**Coloring Heuristic**

To R-Color a Graph (where R is the number of registers available)

1. While any node, n, has < R neighbors:
   - Remove n from the Graph.
   - Push n onto a Stack.

2. If the remaining Graph is non-empty:
   - Compute the Cost of each node.
   - The Cost of a Node (a Live Range) is the number of extra instructions needed if the Node isn't assigned a register, scaled by $10^{\text{loop\_depth}}$.
   - Let $\text{NB}(n)$ = Number of Neighbors of n.
   - Remove that node n that has the smallest Cost(n)/NB(n) value.
   - Push n onto a Stack.
   - Return to Step 1.

3. While Stack is non-empty:
   - Pop n from the Stack.
   - If n’s neighbors are assigned fewer than R colors Then assign n any unassigned color
   - Else leave n uncolored.

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**Example**

```c
int p(int lim1, int lim2) {
    int *T1 = &A[0];
    for (i=0; i<lim1 && *(T1+i)>0;i++){}
    int *T2 = &B[0];
    for (j=0; j<lim2 && *(T2+j)>0;j++){}
    return i+j;
}
```

Do a 3 coloring

<table>
<thead>
<tr>
<th>Lim1</th>
<th>Lim2</th>
<th>T1</th>
<th>T2</th>
<th>(i)</th>
<th>(j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>11</td>
<td>42</td>
</tr>
<tr>
<td>Cost/Neighbors</td>
<td>11/3</td>
<td>11/5</td>
<td>11/3</td>
<td>11/3</td>
<td>42/5</td>
</tr>
</tbody>
</table>

Since no node has fewer than 3 neighbors, we remove a node based on the minimum Cost/Neighbors value.

\(\text{lim2}\) is chosen.

We now have:

Remove (say) \(\text{lim1}\), then \(T1, T2, j\) and \(i\) (order is arbitrary).
The Stack is:

```
  i
 / \
|   |
| T2 |
 \
  j
 / \\  \\
|   |   |
| T1 |
   |
lim1
 / \   |
| lim2 |
 \ 
 i j
```

Assuming the colors we have are R1, R2 and R3, the register assignment we choose is

\[ i:R1, j:R2, T1:R2, T2:R3, \text{lim1}:R3, \text{lim2}:\text{spill} \]

**Color Preferences**

Sometimes we wish to assign a particular register (color) to a selected Live Range (e.g., a parameter or return value) if possible.

We can mark a node in the Interference Graph with a Color Preference.

When we unstack nodes and assign colors, we will avoid choosing color \( c \) if an uncolored neighbor has indicated a preference for it. If only color \( c \) is left, we take it (and ignore the preference).

**Example**

Assume in our previous example that \( \text{lim1} \) has requested register R1 and \( \text{lim2} \) has requested register R2 (because these are the registers the parameters are passed in).

Now when \( i, j \) and \( T2 \) are unstacked, they respect \( \text{lim1}'s \) and \( \text{lim2}'s \) preferences:

\[ i:R3, j:R1, T2:R2, T1:R2, \text{lim1}:R1, \text{lim2}:\text{spill} \]
Using Coloring to Optimize Register Moves

A nice “fringe benefit” of allocating registers via coloring is that we can often optimize away register to register moves by giving the source and target the same color.

Consider

Live in: a, b
t1 = a + b
x = t1
y = x + 1
q = t1
Live out: y, q

We’d like x, t1 and q to get the same color. How do we “force” this?

We can “merge” x, t1 and q together:

Live in: a, b
t1 = a + b
x = t1
y = x + 1
q = t1
Live out: y, q

Now a 2-coloring that optimizes away both register to register moves is trivial.

Reckless Coalescing

Originally, Chaitin suggested merging all move-related nodes that don’t interfere.

This is reckless—the merged node may not be colorable!

(Is it worth a spill to save a move??)

This Graph is 2-colorable before the reckless merge, but not after.

Conservative Coalescing

In response to Chaitin’s reckless coalescing approach, Briggs suggested a more conservative approach.

See “Improvement to Graph Coloring Register Allocation,” P. Briggs et. al., ACM Toplas, May 1994.
Briggs suggested that two move-related nodes should be merged only if the combined source and target node has fewer than $R$ neighbors.

This guarantees that the combined node will be colorable, but may miss some optimization opportunities.

```
    a ------ d
   /     /   |
  b     c   e
    

After a merge of nodes $a$ and $d$, there will be four neighbors, but a 2-coloring is still possible.
```

Iterated Coalescing

This is an intermediate approach, that seeks to be safer than reckless coalescing and more effective than conservative coalescing. It was proposed by George and Appel.

1. Build:
Create an Interference Graph, as usual. Mark source-target pairs with a special move-related arc (denoted as a dashed line).

2. Simplify:
Remove and stack non-move-related nodes with $< R$ neighbors.

3. Coalesce:
Combine move-related pairs that will have $< R$ neighbors after coalescing.

   Repeat steps 2 and 3 until only nodes with $R$ or more neighbors or move-related nodes remain or the graph is empty.

4. Freeze:
If the Interference Graph is non-empty:
   Then If there exists a move-related node with $< R$ neighbors
       Then: “Freeze in” the move and make the node non-move-related.
         Return to Steps 2 and 3.
   Else: Use Chaitin’s Cost/Neighbors criterion to remove and stack a node.
         Return to Steps 2 and 3.

5. Unstack:
Color nodes as they are unstacked as per Chaitin and Briggs.
Example

Assume we want a 4-coloring.
Note that neither j & b nor d & c can be conservatively colored.

We simplify by removing nodes with fewer than 4 neighbors.
We remove and stack: g, h, k, f, e, m

The remaining Interference Graph is

We can now conservatively coalesce the move-related pairs to obtain

These remaining nodes can now be removed and stacked.

We can now unstack and color:
d & c: R1, j & b: R2, m: R3, e: R4, f: R1, k: R3, h: R1, g: R4

No spills were required and both moves were optimized away.