Reading Assignment

• Read “Minimum Cost Interprocedural Register Allocation,” by S. Kurlander et al. (linked from class Web page).
Call Graphs

A Call Graph represents the calling structure of a program.

- Nodes are subprograms (procedures and functions).
- Arcs represent calls between subprograms. An arc from A to B denotes that a call to B appears within A.
- For an indirect call (a function parameter or a function pointer) an arc is added to all potential callees.
Example

```c
main() {
    if (pred(a,b))
        subr1(a)
    else subr2(b);
}

bool pred(int a, int b){
    return a==b; }

subr1(int a){ print(a);}

subr2(int x){ print(2*x);}
```
Wall’s Interprocedural Register Allocator

Operates in two phases:

1. Register candidates are identified at the subprogram level. Each candidate (a single variable or a set of non-interfering live ranges) is compiled as if it won’t get a register. At link-time unnecessary loads and stores are edited away if the candidate is allocated a register.

2. At link-time, when all subprograms are known and available, register candidates are allocated registers.
Identifying Interprocedural Register Sharing

If two subprograms are not connected in the call graph, a register candidate in each can share the same register without any saving or restoring across calls.

A register candidate from `pred`, `subr1` and `subr2` can all share one register.
At the interprocedural level we must answer 2 questions:

1. A local candidate of one subprogram can share a register with candidates of what other subprograms?

2. Which local register candidates will yield the greatest benefit if given a register?

Wall designed his allocator for a machine with 52 registers. This is enough to divide all the registers among the subprograms without any saves or restores at call sites.

With fewer registers, spills, saves and restores will often be needed (if registers are used aggressively within a subprogram).
Restrictions on the Call Graph

Wall limited calls graphs to DAGs since cycles in a call graph imply recursion, which will force saves and restores (why?)

Cost Computations

Each register candidate is given a per-call cost, based on the number of saves and restores that can be removed, scaled by $10^{\text{loop\_depth}}$. This benefit is then multiplied by the expected number of calls, obtained by summing the total number of call sites, scaled by loop nesting depth.
**Grouping Register Candidates**

We now have an estimate of the benefit of allocating a register to a candidate. Call this estimate $\text{cost(candidate)}$.

We estimate potential interprocedural sharing of register candidates by assigning each candidate to a *Group*. All candidates within a group can share a register. No two candidates in any subprogram are in the same group.
Groups are assigned during a reverse depth-first traversal of the call graph.

\texttt{AsgGroup(node n) \{ }
\texttt{\hspace{1cm} if (n is a leaf node)}
\texttt{\hspace{1.5cm} grp = 0}
\texttt{\hspace{1cm} else \{ for (each }c \in \text{ children(n)) \{ }
\texttt{\hspace{2cm} AsgGroup(c) \}}
\texttt{\hspace{1cm} grp = 1 + \text{Max} (\text{Max group used in } c)}
\texttt{\hspace{2cm} c \in \text{ children(n)} \}
\texttt{\hspace{1cm} \}}
\texttt{\hspace{1cm} for (each }r \in \text{ registerCandidates(n)) \{ }
\texttt{\hspace{2cm} assign } r \text{ to } grp
\texttt{\hspace{2cm} grp++ \}}
\texttt{\}}
\texttt{\}}
\texttt{Global variables are assigned to a singleton group.}
Example

At Print: $\text{grp}(i)=0$, $\text{grp}(j)=1$

At subr1: Max grp used in print is 1
$\text{grp}(x)=2$, $\text{grp}(y)=3$

At subr2: Max grp used in print is 1
$\text{grp}(t)=2$

At main: Max grp used in children is 3
$\text{grp}(a)=4$, $\text{grp}(b)=5$, $\text{grp}(c)=6$
If A calls B (directly or indirectly), then none of A's register candidates are in the same group as any of B's register candidates.

This guarantees that A and B will use different registers.

Thus no saves or restores are needed across a call from A to B.
Assigning Registers to Groups

\[ \text{Cost(group)} = \sum_{\text{candidates} \in \text{group}} \text{cost(candidates)} \]

We assign registers to groups based on the cost of each group, using an "auction."

\[
\text{for (r=0; r < RegisterCount; r++) } \{
\text{Let G be the group with the greatest cost that has not yet been assigned a register.}
\text{Assign r to G}
\}\n\]
Example (3 Registers)

<table>
<thead>
<tr>
<th>Group</th>
<th>Members</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>i</td>
<td>40</td>
</tr>
<tr>
<td>1</td>
<td>j</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>x, t</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>y</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>a</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>b</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>c</td>
<td>30</td>
</tr>
</tbody>
</table>
The 3 registers are given to the groups with the highest weight, i (40), c(30), a(20).

Is this optimal?

No! If y and t are grouped together, y and t (cost=25) get the last register.
Recursion

To handle recursion, any call to a subprogram “up” in the call graph must save and restore all registers possibly in use between the caller and callee.

A call from E to B saves r3 to r5.
**Performance**

Wall found interprocedural register allocation to be very effective (given 52 Registers!).

Speedups of 10–28% were reported. Even with only 8 registers, speedups of 5–20% were observed.
Optimal Interprocedural Register Allocation

Wall's approach to interprocedural register allocation isn't optimal because register candidates aren't grouped to achieve maximum benefit.

Moreover, the placement of save and restore code *if needed* isn't considered.

These limitations are addressed by Kurlander in “Minimum Cost Interprocedural Register Allocation.”
Optimal Save-Free Interprocedural Register Allocation

- Allocation is done on a cycle-free call graph.
- Each subprogram has one or more register candidates, $c_i$.
- Each register candidate, $c_i$, has a cost (or benefit), $w_i$, that is the improvement in performance if $c_i$ is given a register. (This $w_i$ value is scaled to include nested loops and expected call frequencies.)
INTERFERENCE BETWEEN REGISTER CANDIDATES

The notion of interference is extended to include interprocedural register candidates:

• Two Candidates in the same subprogram always interfere. (Local non-interfering variables and values have already been grouped into interprocedural register candidates.)

• If subprogram P calls subprogram Q (directly or indirectly) then register candidates within P always interfere with register candidates within Q.
Example

The algorithm can group candidate \( p \) with either \( t \) or \( u \) (since they don't interfere). It can also group candidate \( q \) with either \( t \) or \( u \).

If two registers are available, it must "discover" that assigning \( R1 \) to \( q \& t \), and \( R2 \) to \( m \) is optimal.
Non-interfering register candidates are grouped into registers so as to solve:

\[
\text{Maximize } \sum_{j} w_j \\
\text{subject to } c_j \in \bigcup_{i=1}^{k} R_i
\]

That is, we wish to group sets of non-interfering register candidates into \( k \) registers such that the overall benefit is maximized.

But how do we solve this?
Certainly examining all possible groupings will be prohibitively expensive!
Kurlander solved this problem by mapping it to a known problem in Integer Programming: the Dual Network Flow Problem.

Solution techniques for this problem are well known—libraries of standard solution algorithms exist.

Moreover, this problem can be solved in polynomial time.

That is, it is "easier" than optimal global (intraprocedural) register allocation, which is NP-complete!
Adding Saves & Restores

Wall designed his save–free interprocedural allocator for a machine with 52 registers.

Most computers have far fewer registers, and hence saving and restoring across calls, when profitable, should be allowed.

Kurlander's Technique can be extended to include save/restore costs. If the cost of saving and restoring is less than the benefit of allocating an extra register, saving is done. Moreover, saving is done where it is cheapest (not closest!).
**Example**

```c
main() { ... p(); ...}

p(){ ...
    for (i=0; i<1000000; i++){
        q():
    }
}
```

We first allocate registers in a save-free mode. After all Registers have been allocated, `q` may need additional registers.

Most allocators would add save/restore code at `q`'s call site (or `q`'s prologue and epilogue).

An optimal allocator will place save/restore code at `p`'s call site, freeing a register that `p` doesn't even want (but that `q` does want!)
Extending the Cost Model

- As before, we group register candidates of different subprograms into registers.
- Now only candidates within the same subprogram automatically interfere.
- Saves are placed on the edges of the call graph.
- We aim to solve

\[
\text{Maximize } \sum_{k} w_j - \sum_{e_m \in \text{Edges}} S_m * \text{Saved}_m
\]

where \( S_m \) is the per/register save/restore cost and \( \text{Saved}_m \) is the number of registers saved on edge \( e_m \).
• As registers are saved, they may be reused in child subprograms.

• This optimization problem can be solved as a Network Dual Flow Problem.

• Again, the solution algorithm is polynomial.
Example (One Register)

P₁ gets R₁ save-free for m.
A save on P₁ → P₄ costs 1 and gives a register to n (net profit = 2), so we do it.
A save on P₁ → P₂ for z costs 2, and yields 1, which isn't profitable.
A save on P₂ → P₃ for q costs 4, and yields 3, which isn't profitable.
A save on P₁ → P₂ for q costs 2, and yields 3, which is a net gain.
Handling Global Variables

- Wall's technique handled globals by assuming they interfere with all subprograms and all other globals.
- Kurlander's approach is incremental (and non-optimal):
  
  First, an optimal allocation for \( r \) registers is computed.

  Next, one register is "stolen" and assigned interprocedurally to the most beneficial global.
  (Subprograms that don't use the global can save and restore it locally, allowing local reuse).

  An optimal allocation using \( R-1 \) registers is computed. If this solution plus the shared global is more profitable than the \( R \) register
solution, the global allocation is "locked in."

Next, another register is "stolen" for a global, leaving R-2 for interprocedural allocation.

This process continues until stealing another register for a global isn't profitable.
Why is Optimal
Interprocedural Register Allocation Easier than Optimal IntraProcedural Allocation?

This result seems counter-intuitive. How can allocating a whole program be easier (computationally) than allocating only one subprogram.

Two observations provide the answer:

- Interprocedural allocation assumes some form of local allocation has occurred (to identify register candidates).
- Interprocedural interference is transitive (if A interferes with B and B interferes with C then A interferes with B). But intraprocedural interference isn’t transitive!