Reading Assignment

- Read “Optimal Spilling for CISC Machines with Few Registers,” by Appel and George. (Linked from the class Web page.)
Example

Let’s look at instruction selection for

\[ a = b - 1; \]

where \(a\) is a global int, accessed with a 32 bit address and \(b\) is a local int, accessed as an offset from the frame pointer.

\[
    \begin{array}{c}
        = \\
        \text{int32} \\
        - \\
        s13 \\
        \text{*} \\
        + \\
        r \\
        s13
    \end{array}
\]
We match tree nodes bottom-up. Each node is labeled with the nonterminals it can be reduced to, the production used to produce the nonterminal, and the cost to generate the node (and its children) from the nonterminal.

We match leaves first:
We now work upward, considering operators whose children have been labeled. Again, if an operator can be generated by a nonterminal, we mark the operator with the nonterminal, the production used to generate the operator, and the total cost (including the cost to generate all children).

If a nonterminal can generate the operator using more than one production, the least-cost derivation is chosen.

When we reach the root, the nonterminal with the lowest overall cost is used to generate the tree.
\[ \begin{align*} 
\text{int32} &= s13 \\
\text{Imm:R4:0} \\
\text{Reg:R5:1} \\
\text{Imm:R4:0} \\
\text{Reg:R5:1} \\
\text{UInt:R0:0} \\
\text{Reg:R6:2} \\
\text{Adr:R2:0} \\
\text{Adr:R3:0} \\
\text{Reg:R9:1} \\
\text{Reg:R8:2} \\
\text{Void:R10:4} \\
\end{align*} \]
Note that once we know the production used to generate the root of the tree, we know the productions used to generate each subtree too:

```
int32 = Void:R10:4

UInt:R0:0

int32 = Reg:R8:2

Reg:R9:1 * s13

Reg:R3:0 + Adr

Reg:R1:0

r

s13

Imm:R4:0

Imm:R4:0

Reg:R8:2

s13
```
We generate code by doing a depth-first traversal, generating code for a production after all the production’s children have been processed.

We need to do register allocation too; for our example, a simple on-the-fly generator will suffice.

```
l          l%fp+b, %10
s         %10, 1, %10
b         %hi(a), %g1
st        %10, [%g1+%lo(a)]
```
Had we translated a slightly difference expression,

\[ a = b - 1000000; \]

we would automatically get a different code sequence (because 1000000 is an int32 rather than an s13):

\[
\begin{align*}
    & \text{ld} \quad [%fp+b],%l0 \\
    & \text{sethi} \quad %hi(1000000),%g1 \\
    & \text{or} \quad %g1,%lo(1000000),%l1 \\
    & \text{sub} \quad %l0,%l1,%l0 \\
    & \text{sethi} \quad %hi(a),%g1 \\
    & \text{st} \quad %l0, [%g1+%lo(a)]
\end{align*}
\]
Adding New Rules

Since instruction selectors can be automatically generated, it’s easy to add “extra” rules that handle optimizations or special cases.

For example, we might add the following to handle addition of a left immediate operand or subtraction of 0 from a register:

<table>
<thead>
<tr>
<th>Rule #</th>
<th>Production</th>
<th>Cost</th>
<th>SPARC Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>R11</td>
<td>Reg → + Imm Reg</td>
<td>1</td>
<td>add Reg, Imm, Reg</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R12</td>
<td>Reg → − Reg Zero</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Improving the Speed of Instruction Selection

As we have presented it, instruction selection looks rather slow—for each node in the IR tree, we must match productions, compare costs, and select least-cost productions. Since compilers routinely generate program with tens or hundreds of thousands of instructions, doing a lot of computation to select one instruction (even if it’s the best instruction) could be too slow. Fortunately, this need not be the case. Instruction selection using BURS can be made very fast.
Adding States to B U R G

We can precompute a set of states that represent possible labelings on IR tree nodes. A table of node names and subtree states then is used to select a node’s state. Thus labeling becomes nothing more than repeated table lookup.

For example, we might create a state s0 that corresponds to the labeling \{Reg:R1:0, Adr:R2:0\}.

A state selection function, \textit{label}, defines \(\text{label}(r) = s0\). That is, whenever \(r\) is matched as a leaf, it is to be labeled with \(s0\).

If a node is an operator, \textit{label} uses the name of the operator and the labeling
assigned to its children to choose the operator’s label. For example,
\[ \text{label}(+, s_0, s_1) = s_2 \]
says that a + with children labeled as \( s_0 \) and \( s_1 \) is to be labeled as \( s_2 \).

In theory, that’s all there is to building a fast instruction selector.

We generate possible labelings, encode them as states, and table all combinations of labelings.

But,

how do we know the set of possible labelings is even finite?

In fact, it isn’t!
Normalizing Costs

It is possible to generate states that are identical except for their costs. For example, we might have:

\[ s_1 = \{\text{Reg:R1:0, Adr:R2:0}\}, \]
\[ s_2 = \{\text{Reg:R1:1, Adr:R2:1}\}, \]
\[ s_3 = \{\text{Reg:R1:2, Adr:R2:2}\}, \text{ etc.} \]

Here an important insight is needed—the absolute costs included in states aren’t really essential. Rather, relative costs are what is important. In \( s_1, s_2, \) and \( s_3 \), Reg and Adr have the same cost. Hence the same decision in choosing between Reg and Adr will be made in all three states.
We can limit the number of states needed by normalizing costs within states so that the lowest cost choice is always 0, and other costs are differences (deltas) from the lowest cost choice.

This observation keeps costs bounded within states (except for pathologic cases).

Using additional techniques to further reduce the number of states needed, and the time needed to generate them, fast and compact BURS instruction selectors are achievable. See

“Simple and Efficient BURS Table Generation,” T. Proebsting, 1992 PLDI Conference.
## Example

<table>
<thead>
<tr>
<th>State</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_0)</td>
<td>{Reg:R1:0, Adr:R2:0}</td>
</tr>
<tr>
<td>(s_1)</td>
<td>{Imm:R4:0, Reg:R5:1}</td>
</tr>
<tr>
<td>(s_2)</td>
<td>{adr:R3:0}</td>
</tr>
<tr>
<td>(s_3)</td>
<td>{Reg:R9:0}</td>
</tr>
<tr>
<td>(s_4)</td>
<td>{UInt:R0:0}</td>
</tr>
<tr>
<td>(s_5)</td>
<td>{Reg:R8:0}</td>
</tr>
<tr>
<td>(s_6)</td>
<td>{Void:R10:0}</td>
</tr>
<tr>
<td>(s_7)</td>
<td>{Reg:R7:0}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>Left Child</th>
<th>Right Child</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r)</td>
<td></td>
<td></td>
<td>(s_0)</td>
</tr>
<tr>
<td>(s_{13})</td>
<td></td>
<td></td>
<td>(s_1)</td>
</tr>
<tr>
<td>int32</td>
<td></td>
<td></td>
<td>(s_4)</td>
</tr>
<tr>
<td>+</td>
<td>(s_0)</td>
<td>(s_1)</td>
<td>(s_2)</td>
</tr>
<tr>
<td>*</td>
<td>(s_2)</td>
<td></td>
<td>(s_3)</td>
</tr>
<tr>
<td>-</td>
<td>(s_3)</td>
<td>(s_1)</td>
<td>(s_5)</td>
</tr>
<tr>
<td>-</td>
<td>(s_1)</td>
<td>(s_3)</td>
<td>(s_7)</td>
</tr>
<tr>
<td>=</td>
<td>(s_4)</td>
<td>(s_5)</td>
<td>(s_6)</td>
</tr>
</tbody>
</table>
We start by looking up the state assigned to each leaf. We then work upward, choosing the state of a parent based on the parent’s kind and the states assigned to the children. These are all table lookups, and hence very fast.

At the root, we select the nonterminal and production based on the state assigned to the root (any entry with 0 cost). Knowing the production used at the root tells us the nonterminal used at each child. Each state has only one entry per nonterminal, so knowing a node’s state and the nonterminal used to generate it immediately tells us the production used. Hence identifying the production used for each node is again very fast.
Step 1 (Label leaves with states):

\[
\begin{align*}
\text{int32} &= s^4 \\
\* &= s^{13} \\
+ &= s^0 \\
\times &= s^{13} \\
- &= s^1 \\
- &= s^5 \\
\end{align*}
\]

Step 2 (Propagate states upward):

\[
\begin{align*}
\text{int32} &= s^{13} \\
\* &= s^0 \\
\times &= s^{13} \\
- &= s^5 \\
- &= s^1 \\
\end{align*}
\]
Step 3 (Choose production used at root): R10.

Step 4 (Propagate productions used downward to children):

```
= R10
 R0, int32
    - R8
      R9 * s13 R4
        R3 + s13 R4
          R1 r s13 R4
```
Code Generation for x86 Machines

The x86 presents several special difficulties when generating code.

- There are only 8 architecturally visible registers, and only 6 of these are allocatable. Deciding what values to keep in registers, and for how long, is a difficult, but crucial, decision.

- Operands may be addressed directly from memory in some instructions. Such instructions avoid using a register, but are longer and add to I-cache pressure.
In “Optimal Spilling for CISC Machines with Few Registers,” Appel and George address both of these difficulties.

They use Integer Programming techniques to directly and optimally solve the crucial problem of deciding which live ranges are to be register-resident at each program point. Stores and loads are automatically added to split long live ranges.

Then a variant of Chaitin-style register allocation is used to assign registers to live ranges chosen to be register-resident.

The presentation of this paper, at the 2001 PLDI Conference, is at

www.cs.wisc.edu/~fischer/cs701/cisc.spilling.pdf
Optimistic Coalescing

Given R allocatable registers, Appel and George guarantee that no more than R live ranges are marked as register resident.

This doesn’t always guarantee that an R coloring is possible.

Consider the following program fragment:

```plaintext
x=0;
while (...) {
    y = x+1;
    print(x);
    z = y+1;
    print(y);
    x = z+1;
    print(z);
}
```
At any given point in the loop body only 2 variables are live, but 3 registers are needed (\(x\) interferes with \(y\), \(y\) interferes with \(z\) and \(z\) interferes with \(x\)).

We know that we have enough registers to handle all live ranges marked as register-resident, but we may need to “shuffle” register allocations at certain points.

Thus at one point \(x\) might be allocated R1 and at some other point it might be placed in R2. Such shuffling implies register to register copies, so we’d like to minimize their added cost.
Appel and George suggest allowing changes in register assignments between program points. This is done by creating multiple variable names for a live range \((x_1, x_2, x_3, \ldots)\), one for each program point. Variables are connected by assignments between points. Using coalescing, it is expected that most of the assignments will be optimized away.

Using our earlier example, we have the following code with each variable expanded into 3 segments (one for each assignment). Copies of dead variables are removed to simplify the example:
\texttt{x_3 = 0;}
\texttt{while (...) {}
\texttt{x_1 = x_3;}
\texttt{y_1 = x_1 + 1;}
\texttt{print(x_1);}
\texttt{y_2 = y_1;}
\texttt{z_2 = y_2 + 1;}
\texttt{print(y_2);}
\texttt{z_3 = z_2;}
\texttt{x_3 = z_3 + 1;}
\texttt{print(z_3);}
\texttt{}}
\textbf{Now a 2 coloring is possible:}
\texttt{x_1 : R1, y_1 : R2}
\texttt{z_2 : R1, y_2 : R2}
\texttt{z_3 : R1, x_3 : R2}
\texttt{(and only x_1 = x_3 is retained).}
Appel and George found that iterated coalescing wasn’t effective (too many copies, most of which are useless). Instead they recommend Optimistic Coalescing. The idea is to first do Chaitin-style reckless coalescing of all copies, even if colorability is impaired. Then we do graph coloring register allocation, using the cost of copies as the “spill cost.” As we select colors, a coalesced node that can’t be colored is simply split back to the original source and target variables. Since we always limit the number of live ranges to the number of colors, we know the live ranges must be colorable (with register to register copies sometimes needed).
Using our earlier example, we initially merge $x_1$ and $x_3$, $y_1$ and $y_2$, $z_2$ and $z_3$. We already know this can’t be colored with two registers. All three pairs have the same costs, so we arbitrarily stack $x_1 - x_3$, then $y_1 - y_2$ and finally $z_2 - z_3$.

When we unstack, $z_2 - z_3$ gets R1, and $y_1 - y_2$ gets R2. $x_1 - x_3$ must be split back into $x_1$ and $x_3$. $x_1$ interferes with $y_1 - y_2$ so it gets R1. $x_3$ interferes with $z_2 - z_3$ so it gets R2, and coloring is done.

$x_1$: R1, $y_1$: R2
$z_2$: R1, $y_2$: R2
$z_3$: R1, $x_3$: R2