Normalizing Costs

It is possible to generate states that are identical except for their costs. For example, we might have
\[ s_1 = \{ \text{Reg:R1:0, Adr:R2:0} \}, \]
\[ s_2 = \{ \text{Reg:R1:1, Adr:R2:1} \}, \]
\[ s_3 = \{ \text{Reg:R1:2, Adr:R2:2} \}, \]

Here an important insight is needed—the absolute costs included in states aren't really essential. Rather relative costs are what is important. In \( s_1, s_2, \) and \( s_3, \) Reg and Adr have the same cost. Hence the same decision in choosing between Reg and Adr will be made in all three states.

Example

<table>
<thead>
<tr>
<th>State</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_0 )</td>
<td>{ \text{Reg:R1:0, Adr:R2:0} }</td>
</tr>
<tr>
<td>( s_1 )</td>
<td>{ \text{Imm:R4:0, Reg:R5:1} }</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>{ \text{adr:R3:0} }</td>
</tr>
<tr>
<td>( s_3 )</td>
<td>{ \text{Reg:R9:0} }</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>{ \text{UInt:R0:0} }</td>
</tr>
<tr>
<td>( s_5 )</td>
<td>{ \text{Reg:R8:0} }</td>
</tr>
<tr>
<td>( s_6 )</td>
<td>{ \text{Void:R10:0} }</td>
</tr>
<tr>
<td>( s_7 )</td>
<td>{ \text{Reg:R7:0} }</td>
</tr>
</tbody>
</table>

Node | Left Child | Right Child | Result |
--- | --- | --- | --- |
\( r \) | | | \( s_0 \) |
\( s_{13} \) | | | \( s_1 \) |
\( \text{int32} \) | | | \( s_4 \) |
\( + \) | \( s_0 \) | \( s_1 \) | \( s_2 \) |
\( * \) | \( s_2 \) | | \( s_3 \) |
\( - \) | \( s_3 \) | \( s_1 \) | \( s_5 \) |
\( \text{=} \) | \( s_1 \) | \( s_3 \) | \( s_7 \) |
| \( = \) | \( s_4 \) | \( s_5 \) | \( s_6 \) |
Step 1 (Label leaves with states):

\[ \begin{array}{c}
\text{s4} \\
\text{int32} \\
\frac{}{*} \\
\frac{}{+} \\
\frac{}{r} \\
\frac{}{s13} \\
\frac{}{s1} \\
\end{array} \]

Step 2 (Propagate states upward):

\[ \begin{array}{c}
\text{s6} \\
\text{int32} \\
\frac{}{*} \\
\frac{}{+} \\
\frac{}{r} \\
\frac{}{s13} \\
\frac{}{s1} \\
\end{array} \]

Step 3 (Choose production used at root): R10.

Step 4 (Propagate productions used downward to children):

\[ \begin{array}{c}
\text{R10} \\
\text{int32} \\
\frac{}{*} \\
\frac{}{+} \\
\frac{}{r} \\
\frac{}{s13} \\
\frac{}{s1} \\
\end{array} \]

Code Generation for x86 Machines

The x86 presents several special difficulties when generating code.

- There are only 8 architecturally visible registers, and only 6 of these are allocatable. Deciding what values to keep in registers, and for how long, is a difficult, but crucial, decision.

- Operands may be addressed directly from memory in some instructions. Such instructions avoid using a register, but are longer and add to I-cache pressure.

In “Optimal Spilling for CISC Machines with Few Registers,” Appel and George address both of these difficulties.

They use Integer Programming techniques to directly and optimally solve the crucial problem of deciding which live ranges are to be register-resident at each program point. Stores and loads are automatically added to split long live ranges.

Then a variant of Chaitin-style register allocation is used to assign registers to live ranges chosen to be register-resident.

The presentation of this paper, at the 2001 PLDI Conference, is at

www.cs.wisc.edu/~fischer/cs701/cisc.spilling.pdf
Optimistic Coalescing

Given R allocatable registers, Appel and George guarantee that no more than R live ranges are marked as register resident.
This doesn't always guarantee that an R coloring is possible.
Consider the following program fragment:
```c
x = 0;
while (...) {
    y = x + 1;
    print(x);
    z = y + 1;
    print(y);
    x = z + 1;
    print(z);
}
```

At any given point in the loop body only 2 variables are live, but 3 registers are needed (x interferes with y, y interferes with z and z interferes with x).
We know that we have enough registers to handle all live ranges marked as register-resident, but we may need to “shuffle” register allocations at certain points.
Thus at one point x might be allocated R1 and at some other point it might be placed in R2. Such shuffling implies register to register copies, so we'd like to minimize their added cost.

Appel and George suggest allowing changes in register assignments between program points. This is done by creating multiple variable names for a live range (x₁, x₂, x₃, ...), one for each program point. Variables are connected by assignments between points. Using coalescing, it is expected that most of the assignments will be optimized away.

Using our earlier example, we have the following code with each variable expanded into 3 segments (one for each assignment). Copies of dead variables are removed to simplify the example:
```c
x₃ = 0;
while (...) {
    x₁ = x₃;
    y₁ = x₁ + 1;
    print(x₁);
    y₂ = y₁;
    z₂ = y₂ + 1;
    print(y₂);
    x₃ = z₂;
    print(x₃);
}
```

Now a 2 coloring is possible:
- x₁: R1, y₁: R2
- x₂: R1, y₂: R2
- x₃: R1, x₃: R2
(and only x₁ = x₃ is retained).
Appel and George found that iterated coalescing wasn’t effective (too many copies, most of which are useless). Instead they recommend Optimistic Coalescing. The idea is to first do Chaitin-style reckless coalescing of all copies, even if colorability is impaired.

Then we do graph coloring register allocation, using the cost of copies as the “spill cost.” As we select colors, a coalesced node that can’t be colored is simply split back to the original source and target variables. Since we always limit the number of live ranges to the number of colors, we know the live ranges must be colorable (with register to register copies sometimes needed).

Using our earlier example, we initially merge \( x_1 \) and \( x_3 \), \( y_1 \) and \( y_2 \), \( z_2 \) and \( z_3 \). We already know this can’t be colored with two registers. All three pairs have the same costs, so we arbitrarily stack \( x_1 \rightarrow x_3 \), then \( y_1 \rightarrow y_2 \) and finally \( z_2 \rightarrow z_3 \).

When we unstack, \( z_2 \rightarrow z_3 \) gets R1, and \( y_1 \rightarrow y_2 \) gets R2. \( x_1 \rightarrow x_3 \) must be split back into \( x_1 \) and \( x_3 \). \( x_1 \) interferes with \( y_1 \rightarrow y_2 \) so it gets R1. \( x_3 \) interferes with \( z_2 \rightarrow z_3 \) so it gets R2, and coloring is done.

\[ x_1: R1, \quad y_1: R2 \]
\[ z_2: R1, \quad y_2: R2 \]
\[ z_3: R1, \quad x_3: R2 \]

**Reading Assignment**

- Read pages 1-30 of “Automatic Program Optimization,” by Ron Cytron.
  (Linked from the class Web page.)

**Data Flow Frameworks**

- Data Flow Graph:
  Nodes of the graph are basic blocks or individual instructions.
  Arcs represent flow of control.

  **Forward Analysis:**
  Information flow is the same direction as control flow.

  **Backward Analysis:**
  Information flow is the opposite direction as control flow.

  **Bi-directional Analysis:**
  Information flow is in both directions. (Not too common.)
• Meet Lattice
  Represents solution space for the data flow analysis.

• Meet operation
  (And, Or, Union, Intersection, etc.)
  Combines solutions from predecessors or successors in the control flow graph.

- Transfer Function
  Maps a solution at the top of a node to a solution at the end of the node (forward flow) or Maps a solution at the end of a node to a solution at the top of the node (backward flow).

Example: Available Expressions
This data flow analysis determines whether an expression that has been previously computed may be reused.

Available expression analysis is a forward flow problem—computed expression values flow forward to points of possible reuse.

The best solution is True—the expression may be reused.

The worst solution is False—the expression may not be reused.

The Meet Lattice is:

\[ T \] (Expression is Available)
\[ F \] (Expression is Not Available)

As initial values, at the top of the start node, nothing is available. Hence, for a given expression, \( \text{AvailIn}(b_0) = F \).

We choose an expression, and consider all the variables that contribute to its evaluation. Thus for \( e_1 = a + b - c \), \( a, b \) and \( c \) are \( e_1 \)'s operands.
The transfer function for $e_1$ in block $b$ is defined as:
If $e_1$ is computed in $b$ after any assignments to $e_1$'s operands in $b$
Then $\text{AvailOut}(b) = T$
Elsif any of $e_1$'s operands are changed after the last computation of $e_1$ or $e_1$'s operands are changed without any computation of $e_1$
Then $\text{AvailOut}(b) = F$
Else $\text{AvailOut}(b) = \text{AvailIn}(b)$

The meet operation (to combine solutions) is:

$$\text{AvailIn}(b) = \text{AND} \sum_{p \in \text{Pred}(b)} \text{AvailOut}(p)$$