Application of Depth-First Ordering

- Retreating edges (a necessary component of loops) are easy to identify:
  \[ a \rightarrow b \] is a retreating edge if and only if \( \text{dfo}(b) \leq \text{dfo}(a) \)

- A depth-first ordering in an excellent visit order for solving forward data flow problems. We want to visit nodes in essentially topological order, so that all predecessors of a node are visited (and evaluated) before the node itself is.

Dominators

A CFG node \( M \) dominates \( N \)
\( (M \text{ dom } N) \) if and only if all paths from the start node to \( N \) must pass through \( M \).

A node trivially dominates itself. Thus \( (N \text{ dom } N) \) is always true.

A CFG node \( M \) strictly dominates \( N \)
\( (M \text{ sdom } N) \) if and only if
\( (M \text{ dom } N) \) and \( M \neq N \).
A node can't strictly dominates itself. Thus \( (N \text{ sdom } N) \) is never true.

A CFG node may have many dominators.

![Diagram of CFG nodes and dominators](image)

Node \( F \) is dominated by \( F, E, D \) and \( A \).

Immediate Dominators

If a CFG node has more than one dominator (which is common), there is always a unique “closest” dominator called its immediate dominator.

\( (M \text{ idom } N) \) if and only if
\( (M \text{ sdom } N) \) and
\( (P \text{ sdom } N) \implies (P \text{ dom } M) \)

To see that an immediate dominator always exists (except for the start node) and is unique, assume that node \( N \) is strictly dominated by \( M_1, M_2, \ldots, M_p, P \geq 2 \).

By definition, \( M_1, \ldots, M_p \) must appear on all paths to \( N \), including acyclic paths.
Look at the relative ordering among $M_1$ to $M_p$ on some arbitrary acyclic path from the start node to $N$.
Assume that $M_i$ is “last” on that path (and hence “nearest” to $N$).

If, on some other acyclic path, $M_j \neq M_i$ is last, then we can shorten this second path by going directly from $M_i$ to $N$ without touching any more of the $M_1$ to $M_p$ nodes.
But, this totally removes $M_j$ from the path, contradicting the assumption that ($M_j$ sdom $N$).

**Dominator Trees**

Using immediate dominators, we can create a dominator tree in which $A \rightarrow B$ in the dominator tree if and only if ($A$ idom $B$).

A Dominator Tree is a compact and convenient representation of both the dom and idom relations.
A node in a Dominator Tree dominates all its descendents in the tree, and immediately dominates all its children.

Note that the Dominator Tree of a CFG and its DFST are distinct trees (though they have the same nodes).