Optimistic Coalescing

Given R allocatable registers, Appel and George guarantee that no more than R live ranges are marked as register resident.

This doesn't always guarantee that an R coloring is possible.

Consider the following program fragment:

```c
x=0;
while (...) {
    y = x+1;
    print(x);
    z = y+1;
    print(y);
    x = z+1;
    print(z);
}
```

At any given point in the loop body only 2 variables are live, but 3 registers are needed (x interferes with y, y interferes with z and z interferes with x).

We know that we have enough registers to handle all live ranges marked as register-resident, but we may need to "shuffle" register allocations at certain points.

Thus at one point x might be allocated R1 and at some other point it might be placed in R2. Such shuffling implies register to register copies, so we'd like to minimize their added cost.

Appel and George suggest allowing changes in register assignments between program points. This is done by creating multiple variable names for a live range (x₁, x₂, x₃, ...), one for each program point. Variables are connected by assignments between points. Using coalescing, it is expected that most of the assignments will be optimized away.

Using our earlier example, we have the following code with each variable expanded into 3 segments (one for each assignment). Copies of dead variables are removed to simplify the example:
Appel and George found that iterated coalescing wasn't effective (too many copies, most of which are useless).

Instead they recommend Optimistic Coalescing. The idea is to first do Chaitin-style reckless coalescing of all copies, even if colorability is impaired. Then we do graph coloring register allocation, using the cost of copies as the "spill cost." As we select colors, a coalesced node that can't be colored is simply split back to the original source and target variables. Since we always limit the number of live ranges to the number of colors, we know the live ranges must be colorable (with register to register copies sometimes needed).

Using our earlier example, we initially merge \( x_1 \) and \( x_3 \), \( y_1 \) and \( y_2 \), \( z_2 \) and \( z_3 \). We already know this can't be colored with two registers. All three pairs have the same costs, so we arbitrarily stack \( x_1 \cdot x_3 \), then \( y_1 \cdot y_2 \) and finally \( z_2 \cdot z_3 \).

When we unstack, \( z_2 \cdot z_3 \) gets R1, and \( y_1 \cdot y_2 \) gets R2. \( x_1 \cdot x_3 \) must be split back into \( x_1 \) and \( x_3 \). \( x_1 \) interferes with \( y_1 \cdot y_2 \) so it gets R1. \( x_3 \) interferes with \( z_2 \cdot z_3 \) so it gets R2, and coloring is done.

\[ x_1: R1, \ y_1: R2 \]
\[ z_2: R1, \ y_2: R2 \]
\[ z_3: R1, \ x_3: R2 \]

Data Flow Frameworks

- Data Flow Graph:
  Nodes of the graph are basic blocks or individual instructions.
  Arcs represent flow of control.

Forward Analysis:
  Information flow is the same direction as control flow.

Backward Analysis:
  Information flow is the opposite direction as control flow.

Bi-directional Analysis:
  Information flow is in both directions. (Not too common.)
Meet Lattice
Represents solution space for the data flow analysis.

Meet operation
(And, Or, Union, Intersection, etc.)
Combines solutions from predecessors or successors in the control flow graph.

Transfer Function
Maps a solution at the top of a node to a solution at the end of the node (forward flow)
or Maps a solution at the end of a node to a solution at the top of the node (backward flow).

Example: Available Expressions
This data flow analysis determines whether an expression that has been previously computed may be reused.

Available expression analysis is a forward flow problem—computed expression values flow forward to points of possible reuse.

The best solution is True—the expression may be reused.

The worst solution is False—the expression may not be reused.

The Meet Lattice is:
- T (Expression is Available)
- F (Expression is Not Available)

As initial values, at the top of the start node, nothing is available. Hence, for a given expression, AvailIn(b0) = F

We choose an expression, and consider all the variables that contribute to its evaluation. Thus for \( e_1 = a + b - c \), a, b and c are \( e_1 \)'s operands.
The transfer function for \( e_1 \) in block \( b \) is defined as:

If \( e_1 \) is computed in \( b \) after any assignments to \( e_1 \)'s operands in \( b \) Then \( \text{AvailOut}(b) = T \)
Elsif any of \( e_1 \)'s operands are changed after the last computation of \( e_1 \) or \( e_1 \)'s operands are changed without any computation of \( e_1 \) Then \( \text{AvailOut}(b) = F \)
Else \( \text{AvailOut}(b) = \text{AvailIn}(b) \)

The meet operation (to combine solutions) is:

\[
\text{AvailIn}(b) = \text{AND} \left( \text{AvailOut}(p) \mid p \in \text{Pred}(b) \right)
\]

**Example:** \( e_1 = v+w \)

Circularities Require Care

Since data flow values can depend on themselves (because of loops), care is required in assigning initial “guesses” to unknown values. Consider

If the flow value on the loop backedge is initially set to false, it can never become true. (Why?) Instead we should use True, the identity for the AND operation.