functions defined in the structure body, using the specific le definition provided.
Now we just define a functor that creates a sorting structure based on an order structure:

functor MakeSorting(O:Order) =
  struct
    open O; (* makes le available*)
    fun split [] = ([],[])
    | split [a] = ([a],[])
    | split (a::b::rest) =
      let val (left,right) = split rest in
        (a::left,b::right)
      end;
    fun merge([],[]) = []
    | merge([],hd::tl) = hd::tl
    | merge(hd::tl,[]) = hd::tl
    | merge(hd::tl,h::t) =
      if le(hd,h)
        then hd::merge(tl,h::t)
        else h::merge(hd::tl,t)
    end;
    fun sort [] = []
    | sort([a]) = [a]
    | sort(a::b::rest) =
      let val (left,right) = split(a::b::rest) in
        merge(sort(left),
             sort(right))
      end;
    fun inOrder [] = true
    | inOrder [a] = true
    | inOrder (a::b::rest) =
      le(a,b) andalso
      inOrder (b::rest);
  end;

fun sort [] = []
  | sort([a]) = [a]
  | sort(a::b::rest) =
    let val (left,right) = split(a::b::rest) in
      merge(sort(left),
           sort(right))
    end;

fun inOrder [] = true
  | inOrder [a] = true
  | inOrder (a::b::rest) =
    le(a,b) andalso
    inOrder (b::rest);
end;

Now

structure IntSorting = MakeSorting(IntOrder);
creates a custom structure for sorting integers:

IntSorting.sort [3,0,-22,8];
val it = [~22,0,3,8] : elem list

To sort strings, we just define a structure containing an le defined for strings with order as its signature (i.e., type) and pass it to MakeSorting:

structure StrOrder:Order =
  struct
    type elem = string
    fun le(a:string,b) = a <= b;
  end;

structure StrSorting = MakeSorting(StrOrder);
StrSorting.sort ("cc","abc","xyz");
val it = ["abc","cc","xyz"] : StrOrder.elem list
StrSorting.inOrder ("cc","abc","xyz");
val it = false : bool
StrSorting.inOrder (3,0,-22,8));
stdIn:593.1-593.32 Error: operator and operand don’t agree
[literal]
  operator domain: StrOrder.elem list
  operand domain: int list
  in expression:
    StrSorting.inOrder (3 :: 0 :: ~22 :: <exp> :: <exp>)