

# **CS 701**

**Charles N. Fischer**

**Fall 2014**

**<http://www.cs.wisc.edu/~fischer/cs701.html>**

# **In Memoriam**



**Susan B. Horwitz**

**1955 - 2014**

# **Class Meets**

**Mondays and Wednesdays,  
11:00 — 12:15**

**2540 Engineering Hall**

# **Instructor**

**Charles N. Fischer**

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**Office Hours:**

**10:30 - Noon, Tuesdays and  
Thursdays, or by appointment**

# Key Dates

- **September 22: Project 1 due**
- **October 15: Project 2 due**
- **November 5: Project 3 due**
- **November 14,  
5:30-7:30: Midterm**
- **December 12: Project 4 due**
- **December 14: Final Exam, time to be  
determined**

# **Class Text**

- **Crafting a Compiler**  
**Fischer, Cytron, LeBlanc**  
**ISBN-10: 0136067050**  
**ISBN-13: 9780136067054**  
**Publisher: Addison-Wesley**
- **Handouts and Web-based reading will also be used.**

# **Reading Assignment**

- **Section 14.1 - 14.2.2 of CaC**
- **Pages 1 - 30 of “Automatic Program Optimization”**
- **Assignment 1**

# Class Notes

- **The lecture notes used in each lecture will be made available prior to that lecture on the class Web page (under the “Lecture Nodes” link).**

# Piazza

**Piazza is an interactive online platform used to share class-related information. We recommend you use it to ask questions and track course-related information. If you are enrolled (or on the waiting list) you should have already received an email invitation to participate (about one week ago).**

# Instructional Computers

**We have access to departmental 64 bit Linux boxes (macaroni-01 to macaroni-09) for general class-related computing. These machines have access to LLVM 3.3 at**

**`/unsup/llvm-3.3`**

**If you have access to a Linux box in your office connected to AFS, it will probably work fine for class projects.**

# **CS701 Projects**

- 1. Introduction to LLVM and Simple Local Optimization**
- 2. Dataflow Analysis and Optimization**
- 3. Natural Loops and Loop-Invariant Code Motion**
- 4. Graph Coloring Register Allocation**

# Academic Misconduct Policy

- You must do your assignments—no copying or sharing of solutions.
- You may discuss general concepts and Ideas, especially on Piazza.
- All cases of Misconduct *must* be reported.
- Penalties may be **severe**.

# Partnership Policy

**Projects may be done individually  
*or* by two person teams (your  
choice).**

# Guest Lecturers

- 1. Tom Reps**
- 2. Somesh Jha**
- 3. Ben Liblit**

# Overview of Course Topics

## 1. Register Allocation

### Local Allocation

**Avoid unnecessary loads and stores within a *basic block*. Remember and reuse register contents.**

**Consider effects of *aliasing*.**

### Global Allocation

**Allocate registers within a single subprogram. Choose “most profitable” values. Map several values to the *same* register.**

### Interprocedural Allocation

**Avoid saves and restores across calls. Share globals in registers.**

## 2. Code Scheduling

**We can reorder code to reduce latencies and to maximize ILP (*Instruction Level Parallelism*). We must respect *data dependencies* and *control dependencies*.**

<b>ld</b> [a],%r1	<b>ld</b> [a],%r1
<b>add</b> %r1,1,%r2	<b>mov</b> 3,%r3
<b>mov</b> 3,%r3	<b>add</b> %r1,1,%r2
<b>(before)</b>	<b>(after)</b>

### **3. Automatic Instruction Selection**

**How do we map an IR (*Intermediate Representation*) into Machine Instructions?**

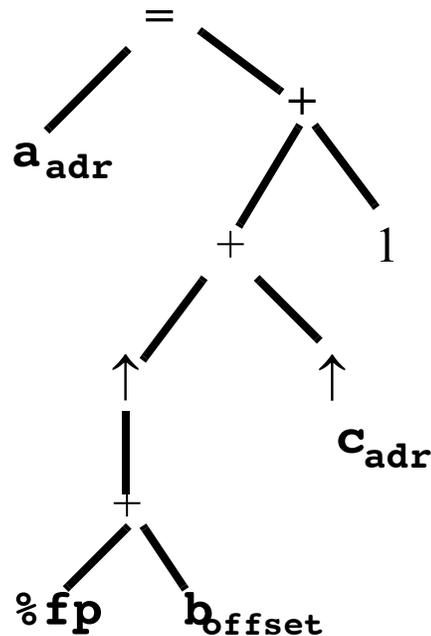
**Can we guarantee the *best* instruction sequence?**

**Idea—Match instruction patterns (represented as trees) against an IR that is a low-level tree. Each match is a generated instruction; the best overall match is the best instruction sequence.**

**Example:**

**a=b+c+1;**

**In IR tree form:**



**Generated code:**

```
ld [%fp+boffset], %r1
```

```
ld [cadr], %r2
```

```
add %r1, %r2, %r3
```

```
add %r3, 1, %r4
```

```
st %r4, [aadr]
```

**Why use four *different* registers?**

## 4. Peephole Optimization

Inspect generated code sequences and replace pairs/triples/tuples with better alternatives.

<code>ld [a],%r1</code>	<code>ld [a],%r1</code>
<code>mov const,%r2</code>	<code>add %r1,const,%r3</code>
<code>add %r1,%r2,%r3</code>	
(before)	(after)

<code>mov 0,%r1</code>	<code>OP %g0,%r2,%r3</code>
<code>OP %r1,%r2,%r3</code>	
(before)	(after)

But why not just generate the better code sequence to begin with?

## 5. Cache Improvements

**We want to access data & instructions from the L1 cache whenever possible; misses into the L2 cache (or memory) are *expensive!***

**We will layout data and program code with consideration of cache sizes and access properties.**

## 6. Local & Global Optimizations

**Identify unneeded or redundant code.**

**Decide where to place code.**

**Worry about debugging issues (how reliable are current values and source line numbers after optimization?)**

## 7. Program representations

- **Control Flow Graphs**
- **Program Dependency Graphs**
- **Static Single Assignment Form (SSA)**

**Each program variable is assigned to in only *one* place.**

**After an assignment  $x_i = y_j$ , the relation  $x_i = y_j$  *always* holds.**

**Example:**

<code>if (a)</code>	<code>if (a)</code>
<code>x = 1</code>	<code>x<sub>1</sub> = 1</code>
<code>else x = 2;</code>	<code>else x<sub>2</sub> = 2;</code>
<code>print(x)</code>	<code>x<sub>3</sub> = <math>\phi(x_1, x_2)</math></code>
	<code>print(x<sub>3</sub>)</code>

## 8. Data Flow Analysis

**Determine invariant properties of subprograms; analysis can be extended to entire programs.**

**Model abstract execution.**

**Prove correctness and efficiency properties of analysis algorithms.**

## 9. Points-To Analysis

**All compiler analyses and optimizations are limited by the potential effects of assignments through pointers and references.**

**Thus in C:**

```
b = 1;
```

```
*p = 0;
```

```
print(b);
```

**is 1 or 0 printed?**

**Similarly, in Java:**

```
a[1] = 1;
```

```
b[1] = 0;
```

```
print(a[1]);
```

**is 1 or 0 printed?**

**Points-to analysis aims to determine what variables or heap objects a pointer or reference may access. Exact analysis is impossible (why?). But fast and reasonably accurate analyses are known.**

# Review of Compiler Optimizations

## 1. Redundant Expression Elimination (Common Subexpression Removal)

Use an address or value that has been previously computed. Consider control and data dependencies.

## 2. Partially Redundant Expression (PRE) Elimination

A variant of Redundant Expression Elimination. If a value or address is redundant along *some* execution paths, add computations to other paths to create a fully redundant expression (which is then removed).

Example:

```
if (i > j)
    a[i] = a[j];
a[i] = a[i] * 2;
```

### **3. Constant Propagation**

**If a variable is known to contain a particular constant value at a particular point in the program, replace references to the variable at that point with the constant value.**

### **4. Copy Propagation**

**After the assignment of one variable to another, a reference to one variable may be replaced with the value of the other variable (until one or the other of the variables is reassigned).**

**(This may also “set up” dead code elimination. Why?)**

## **5. Constant Folding**

**An expression involving constant (literal) values may be evaluated and simplified to a constant result value. Particularly useful when constant propagation is performed.**

## **6. Dead Code Elimination**

**Expressions or statements whose values or effects are unused may be eliminated.**

## **7. Loop Invariant Code Motion**

**An expression that is *invariant* in a loop may be moved to the loop's header, evaluated once, and reused within the loop.**

***Safety and profitability* issues may be involved.**

## **8. Scalarization (Scalar Replacement)**

**A field of a structure or an element of an array that is repeatedly read or written may be copied to a local variable, accessed using the local, and later (if necessary) copied back.**

**This optimization allows the local variable (and in effect the field or array component) to be allocated to a register.**

## **9. Local Register Allocation**

**Within a *basic block* (a straight line sequence of code) track register contents and reuse variables and constants from registers.**

## **10. Global Register Allocation**

**Within a subprogram, frequently accessed variables and constants are allocated to registers. Usually there are *many more* register candidates than available registers.**

## **11. Interprocedural Register Allocation**

**Variables and constants accessed by more than one subprogram are allocated to registers. This can *greatly* reduce call/return overhead.**

## **12. Register Targeting**

**Compute values directly into the intended target register.**

## **13. Interprocedural Code Motion**

**Move instructions across subprogram boundaries.**

## **14. Call Inlining**

**At the site of a call, insert the body of a subprogram, with actual parameters initializing formal parameters.**

## 15. Code Hoisting and Sinking

If the same code sequence appears in two or more alternative execution paths, the code may be *hoisted* to a common ancestor or *sunk* to a common successor. (This reduces code size, but does not reduce instruction count.)

## 16. Loop Unrolling

Replace a loop body executed  $N$  times with an expanded loop body consisting of  $M$  copies of the loop body. This expanded loop body is executed  $N/M$  times, reducing loop overhead and increasing optimization possibilities within the expanded loop body.

## 17. Software Pipelining

**A value needed in iteration  $i$  of a loop is computed during iteration  $i-1$  (or  $i-2, \dots$ ). This allows long latency operations (floating point divides and square roots, low hit-ratio loads) to execute in parallel with other operations. Software pipelining is sometimes called *symbolic loop unrolling*.**

## 18. Strength Reduction

**Replace an expensive instruction with an equivalent but cheaper alternative. For example a division may be replaced by multiplication of a reciprocal, or a list append may be replaced by cons operations.**

## 19. Data Cache Optimizations

- **Locality Optimizations**

**Cluster accesses of data values both spatially (within a cache line) and temporally (for repeated use).**

*Loop interchange and loop tiling improve temporal locality.*

- **Conflict Optimizations**

**Adjust data locations so that data used consecutively and repeatedly don't share the same cache location.**

## 20. Instruction Cache Optimizations

**Instructions that are repeatedly executed should be accessed from the instruction cache rather than the secondary cache or memory. Loops and “hot” instruction sequences should fit within the cache.**

**Temporally close instruction sequences should not map to conflicting cache**

# Basic Blocks

**A basic block is a linear sequence of instructions containing no branches except at the very end.**

**A basic block is always executed sequentially as a unit.**

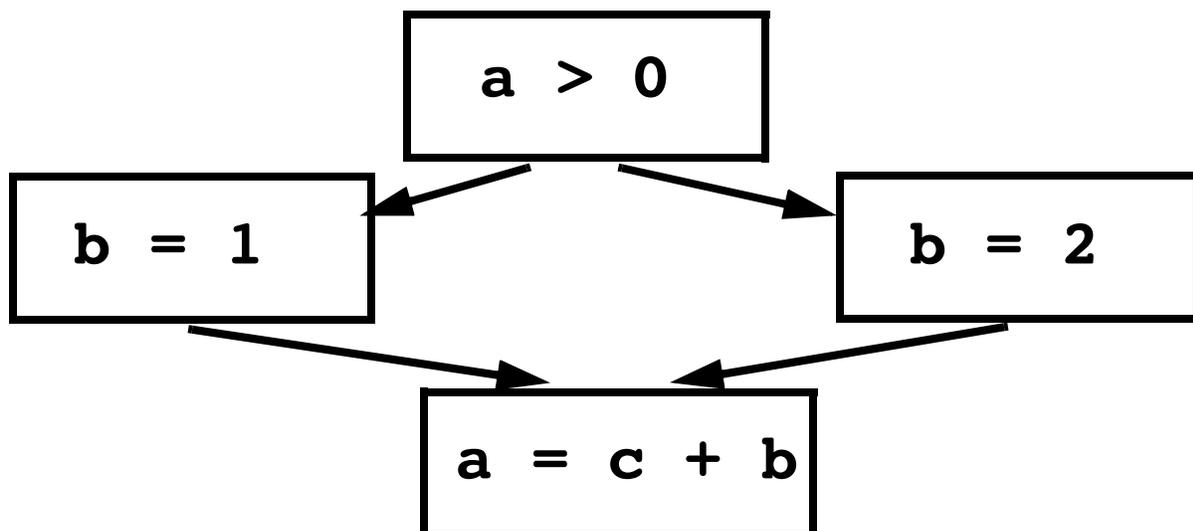
# Control Flow Graphs

**A Control Flow Graph (CFG) models possible execution paths through a program.**

**Nodes are basic blocks and arcs are potential transfers of control.**

**For example,**

```
if (a > 0)  
    b = 1;  
else b = 2;  
a = c + b;
```



**For a Basic Block  $b$ :**

**Let  $\text{Preds}(b)$  = the set of basic blocks that are Immediate Predecessors of  $b$  in the CFG.**

**Let  $\text{Succ}(b)$  = the set of basic blocks that are Immediate Successors to  $b$  in the CFG.**

# Data Flow Problems

**A data flow problem is a program analysis computed on a control flow graph.**

**A data flow problem may be *forward* (following a program's control flow) or *reverse* (opposite a program's control flow).**

**Informally, forward analyses “remember the past” while reverse analyses “predict the future.”**

**Some analyses determine that an event *may* have occurred, while others determine that an event *must* have occurred.**

**Some analyses compute a set of values, while others are Boolean-valued.**

**Two important data flow problems are *Reaching Definitions* and *Liveness*.**

**For a given use of a variable  $v$  reaching definitions tell us which assignments to  $v$  may reach (affect) the current value of  $v$ . Reaching definition analysis is useful in both optimization and debugging.**

**Liveness analysis tells us at a particular point in a program whether the current value of variable  $v$  will ever be used. A variable that is not live is dead. A dead value need not be kept in memory, or perhaps even be computed.**

# Reaching Definitions

**For a Basic Block  $b$  and Variable  $V$ :**

**Let  $\text{DefsIn}(b)$  = the set of basic blocks that contain definitions of  $V$  that reach (may be used in) the beginning of Basic Block  $b$ .**

**Let  $\text{DefsOut}(b)$  = the set of basic blocks that contain definitions of  $V$  that reach (may be used in) the end of Basic Block  $b$ .**

**The sets  $\text{Preds}$  and  $\text{Succ}$  are derived from the structure of the CFG.**

**They are given as part of the definition of the CFG.**

**DefsIn and DefsOut must be computed, using the following rules:**

**1. If Basic Block  $b$  contains a definition of  $V$  then**

$$\mathbf{DefsOut}(b) = \{b\}$$

**2. If there is no definition to  $V$  in  $b$  then**  
**DefsOut( $b$ ) = DefsIn( $b$ )**

**3. For the First Basic Block,  $b_0$ :**

$$\mathbf{DefsIn}(b_0) = \phi$$

**4. For all Other Basic Blocks**

$$\mathbf{DefsIn}(b) = \bigcup_{p \in \text{Preds}(b)} \mathbf{DefsOut}(p)$$

# Liveness Analysis

**For a Basic Block  $b$  and Variable  $V$ :**

**LiveIn( $b$ ) = true if  $V$  is Live (will be used before it is redefined) at the beginning of  $b$ .**

**LiveOut( $b$ ) = true if  $V$  is Live (will be used before it is redefined) at the end of  $b$ .**

**LiveIn and LiveOut are computed, using the following rules:**

**1. If Basic Block  $b$  has no successors then**

**LiveOut( $b$ ) = false**

**2. For all Other Basic Blocks**

**LiveOut( $b$ ) =  $\bigvee_{s \in \text{Succ}(b)} \text{LiveIn}(s)$**

**3. LiveIn(b) =**

**If V is used before it is defined in  
Basic Block b**

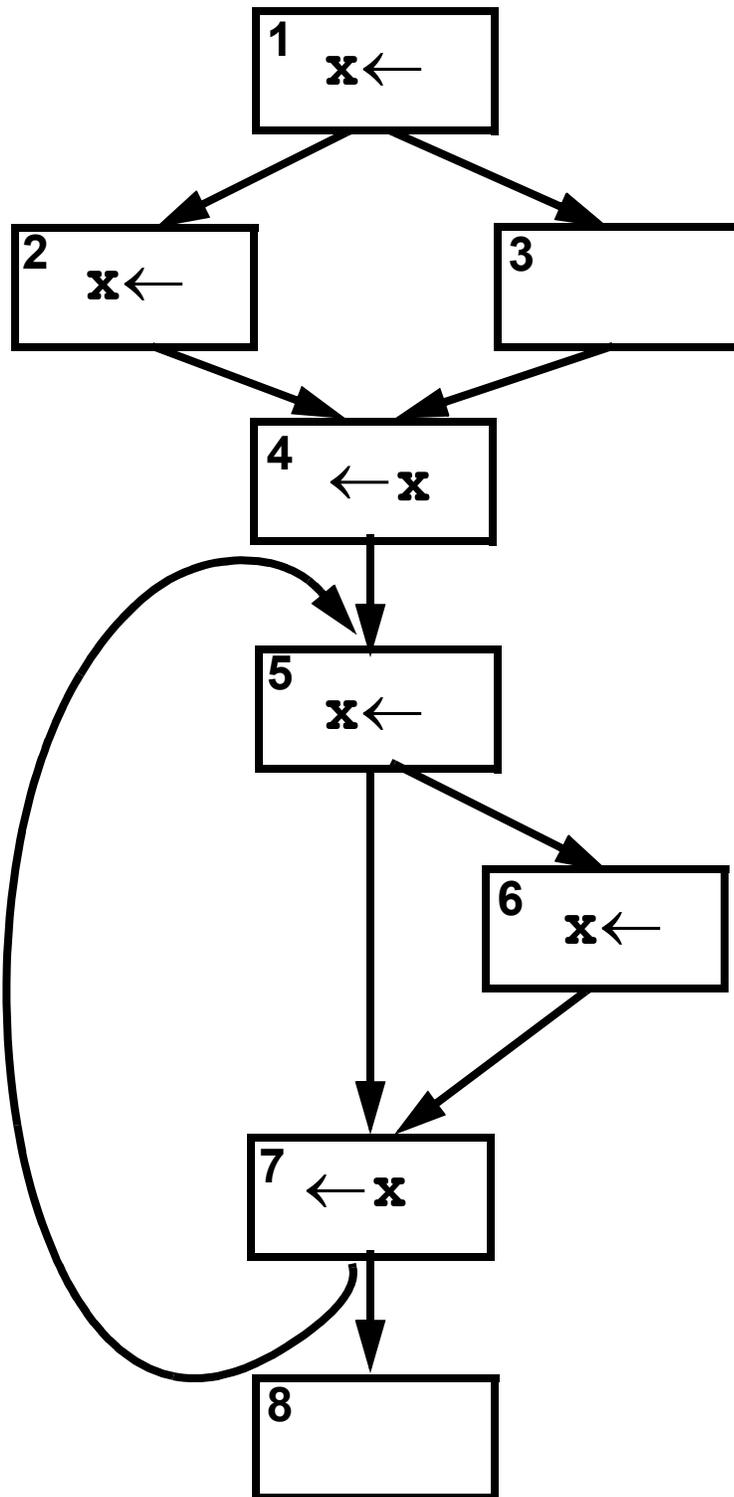
**Then true**

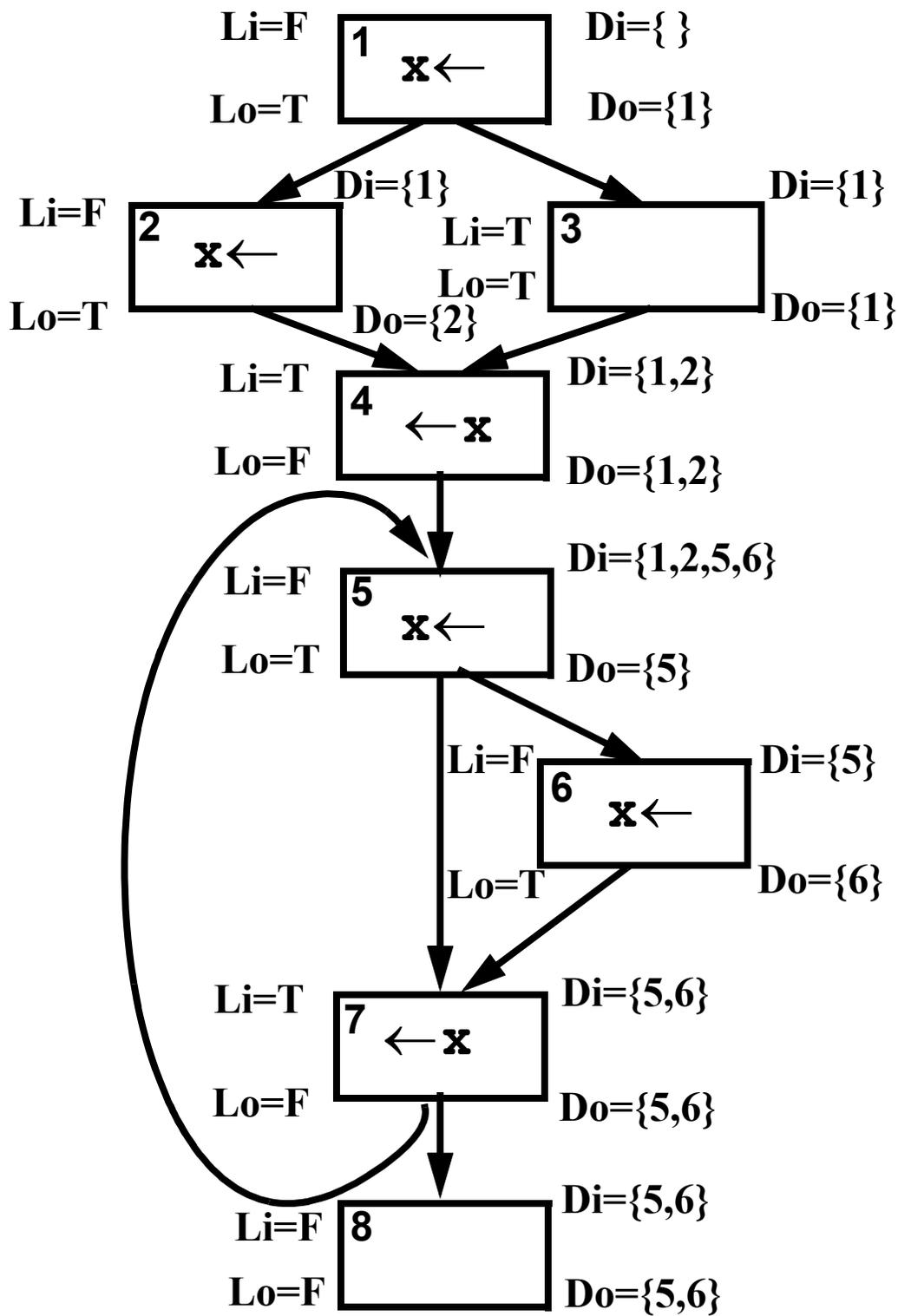
**Elsif V is defined before it is  
used in Basic Block b**

**Then false**

**Else LiveOut(b)**

# Example





# Reading Assignment

- **Section 14.3 - 14.4 of CaC**

# Data Flow Frameworks

- **Data Flow Graph:**

**Nodes of the graph are basic blocks or individual instructions.**

**Arcs represent flow of control.**

**Forward Analysis:**

**Information flow is the same direction as control flow.**

**Backward Analysis:**

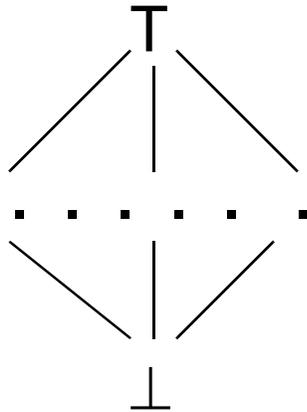
**Information flow is the opposite direction as control flow.**

**Bi-directional Analysis:**

**Information flow is in both directions. (Not too common.)**

- **Meet Lattice**

**Represents solution space for the data flow analysis.**



- **Meet operation**

**(And, Or, Union, Intersection, etc.)**

**Combines solutions from predecessors or successors in the control flow graph.**

- **Transfer Function**

**Maps a solution at the top of a node to a solution at the end of the node (forward flow)**

*or*

**Maps a solution at the end of a node to a solution at the top of the node (backward flow).**

# **Example: Available Expressions**

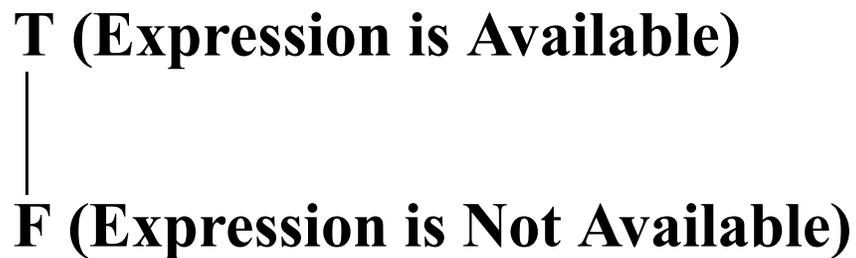
**This data flow analysis determines whether an expression that has been previously computed may be reused.**

**Available expression analysis is a forward flow problem—computed expression values flow forward to points of possible reuse.**

**The best solution is True—the expression may be reused.**

**The worst solution is False—the expression may not be reused.**

## The Meet Lattice is:



**As initial values, at the top of the start node, nothing is available. Hence, for a given expression,  $\text{AvailIn}(b_0) = F$**

**We choose an expression, and consider all the variables that contribute to its evaluation.**

**Thus for  $e_1 = a + b - c$ ,  $a$ ,  $b$  and  $c$  are  $e_1$ 's *operands*.**

**The transfer function for  $e_1$  in block  $b$  is defined as:**

**If  $e_1$  is computed in  $b$  after any assignments to  $e_1$ 's operands in  $b$**

**Then  $\text{AvailOut}(b) = T$**

**Elsif any of  $e_1$ 's operands are changed**

**after the last computation of  $e_1$  or  $e_1$ 's operands are changed without any computation of  $e_1$**

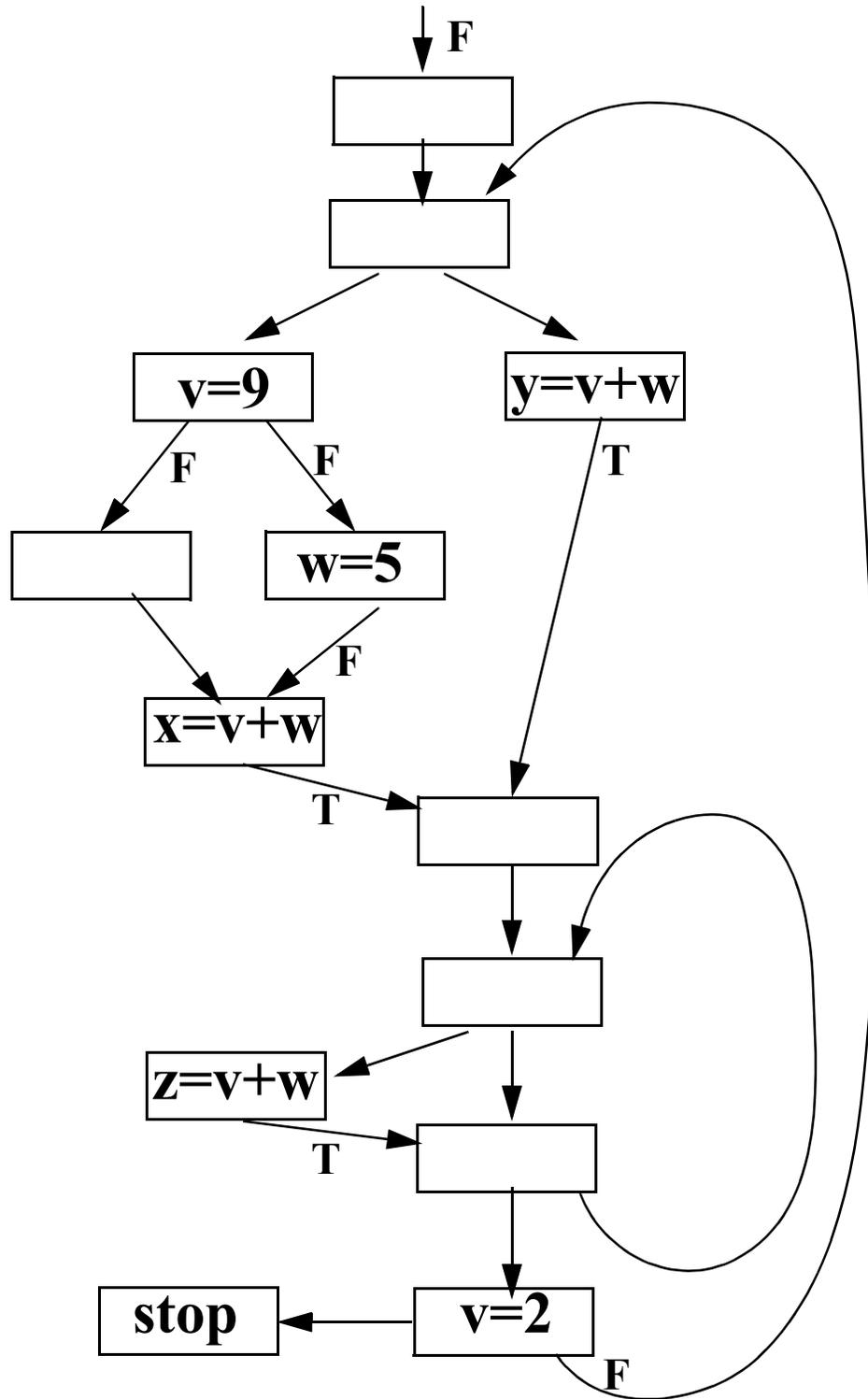
**Then  $\text{AvailOut}(b) = F$**

**Else  $\text{AvailOut}(b) = \text{AvailIn}(b)$**

**The meet operation (to combine solutions) is:**

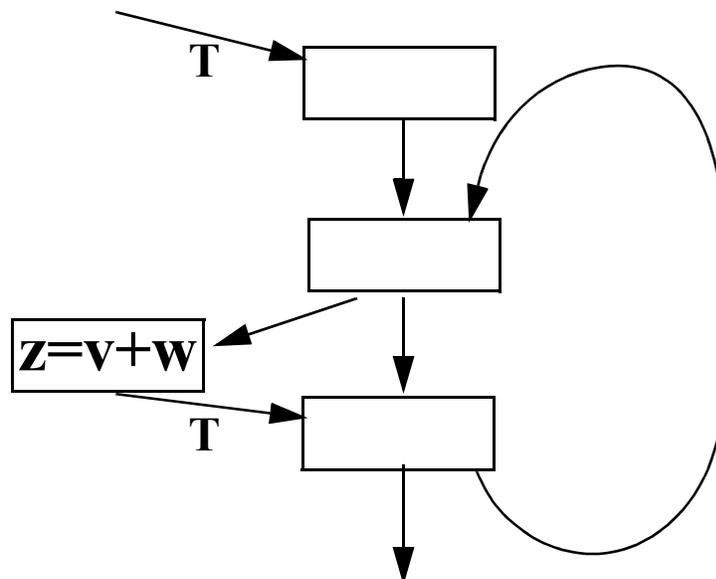
$$\text{AvailIn}(b) = \text{AND}_{p \in \text{Pred}(b)} \text{AvailOut}(p)$$

# Example: $e_1 = v + w$



# Circularities Require Care

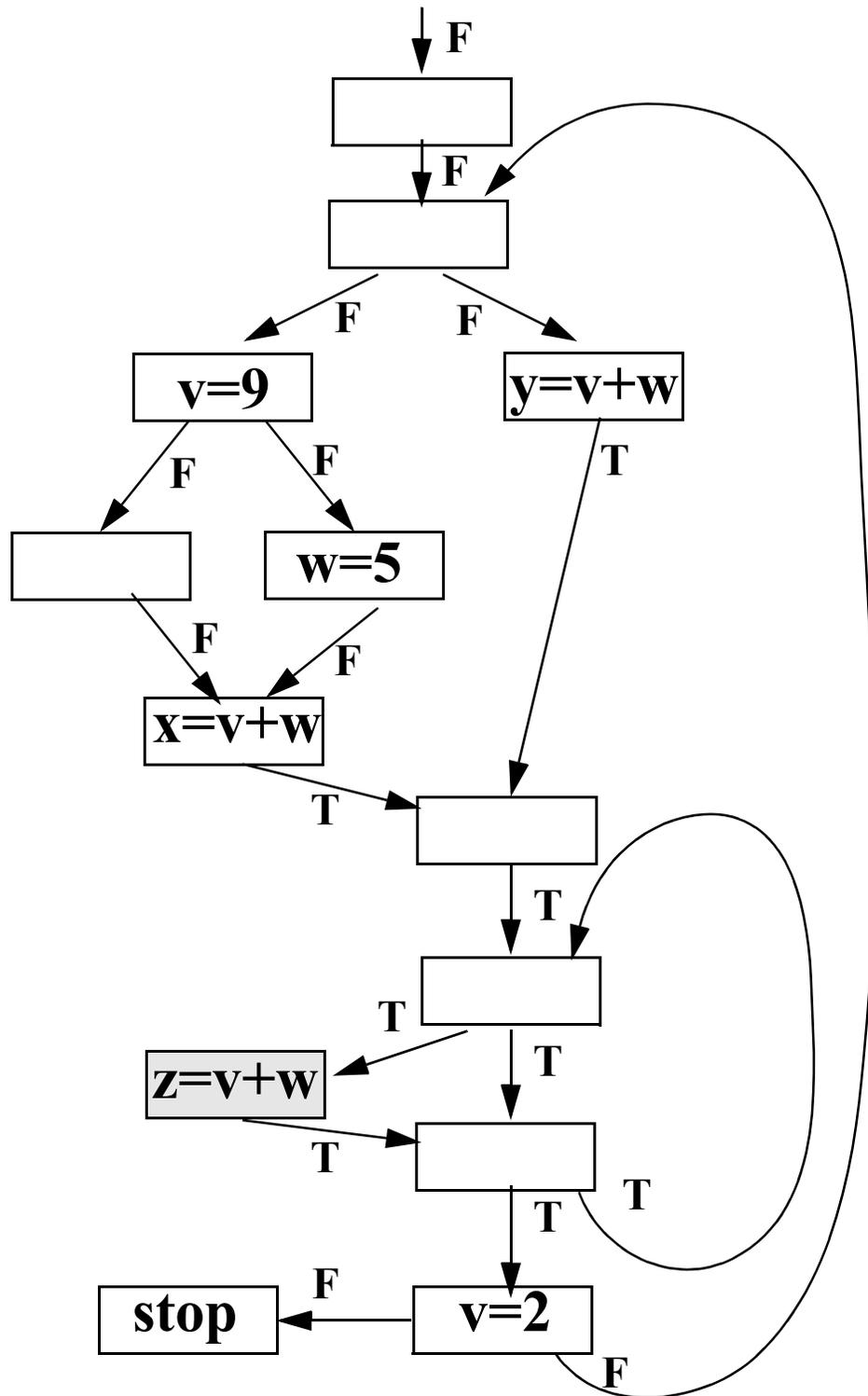
Since data flow values can depend on themselves (because of loops), care is required in assigning initial “guesses” to unknown values.



**Consider**

**If the flow value on the loop backedge is initially set to false, it can never become true. (Why?)**

**Instead we should use True, the *identity* for the AND operation.**



# Very Busy Expressions

**This is an interesting variant of available expression analysis.**

**An expression is *very busy* at a point if it is *guaranteed* that the expression will be computed at some time in the future.**

**Thus starting at the point in question, the expression must be reached before its value changes.**

**Very busy expression analysis is a backward flow analysis, since it propagates information about future evaluations backward to “earlier” points in the computation.**

**The meet lattice is:**

**T (Expression is Very Busy)**



**F (Expression is Not Very Busy)**

**As initial values, at the end of all  
exit nodes, nothing is very busy.**

**Hence, for a given expression,**

**$\text{VeryBusyOut}(b_{\text{last}}) = \mathbf{F}$**

**The transfer function for  $e_1$  in block  $b$  is defined as:**

**If  $e_1$  is computed in  $b$  before any of its operands**

**Then  $\text{VeryBusyIn}(b) = T$**

**Elsif any of  $e_1$ 's operands are changed**

**before  $e_1$  is computed**

**Then  $\text{VeryBusyIn}(b) = F$**

**Else  $\text{VeryBusyIn}(b) = \text{VeryBusyOut}(b)$**

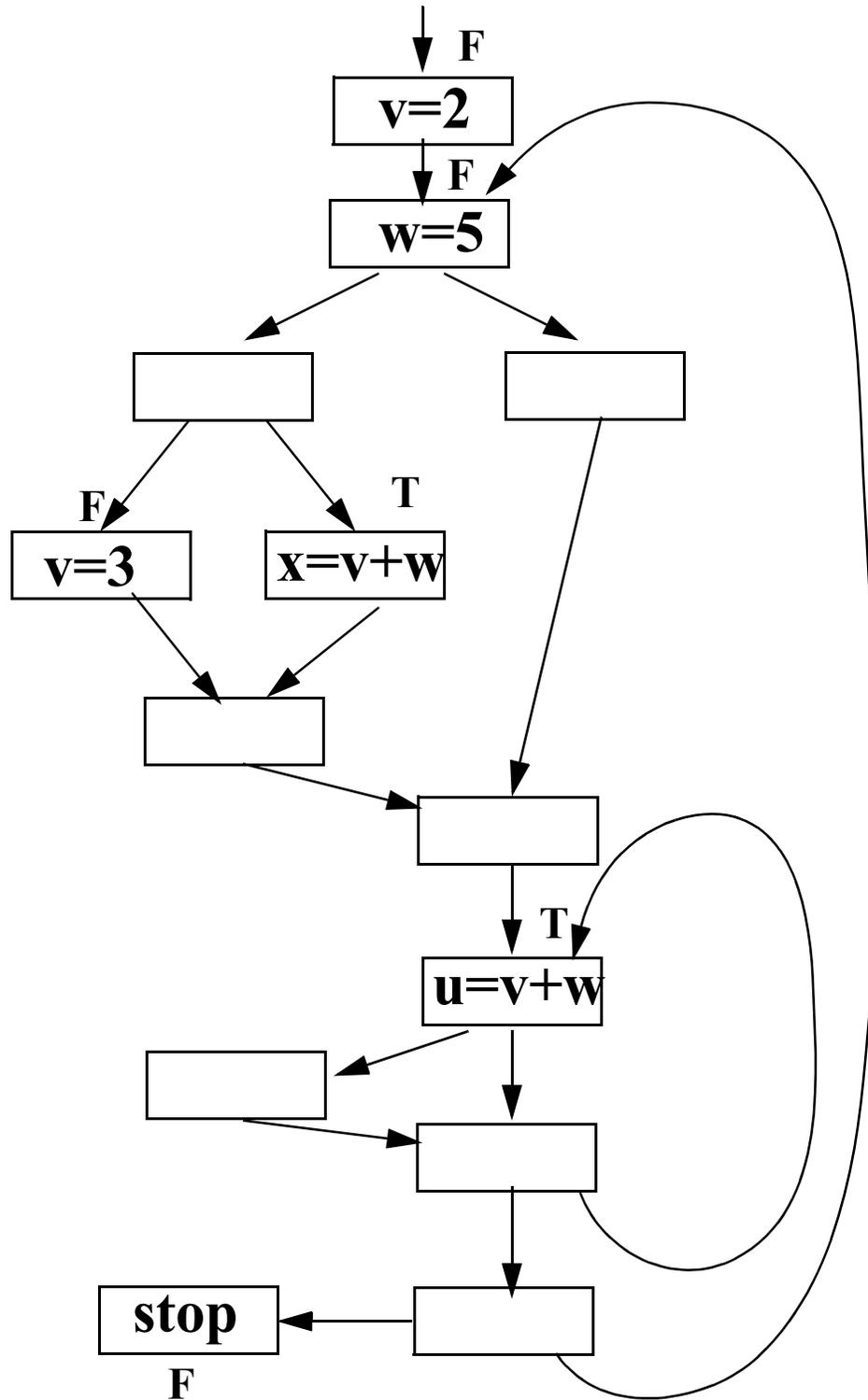
**The meet operation (to combine solutions) is:**

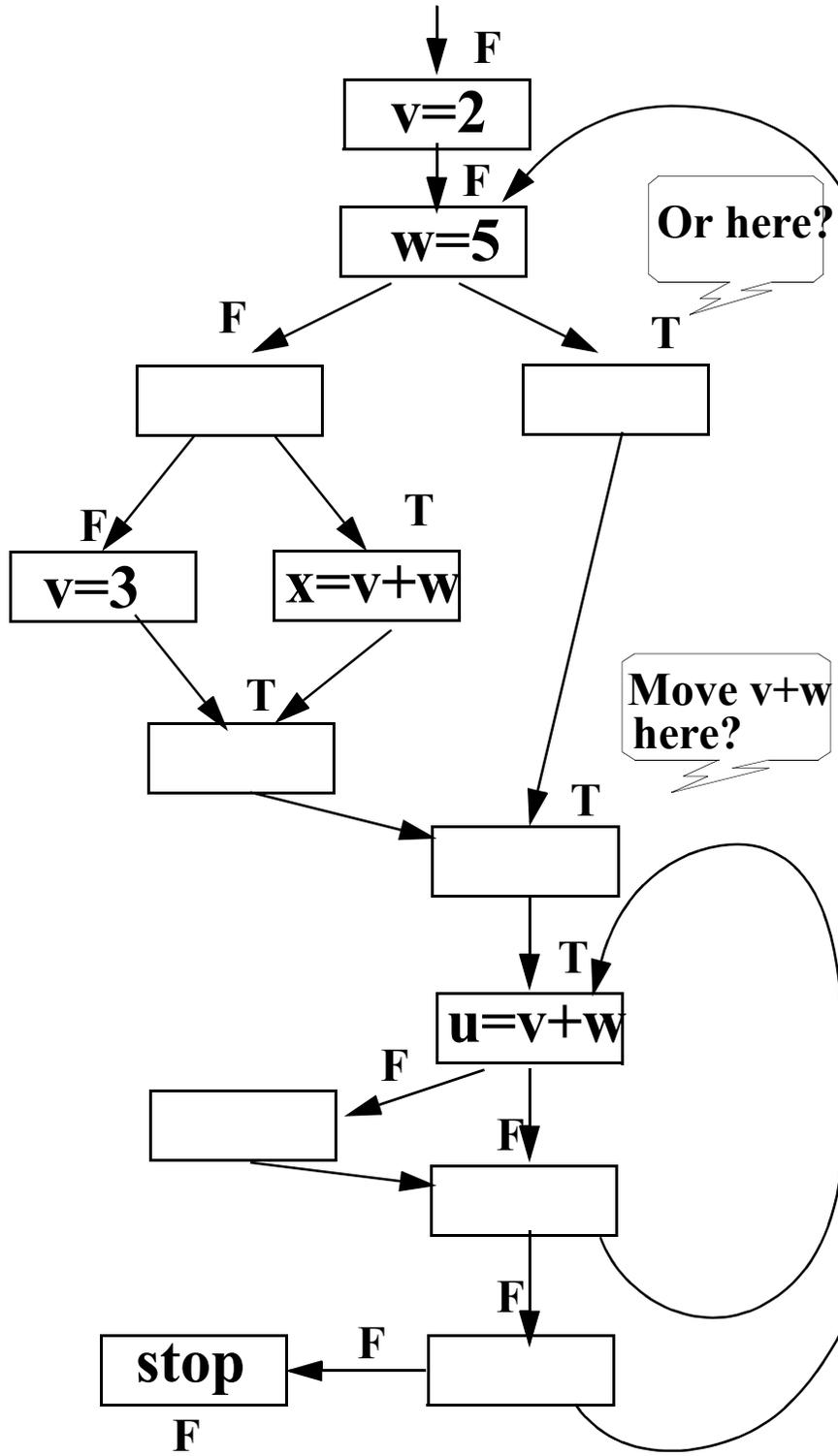
**$\text{VeryBusyOut}(b) =$**

**$\text{ANDVeryBusyIn}(s)$**

**$s \in \text{Succ}(b)$**

# Example: $e_1 = v + w$





# Identifying Identical Expressions

**We can hash expressions, based on hash values assigned to operands and operators. This makes recognizing potentially redundant expressions straightforward.**

**For example, if  $H(a) = 10$ ,  $H(b) = 21$  and  $H(+)$  = 5, then (using a simple product hash),  
 $H(a+b) = 10 \times 21 \times 5 \text{ Mod TableSize}$**

# Effects of Aliasing and Calls

**When looking for assignments to operands, we must consider the effects of pointers, formal parameters and calls.**

**An assignment through a pointer (e.g, `*p = val`) *kills* all expressions dependent on variables `p` might point too. Similarly, an assignment to a formal parameter kills all expressions dependent on variables the formal might be bound to.**

**A call kills all expressions dependent on a variable changeable during the call.**

**Lacking careful alias analysis, pointers, formal parameters and calls can kill all (or most) expressions.**

# **Very Busy Expressions and Loop Invariants**

**Very busy expressions are ideal candidates for invariant loop motion.**

**If an expression, invariant in a loop, is also very busy, we know it must be used in the future, and hence evaluation outside the loop must be worthwhile.**

```

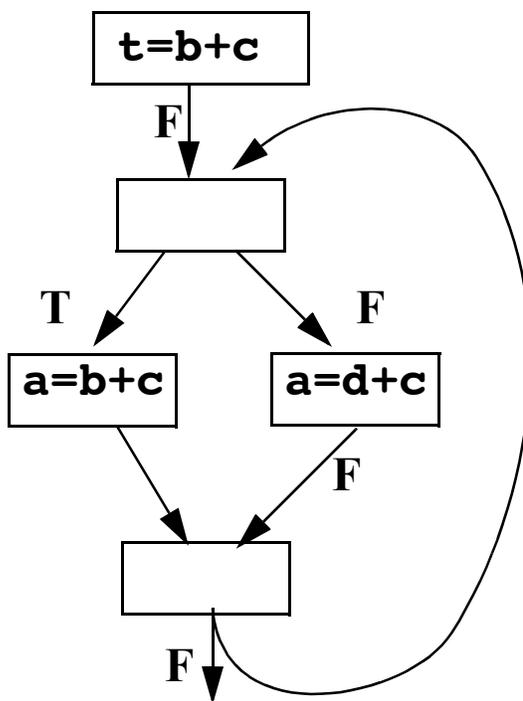
for (...) {
  if (...)
    a=b+c;
  else a=d+c;}

```

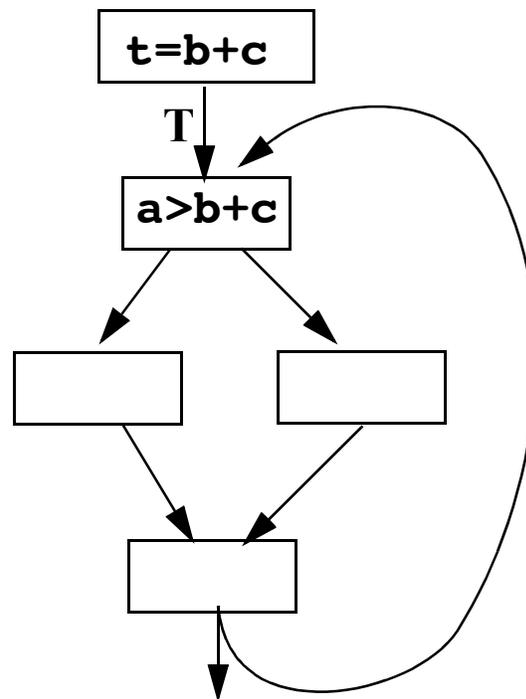
```

for (...) {
  if (a>b+c)
    x=1;
  else x=0;}

```



**b+c is not very busy  
at loop entrance**



**b+c is very busy  
at loop entrance**

# Reaching Definitions

**We have seen reaching definition analysis formulated as a set-valued problem. It can also be formulated on a per-definition basis.**

**That is, we ask “What blocks does a particular definition to  $v$  reach?”**

**This is a boolean-valued, forward flow data flow problem.**

**Initially,  $\text{DefIn}(b_0) = \text{false}$ .**

**For basic block  $b$ :**

**$\text{DefOut}(b) =$**

**If the definition being analyzed is  
the last definition to  $v$  in  $b$**

**Then True**

**Elsif any other definition to  $v$**

**occurs**

**in  $b$**

**Then False**

**Else  $\text{DefIn}(b)$**

**The meet operation (to combine  
solutions) is:**

$$\text{DefIn}(b) = \text{OR}_{p \in \text{Pred}(b)} \text{DefOut}(p)$$

**To get all reaching definition, we do  
a series of single definition analyses.**

# Live Variable Analysis

This is a boolean-valued, backward flow data flow problem.

Initially,  $\text{LiveOut}(b_{\text{last}}) = \text{false}$ .

For basic block  $b$ :

$\text{LiveIn}(b) =$

If the variable is used before it is defined in  $b$

Then True

Elsif it is defined before it is used in  $b$

Then False

Else  $\text{LiveOut}(b)$

The meet operation (to combine solutions) is:

$$\text{LiveOut}(b) = \text{OR}_{s \in \text{Succ}(b)} \text{LiveIn}(s)$$

# Bit Vectoring Data Flow Problems

The four data flow problems we have just reviewed all fit within a *single* framework.

Their solution values are Booleans (bits).

The meet operation is And or OR.

The transfer function is of the general form

$$\text{Out}(b) = (\text{In}(b) - \text{Kill}_b) \cup \text{Gen}_b$$

or

$$\text{In}(b) = (\text{Out}(b) - \text{Kill}_b) \cup \text{Gen}_b$$

where  $\text{Kill}_b$  is true if a value is “killed” within  $b$  and  $\text{Gen}_b$  is true if a value is “generated” within  $b$ .

**In Boolean terms:**

**$\text{Out}(b) = (\text{In}(b) \text{ AND Not Kill}_b) \text{ OR Gen}_b$**

**or**

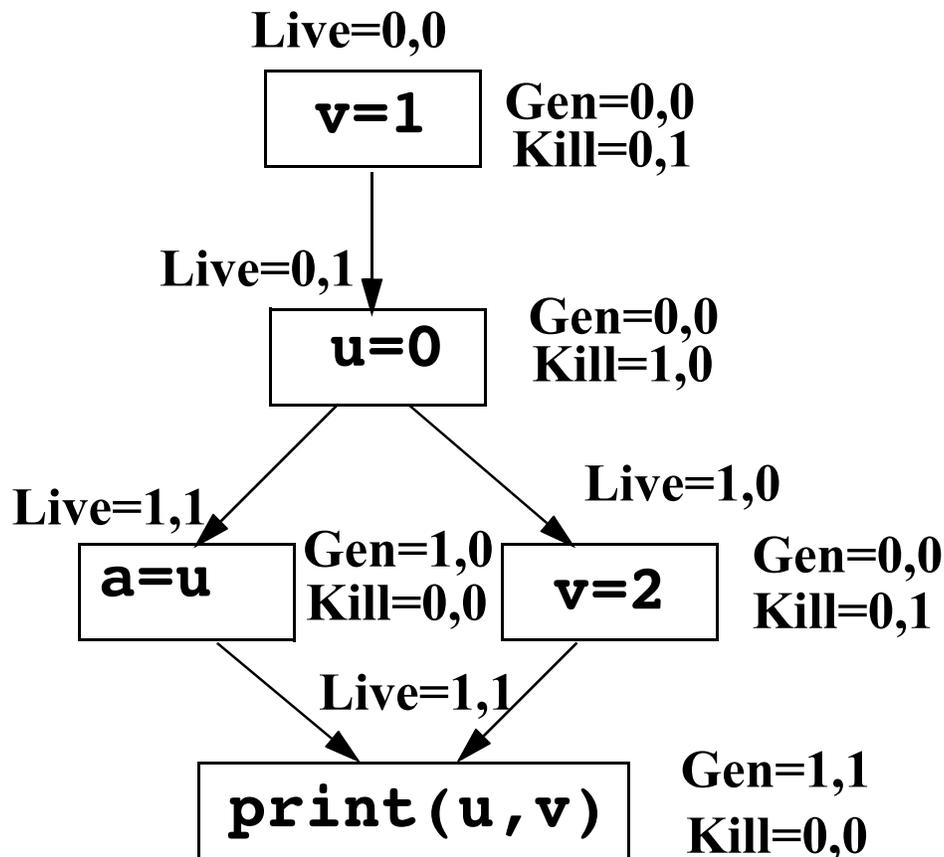
**$\text{In}(b) = (\text{Out}(b) \text{ AND Not Kill}_b) \text{ OR Gen}_b$**

**An advantage of a bit vectoring data flow problem is that we can do a series of data flow problems “in parallel” using a bit vector.**

**Hence using ordinary word-level ANDs, ORs, and NOTs, we can solve 32 (or 64) problems simultaneously.**

# Example

Do live variable analysis for  $u$  and  $v$ , using a 2 bit vector:



We expect *no variable* to be live at the start of  $b_0$ . (Why?)

# Depth-First Spanning Trees

**Sometimes we want to “cover” the nodes of a control flow graph with an acyclic structure.**

**This allows us to visit nodes once, without worrying about cycles or infinite loops.**

**Also, a careful visitation order can approximate forward control flow (very useful in solving forward data flow problems).**

**A Depth-First Spanning Tree (DFST) is a tree structure that covers the nodes of a control flow graph, with the start node serving as root of the DFST.**

# Building a DFST

**We will visit CFG nodes in depth-first order, keeping arcs if the visited node hasn't be reached before.**

**Create a DFST,  $T$ , from a CFG,  $G$ :**

- 1.  $T \leftarrow$  empty tree**
- 2. Mark all nodes in  $G$  as “unvisited.”**
- 3. Call DF(start node)**

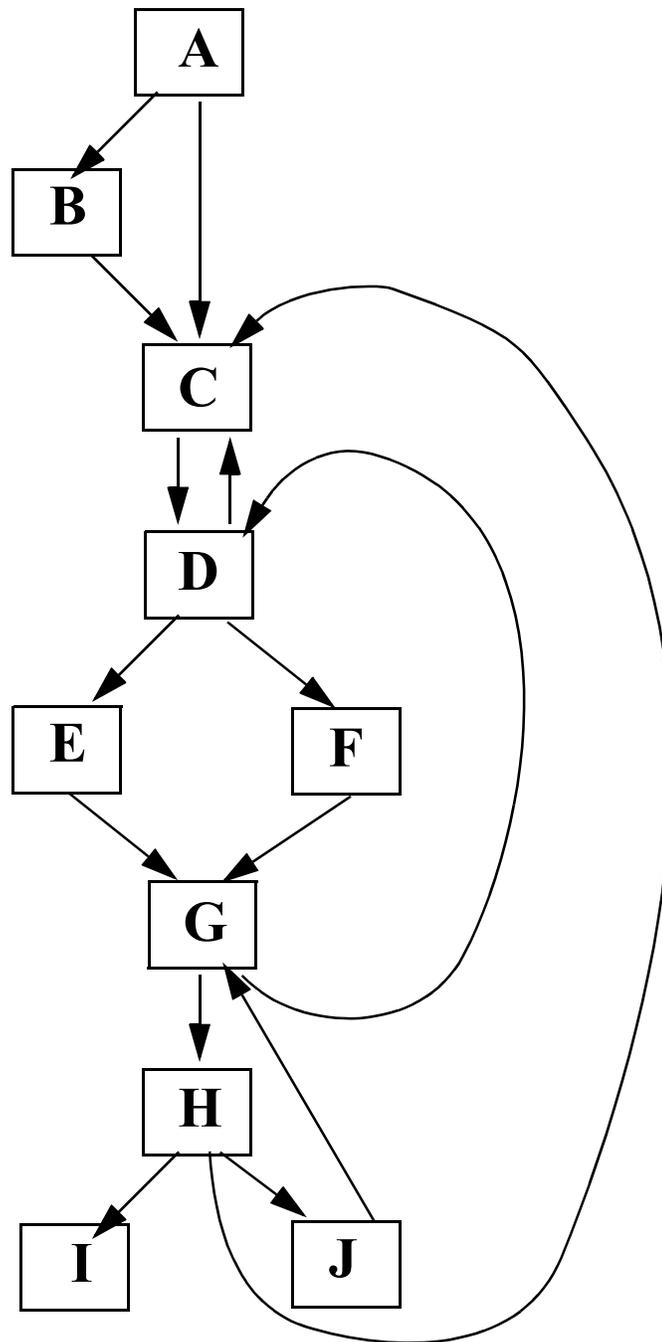
**DF (node) {**

- 1. Mark node as visited.**
- 2. For each successor,  $s$ , of node in  $G$ :**

**If  $s$  is unvisited**

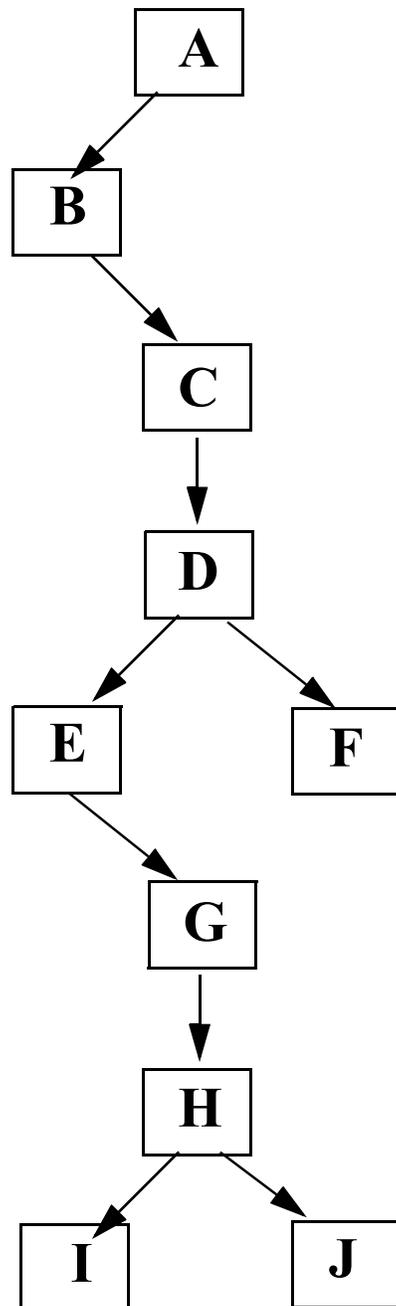
- (a) Add node  $\rightarrow s$  to  $T$**
- (b) Call DF( $s$ )**

# Example



**Visit order is A, B, C, D, E, G, H, I, J, I**

# The DFST is



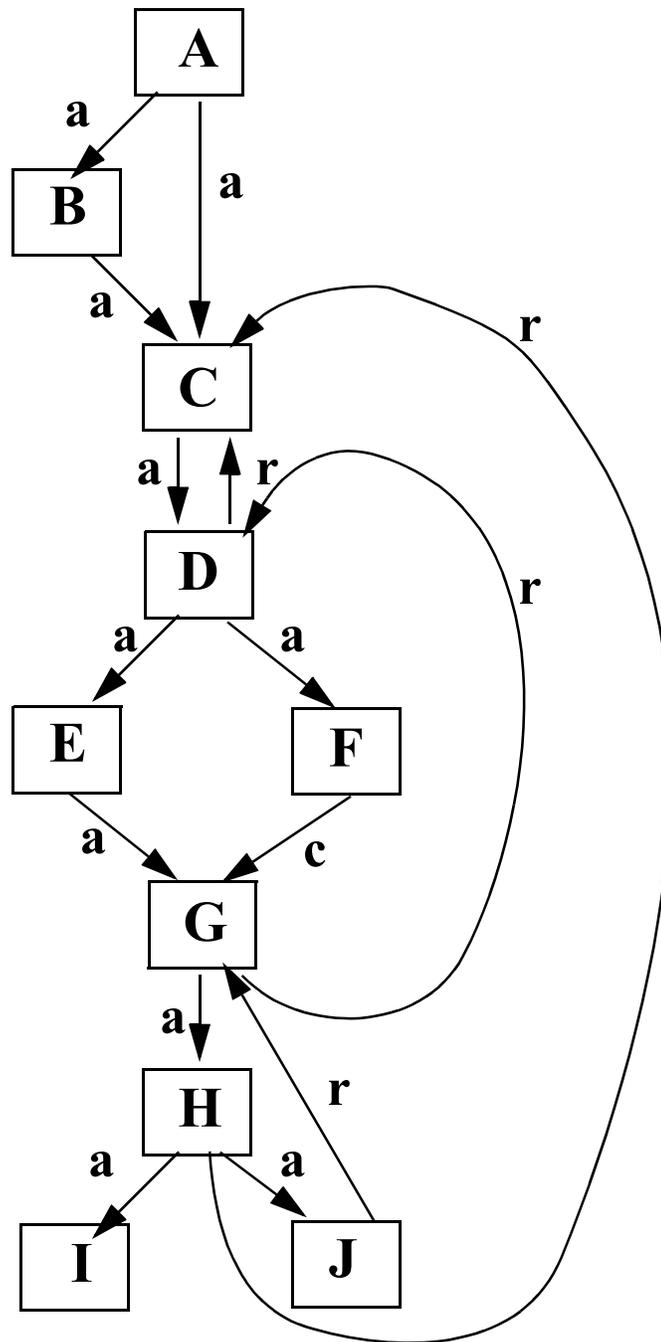
# Categorizing Arcs using a DFST

**Arcs in a CFG can be categorized by examining the corresponding DFST.**

**An arc  $A \rightarrow B$  in a CFG is**

- (a) An *Advancing Edge* if B is a proper descendent of A in the DFST.**
- (b) A *Retreating Edge* if B is an ancestor of A in the DFST. (This includes the  $A \rightarrow A$  case.)**
- (c) A *Cross Edge* if B is neither a descendent nor an ancestor of A in the DFST.**

# Example



# Depth-First Order

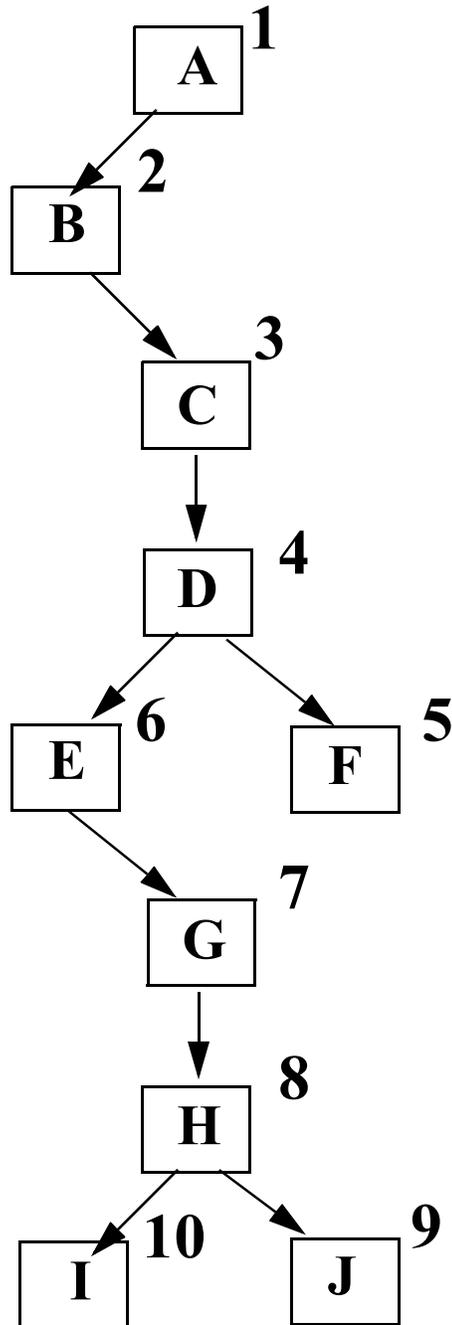
Once we have a DFST, we can label nodes with a *Depth-First Ordering* (DFO).

Let  $i$  = the number of nodes in a CFG (= the number of nodes in its DFST).

```
DFO(node) {  
    For (each successor  $s$  of node) do  
        DFO( $s$ );  
    Mark node with  $i$ ;  
     $i--$ ;  
}
```

# Example

The number of nodes = 10.



# Application of Depth-First Ordering

- *Retreating edges* (a necessary component of loops) are easy to identify:  
 $a \rightarrow b$  is a retreating edge if and only if  $dfo(b) \leq dfo(a)$
- A depth-first ordering is an excellent *visit order* for solving forward data flow problems. We want to visit nodes in essentially topological order, so that all predecessors of a node are visited (and evaluated) before the node itself is.

# Dominators

A CFG node  $M$  *dominates*  $N$  ( $M \text{ dom } N$ ) if and only if *all* paths from the start node to  $N$  *must* pass through  $M$ .

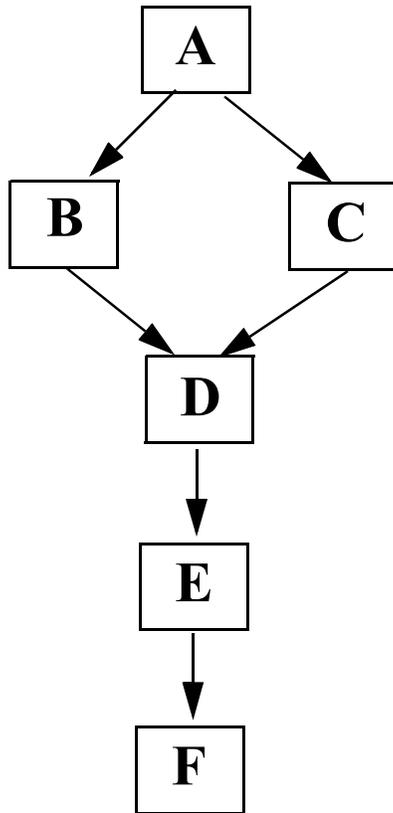
A node trivially dominates itself.  
Thus  $(N \text{ dom } N)$  is always true.

A CFG node  $M$  *strictly dominates*  $N$  ( $M \text{ sdom } N$ ) if and only if  $(M \text{ dom } N)$  and  $M \neq N$ .

A node can't strictly dominates itself.

Thus  $(N \text{ sdom } N)$  is never true.

**A CFG node may have many dominators.**



**Node F is dominated by F, E, D and A.**

# Immediate Dominators

**If a CFG node has more than one dominator (which is common), there is always a unique “closest” dominator called its *immediate dominator*.**

**(M idom N) if and only if  
(M sdom N) and  
(P sdom N)  $\Rightarrow$  (P dom M)**

**To see that an immediate dominator always exists (except for the start node) and is unique, assume that node N is strictly dominated by  $M_1, M_2, \dots, M_p, P \geq 2$ .**

**By definition,  $M_1, \dots, M_p$  must appear on *all* paths to N, including acyclic paths.**

**Look at the relative ordering among  $M_1$  to  $M_p$  on some arbitrary acyclic path from the start node to  $N$ .**

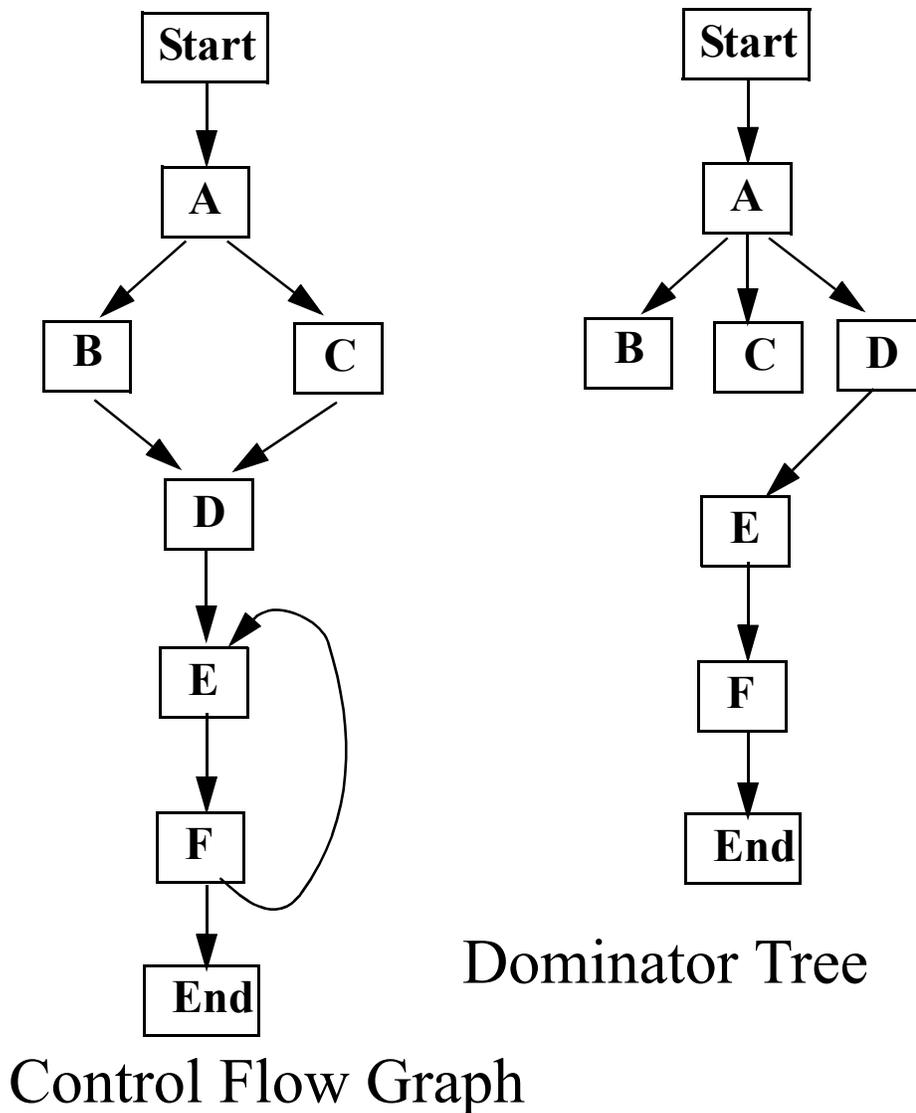
**Assume that  $M_i$  is “last” on that path (and hence “nearest” to  $N$ ).**

**If, on some other acyclic path,  $M_j \neq M_i$  is last, then we can shorten this second path by going directly from  $M_j$  to  $N$  without touching any more of the  $M_1$  to  $M_p$  nodes.**

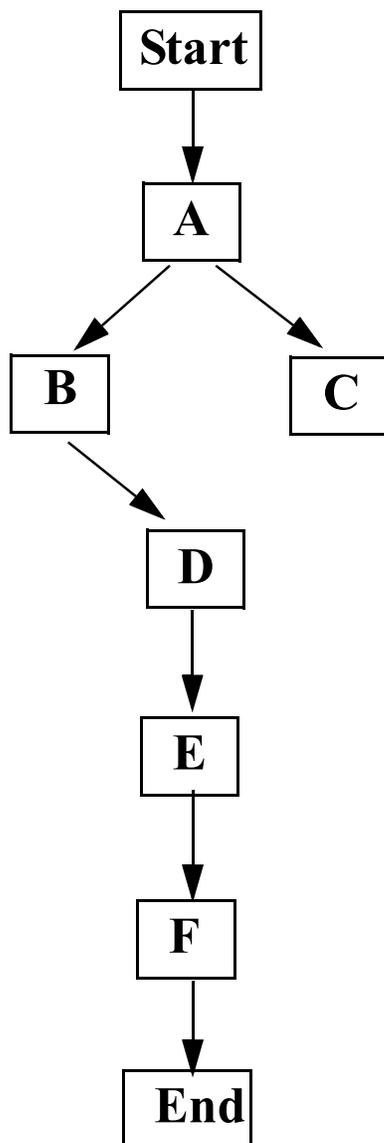
**But, this totally removes  $M_j$  from the path, contradicting the assumption that  $(M_j \text{ sdom } N)$ .**

# Dominator Trees

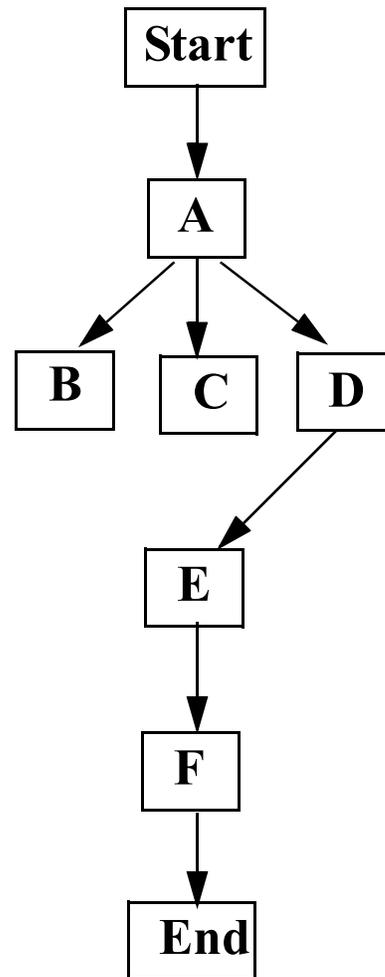
Using immediate dominators, we can create a *dominator tree* in which  $A \rightarrow B$  in the dominator tree if and only if (A idom B).



**Note that the Dominator Tree of a CFG and its DFST are distinct trees (though they have the same nodes).**



**Depth-First Spanning Tree**



**Dominator Tree**

**A Dominator Tree is a compact and convenient representation of both the dom and idom relations.**

**A node in a Dominator Tree dominates all its descendants in the tree, and immediately dominates all its children.**

# Computing Dominators

**Dominators can be computed as a Set-valued Forward Data Flow Problem.**

**If a node  $N$  dominates all of node  $M$ 's predecessors, then  $N$  appears on all paths to  $M$ . Hence  $(N \text{ dom } M)$ .**

**Similarly, if  $M$  *doesn't* dominate all of  $M$ 's predecessors, then there is a path to  $M$  that doesn't include  $M$ . Hence**

**$\neg(N \text{ dom } M)$ .**

**These observations give us a “data flow equation” for dominator sets:**

$$\text{dom}(N) = \{N\} \cup \bigcap_{M \in \text{Pred}(N)} \text{dom}(M)$$

**The analysis domain is the lattice of all subsets of nodes. Top is the set of all nodes; bottom is the empty set. The ordering relation is subset.**

**The meet operation is intersection.**

**The Initial Condition is that**

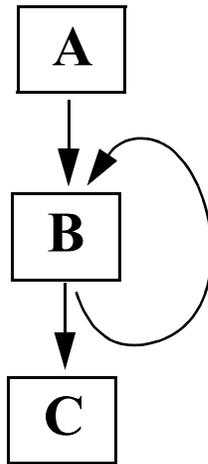
$$\mathbf{DomIn(b_0) = \phi}$$

$$\mathbf{DomIn(b) = \bigcap_{c \in \text{Pred}(b)} \text{DomOut}(c)}$$

$$\mathbf{DomOut(b) = \text{DomIn}(b) \cup \{b\}}$$

# Loops Require Care

**Loops in the Control Flow Graph induce circularities in the Data Flow equations for Dominators. In**



**we have the rule  $\text{dom}(\mathbf{B}) =$   
 $\text{DomOut}(\mathbf{B}) =$   
 $\text{DomIn}(\mathbf{B}) \cup \{\mathbf{B}\} =$   
 $\{\mathbf{B}\} \cup (\underline{\text{DomOut}(\mathbf{B})} \cap \text{DomOut}(\mathbf{A}))$**

**If we choose  $\text{DomOut}(\mathbf{B}) = \phi$   
initially, we get  $\text{DomOut}(\mathbf{B}) =$   
 $\{\mathbf{B}\} \cup (\phi \cap \text{DomOut}(\mathbf{A})) = \{\mathbf{B}\}$   
which is *wrong*.**

**Instead, we should use the Universal Set (of all nodes) which is the *identity* for  $\cap$ .**

**Then we get  $\text{DomOut}(B) = \{B\} \cup (\{\text{all nodes}\} \cap \text{DomOut}(A)) = \{B\} \cup \text{DomOut}(A)$  which is correct.**

# **A Worklist Algorithm for Dominators**

**The data flow equations we have developed for dominators can be evaluated using a simple Worklist Algorithm.**

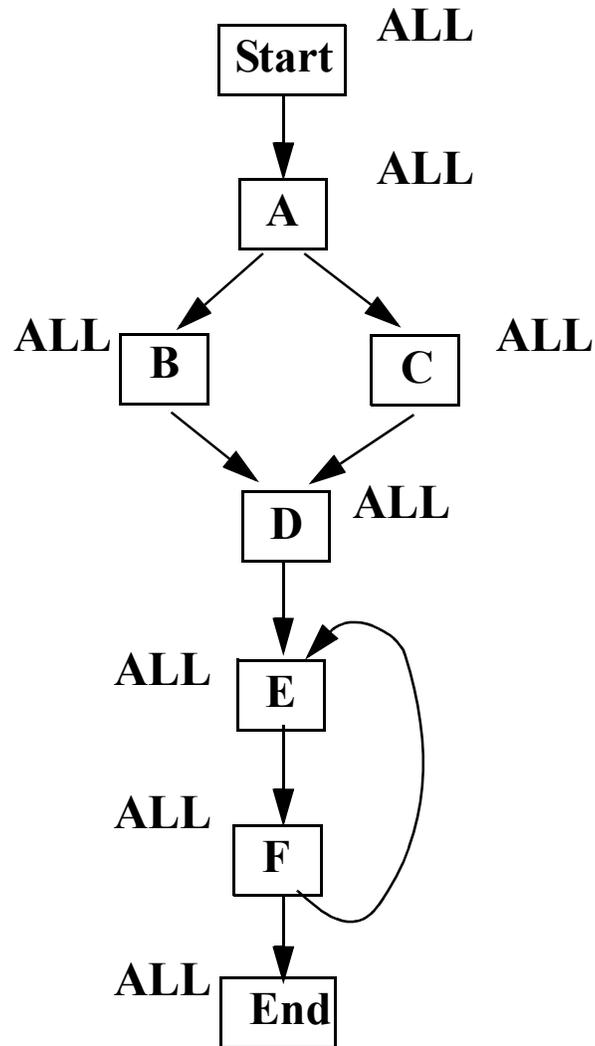
**Initially, each node's dominator set is set to the set of all nodes. We add the start node to our worklist.**

**For each node on the worklist, we reevaluate its dominator set. If the set changes, the updated dominator set is used, and all the node's successors are added to the worklist (so that the updated dominator set can be propagated).**

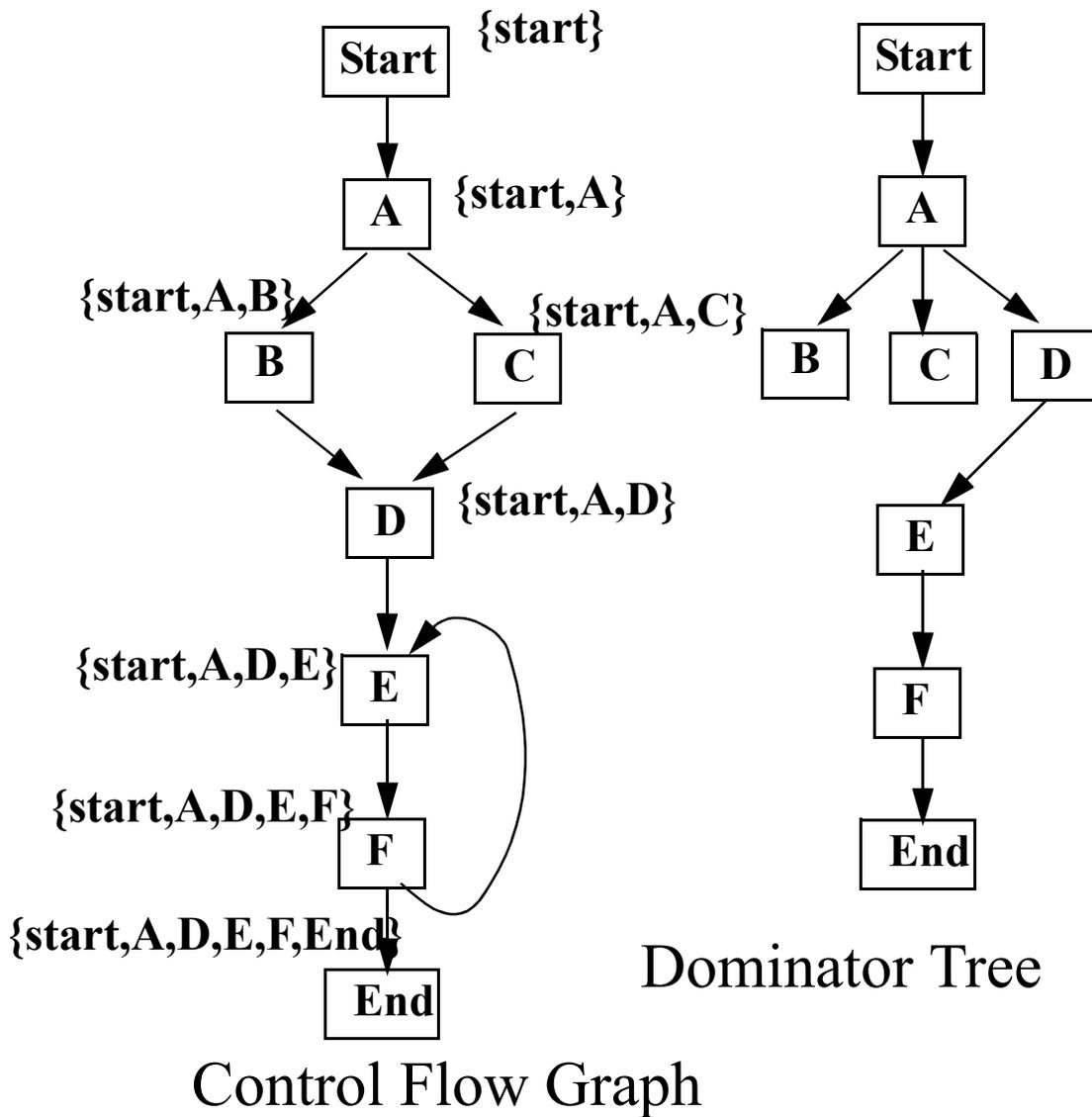
**The algorithm terminates when the worklist is empty, indicating that a stable solution has been found.**

```
Compute Dominators() {  
  For (each n ∈ NodeSet)  
    Dom(n) = NodeSet  
  WorkList = {StartNode}  
  While (WorkList ≠ ∅) {  
    Remove any node Y from  
WorkList  
    New = {Y} U  $\bigcap_{X \in \text{Pred}(Y)} \text{Dom}(X)$   
    If New ≠ Dom(Y) {  
      Dom(Y) = New  
      For (each Z ∈ Succ(Y))  
        WorkList = WorkList U {Z}  
    }}  
  }}
```

# Example



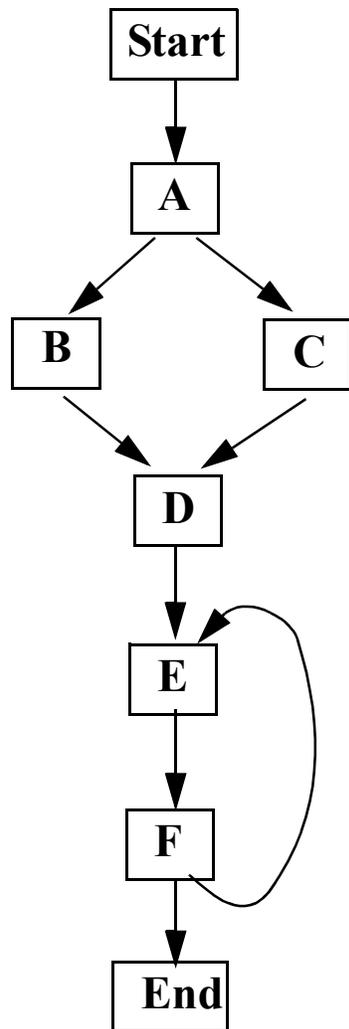
**Initially the WorkList = {Start}.**  
**Be careful when  $\text{Pred}(\text{Node}) = \phi$ .**



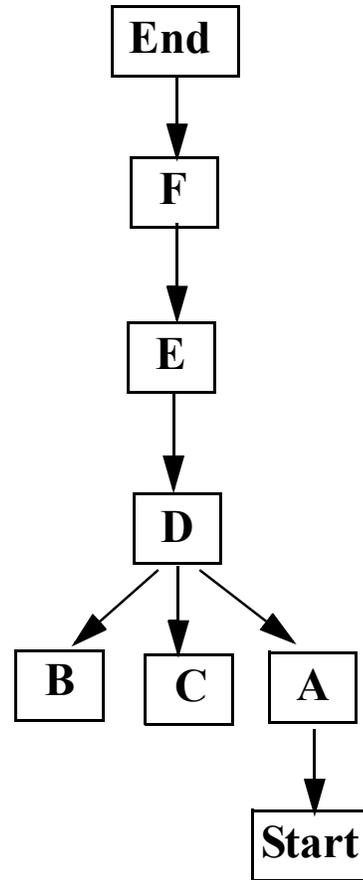
# Postdominance

**A block  $Z$  *postdominates* a block  $Y$  ( $Z$  pdom  $Y$ ) if and only if all paths from  $Y$  to an exit block must pass through  $Z$ . Notions of immediate postdominance and a postdominator tree carry over.**

**Note that if a CFG has a single exit node, then postdominance is equivalent to dominance if flow is reversed (going from the exit node to the start node).**



Control Flow Graph



Postdominator Tree

# Reading Assignment

- **Section 14.5 - 14.7 of CaC**
- **Pages 31 - 63 of “Automatic Program Optimization”**
- **Assignment 2**

# Dominance Frontiers

**Dominators and postdominators tell us which basic block must be executed prior to, or after, a block N.**

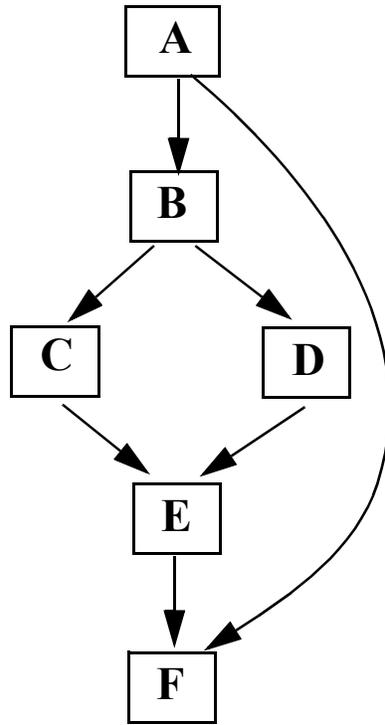
**It is interesting to consider blocks “just before” or “just after” blocks we’re dominated by, or blocks we dominate.**

**The Dominance Frontier of a basic block N,  $DF(N)$ , is the set of all blocks that are immediate successors to blocks dominated by N, but which aren’t themselves strictly dominated by N.**

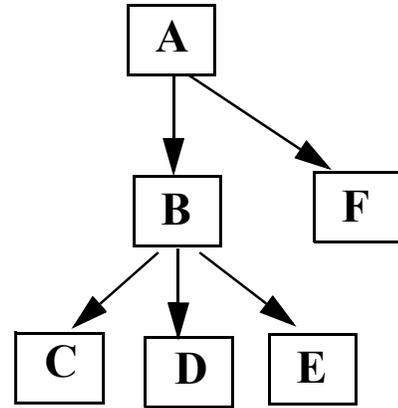
$$\mathbf{DF(N) = \{Z \mid M \rightarrow Z \ \& \ (N \text{ dom } M) \ \& \ \neg(N \text{ sdom } Z)\}}$$

**The dominance frontier of N is the set of blocks that are not dominated N and which are “first reached” on paths from N.**

# Example



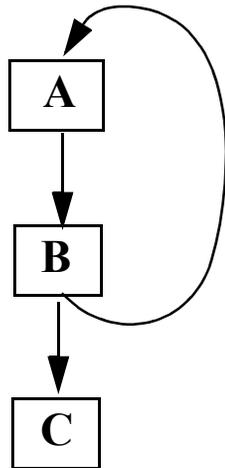
Control Flow Graph



Dominator Tree

Block	A	B	C	D	E	F
Dominance	$\phi$	{F}	{E}	{E}	{F}	$\phi$
Frontier						

## A block can be in its own Dominance Frontier:



Here,  $DF(A) = \{A\}$

Why? Reconsider the definition:

$$DF(N) = \{Z \mid M \rightarrow Z \ \& \ (N \text{ dom } M) \ \& \ \neg(N \text{ sdom } Z)\}$$

Now **B** is dominated by **A** and  $B \rightarrow A$ .

Moreover, **A** does not *strictly* dominate itself. So, it meets the definition.

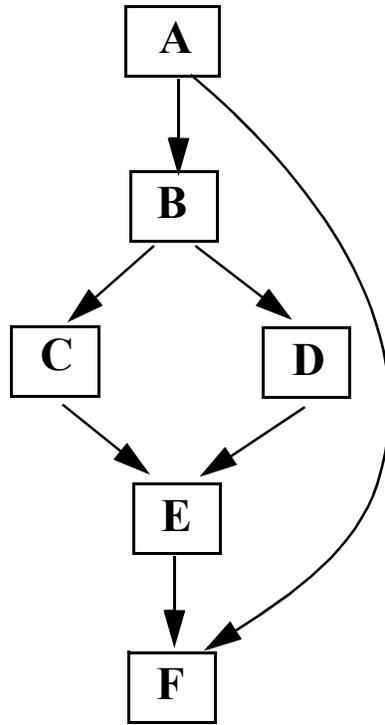
# Postdominance Frontiers

**The Postdominance Frontier of a basic block  $N$ ,  $PDF(N)$ , is the set of all blocks that are immediate predecessors to blocks postdominated by  $N$ , but which aren't themselves postdominated by  $N$ .**

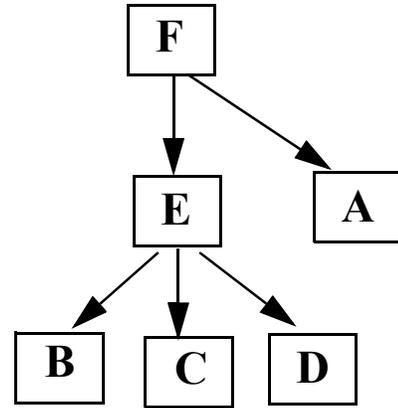
$$PDF(N) = \{Z \mid Z \rightarrow M \ \& \ (N \text{ pdom } M) \ \& \ \neg(N \text{ pdom } Z)\}$$

**The postdominance frontier of  $N$  is the set of blocks closest to  $N$  where a choice was made of whether to reach  $N$  or not.**

# Example



Control Flow Graph



Postominator Tree

Block	A	B	C	D	E	F
Postdominance	$\phi$	{A}	{B}	{B}	{A}	$\phi$
Frontier						

# Control Dependence

**Since CFGs model flow of control, it is useful to identify those basic blocks whose execution is controlled by a branch decision made by a predecessor.**

**We say  $Y$  is *control dependent* on  $X$  if, reaching  $X$ , choosing one out arc will force  $Y$  to be reached, while choosing another arc out of  $X$  allows  $Y$  to be avoided.**

**Formally,  $Y$  is control dependent on  $X$  if and only if,**

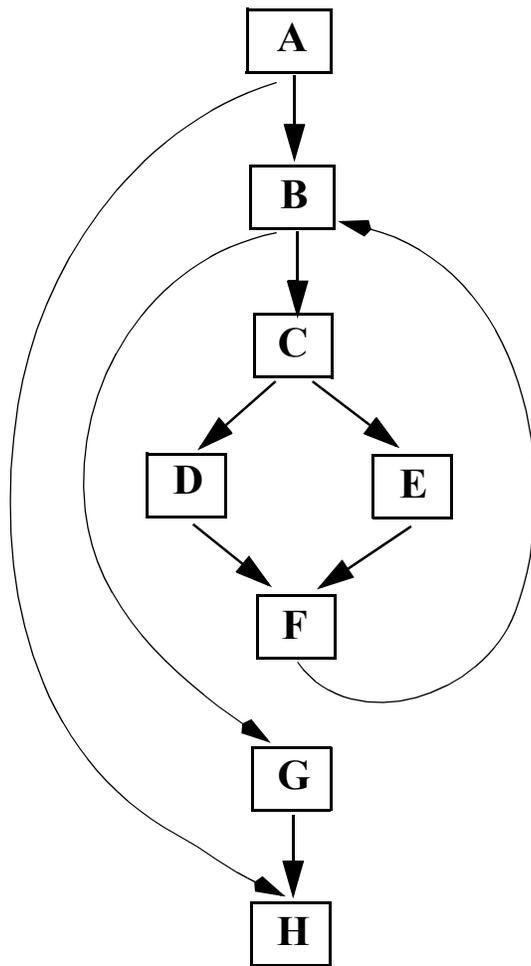
- (a)  $Y$  postdominates a successor of  $X$ .**
- (b)  $Y$  does not postdominate all successors of  $X$ .**

**$X$  is the most recent block where a choice was made to reach  $Y$  or not.**

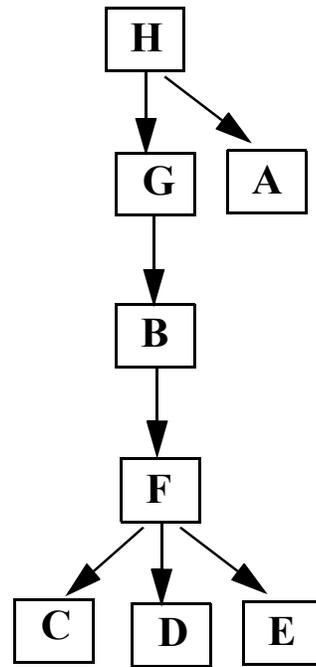
# Control Dependence Graph

**We can build a *Control Dependence Graph* that shows (in graphical form) all Control Dependence relations.**

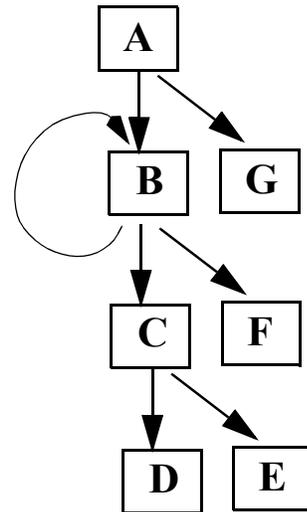
**(A Block *can be* Control Dependent on itself.)**



**Control Flow Graph**



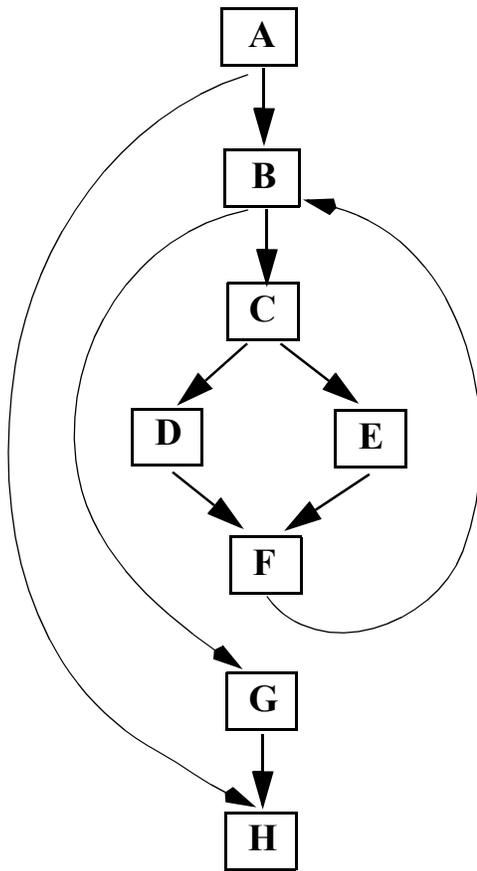
**Postominator Tree**



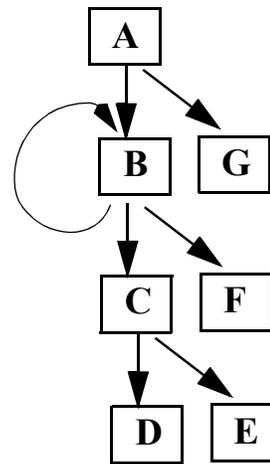
**Control Dependence Graph**

**What happened to H in the CD Graph?**

## Let's reconsider the CD Graph:



Control Flow Graph



Control Dependence Graph

**Blocks C and F, as well as D and E, seem to have the same control dependence relations with their parent. But this isn't so!**

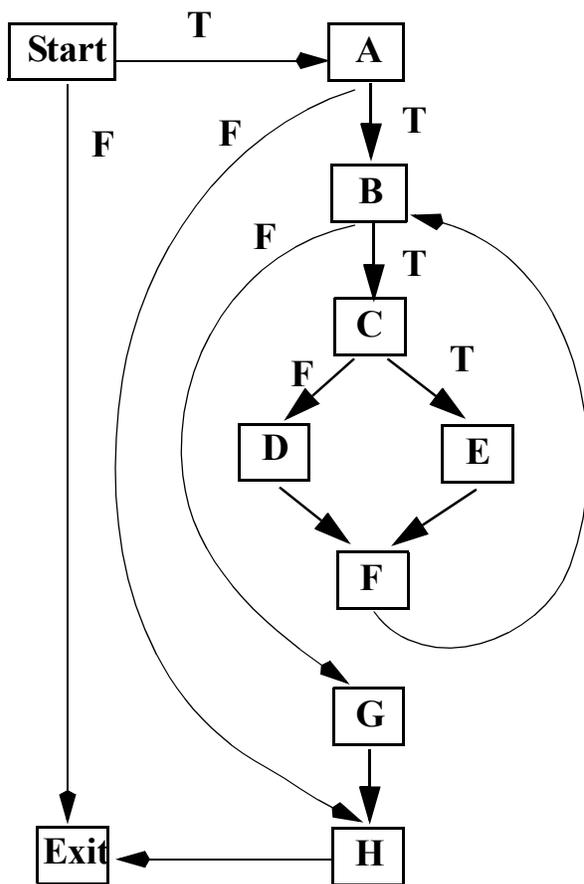
**C and F *are* control equivalent, but D and E are *mutually exclusive*!**

# Improving the Representation of Control Dependence

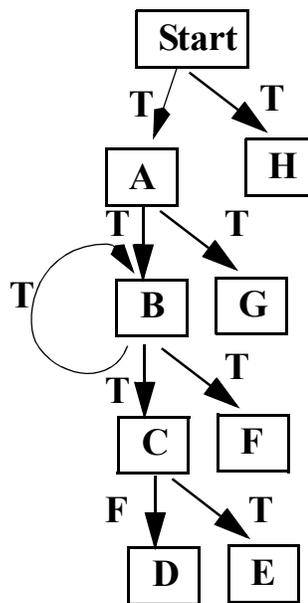
**We can label arcs in the CFG and the CD Graph with the condition (T or F or some switch value) that caused the arc to be selected for execution.**

**This labeling then shows the conditions that lead to the execution of a given block.**

**To allow the exit block to appear in the CD Graph, we can also add “artificial” start and exit blocks, linked together.**



Control Flow Graph



Control Dependence Graph

**C and F have the same Control Dependence relations. They are part of the same extended basic block.**

**But D and E aren't identically control dependent. A and H are control equivalent, as are B and G.**

# Data Flow Frameworks Revisited

**Recall that a Data Flow problem is characterized as:**

- (a) A Control Flow Graph**
- (b) A Lattice of Data Flow values**
- (c) A Meet operator to join solutions from Predecessors or Successors**
- (d) A Transfer Function**  
**Out =  $f_b(\text{In})$  or In =  $f_b(\text{Out})$**

# Value Lattice

**The lattice of values is usually a *meet semilattice* defined by:**

**A: a set of values**

**T and  $\perp$  (“top” and “bottom”):  
distinguished values in the lattice**

**$\leq$ : A reflexive partial order relating  
values in the lattice**

**$\wedge$ : An associative and commutative  
meet operator on lattice values**

# Lattice Axioms

The following axioms apply to the lattice defined by  $\mathbf{A}$ ,  $\mathbf{T}$ ,  $\perp$ ,  $\leq$  and  $\wedge$ :

$$\mathbf{a} \leq \mathbf{b} \Leftrightarrow \mathbf{a} \wedge \mathbf{b} = \mathbf{a}$$

$$\mathbf{a} \wedge \mathbf{a} = \mathbf{a}$$

$$(\mathbf{a} \wedge \mathbf{b}) \leq \mathbf{a}$$

$$(\mathbf{a} \wedge \mathbf{b}) \leq \mathbf{b}$$

$$(\mathbf{a} \wedge \mathbf{T}) = \mathbf{a}$$

$$(\mathbf{a} \wedge \perp) = \perp$$

# Monotone Transfer Function

**Transfer Functions,  $f_b:L \rightarrow L$**   
**(where  $L$  is the Data Flow Lattice)**  
**are normally required to be**  
**monotone.**

**That is  $x \leq y \Rightarrow f_b(x) \leq f_b(y)$ .**

**This rule states that a “worse” input**  
**can’t produce a “better” output.**

**Monotone transfer functions allow**  
**us to guarantee that data flow**  
**solutions are stable.**

**If we had  $f_b(T) = \perp$  and  $f_b(\perp)=T$ ,**  
**then solutions might oscillate**  
**between  $T$  and  $\perp$  indefinitely.**

**Since  $\perp \leq T$ ,  $f_b(\perp)$  should be  $\leq f_b(T)$ .**  
**But  $f_b(\perp) = T$  which is not  $\leq f_b(T) =$**   
 **$\perp$ . Thus  $f_b$  isn’t monotone.**

# Dominators fit the Data Flow Framework

**Given a set of Basic Blocks,  $N$ , we have:**

**$A$  is  $2^N$  (all subsets of Basic Blocks).**

**$T$  is  $N$ .**

**$\perp$  is  $\phi$ .**

**$a \leq b \equiv a \subseteq b$ .**

**$f_Z(\text{in}) = \text{In} \cup \{Z\}$**

**$\wedge$  is  $\cap$  (set intersection).**

**The required axioms are satisfied:**

$$\mathbf{a \subseteq b \Leftrightarrow a \cap b = a}$$

$$\mathbf{a \cap a = a}$$

$$\mathbf{(a \cap b) \subseteq a}$$

$$\mathbf{(a \cap b) \subseteq b}$$

$$\mathbf{(a \cap N) = a}$$

$$\mathbf{(a \cap \phi) = \phi}$$

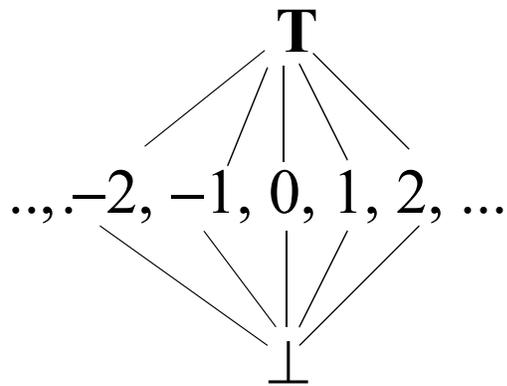
**Also  $f_Z$  is monotone since**

$$\mathbf{a \subseteq b \Rightarrow a \cup \{Z\} \subseteq b \cup \{Z\} \Rightarrow f_Z(a) \subseteq f_Z(b)}$$

# Constant Propagation

**We can model Constant Propagation as a Data Flow Problem. For each scalar integer variable, we will determine whether it is known to hold a particular constant value at a particular basic block.**

**The value lattice is**



**T represents a variable holding a constant, whose value is not yet known.**

**$i$  represents a variable holding a known constant value.**

**$\perp$  represents a variable whose value is non-constant.**

**This analysis is complicated by the fact that variables interact, so we can't just do a series of independent one variable analyses.**

**Instead, the solution lattice will contain functions (or vectors) that map each variable in the program to its constant status ( $T$ ,  $\perp$ , or some integer).**

**Let  $V$  be the set of all variables in a program.**

**Let  $t : V \rightarrow N \cup \{T, \perp\}$**

**$t$  is the set of all total mappings from  $V$  (the set of variables) to  $N \cup \{T, \perp\}$  (the lattice of “constant status” values).**

**For example,  $t_1 = (T, 6, \perp)$  is a mapping for three variables (call them  $A$ ,  $B$  and  $C$ ) into their constant status.  $t_1$  says  $A$  is considered a constant, with value as yet undetermined.  $B$  holds the value 6, and  $C$  is non-constant.**

**We can create a lattice composed of  $t$  functions:**

$$t_T(V) = T \ (\forall V) \ (t_T = (T, T, T, \dots))$$

$$t_{\perp}(V) = \perp \ (\forall V) \ (t_{\perp} = (\perp, \perp, \perp, \dots))$$

$$\mathbf{t}_a \leq \mathbf{t}_b \Leftrightarrow \forall \mathbf{v} \mathbf{t}_a(\mathbf{v}) \leq \mathbf{t}_b(\mathbf{v})$$

**Thus  $(\mathbf{1}, \perp) \leq (\mathbf{T}, \mathbf{3})$**

**since  $\mathbf{1} \leq \mathbf{T}$  and  $\perp \leq \mathbf{3}$ .**

**The meet operator  $\wedge$  is applied  
*componentwise*:**

$$\mathbf{t}_a \wedge \mathbf{t}_b = \mathbf{t}_c$$

**where  $\forall \mathbf{v} \mathbf{t}_c(\mathbf{v}) = \mathbf{t}_a(\mathbf{v}) \wedge \mathbf{t}_b(\mathbf{v})$**

**Thus  $(\mathbf{1}, \perp) \wedge (\mathbf{T}, \mathbf{3}) = (\mathbf{1}, \perp)$**

**since  $\mathbf{1} \wedge \mathbf{T} = \mathbf{1}$  and  $\perp \wedge \mathbf{3} = \perp$ .**

**The lattice axioms hold:**

**$t_a \leq t_b \Leftrightarrow t_a \wedge t_b = t_a$  (since this axiom holds for each component)**

**$t_a \wedge t_a = t_a$  (trivially holds)**

**$(t_a \wedge t_b) \leq t_a$  (per variable def of  $\wedge$ )**

**$(t_a \wedge t_b) \leq t_b$  (per variable def of  $\wedge$ )**

**$(t_a \wedge t_T) = t_a$  (true for all components)**

**$(t_a \wedge t_\perp) = t_\perp$  (true for all components)**

# The Transfer Function

**Constant propagation is a forward flow problem, so  $C_{out} = f_b(C_{in})$**

**$C_{in}$  is a function,  $t(v)$ , that maps variables to  $T, \perp$ , or an integer value  $f_b(t(v))$  is defined as:**

**(1) Initially, let  $t'(v) = t(v)$  ( $\forall v$ )**

**(2) For each assignment statement  $v = e(w_1, w_2, \dots, w_n)$**

**in  $b$ , in order of execution, do:**

**If any  $t'(w_i) = \perp$  ( $1 \leq i \leq n$ )**

**Then set  $t'(v) = \perp$  (strictness)**

**Elsif any  $t'(w_i) = T$  ( $1 \leq i \leq n$ )**

**Then set  $t'(v) = T$  (delay eval of  $v$ )**

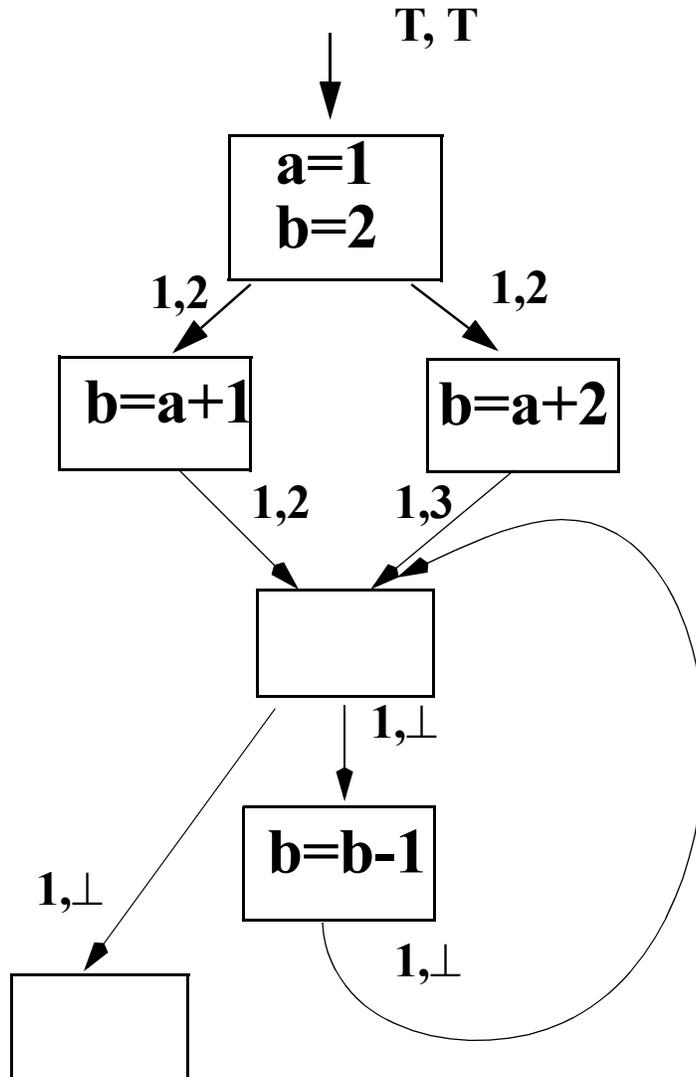
**Else  $t'(v) = e(t'(w_1), t'(w_2), \dots)$**

**(3)  $C_{out} = t'(v)$**

**Note that in valid programs, we don't use uninitialized variables, so variables mapped to T should only occur prior to initialization.**

**Initially, all variables are mapped to T, indicating that initially their constant status is unknown.**

# Example



# Distributive Functions

From the properties of  $\wedge$  and  $f$ 's monotone property, we can show

$$f(a \wedge b) \leq f(a) \wedge f(b)$$

To see this note that

$$a \wedge b \leq a, a \wedge b \leq b \Rightarrow$$

$$f(a \wedge b) \leq f(a), f(a \wedge b) \leq f(b) \quad (*)$$

Now we can establish that

$$x \leq y, x \leq z \Rightarrow x \leq y \wedge z \quad (**)$$

To see that **(\*\*)** holds, note that

$$x \leq y \Rightarrow x \wedge y = x$$

$$x \leq z \Rightarrow x \wedge z = x$$

$$(y \wedge z) \wedge x \leq y \wedge z$$

$$(y \wedge z) \wedge x = (y \wedge z) \wedge (x \wedge x) =$$

$$(y \wedge x) \wedge (z \wedge x) = x \wedge x = x$$

Thus  $x \leq y \wedge z$ , establishing **(\*\*)**.

**Now substituting  $f(a \wedge b)$  for  $x$ ,  
 $f(a)$  for  $y$  and  $f(b)$  for  $z$  in (\*\*) and  
using (\*) we get  
 $f(a \wedge b) \leq f(a) \wedge f(b)$ .**

**Many Data Flow problems have  
flow equations that satisfy the  
*distributive property*:**

$$\mathbf{f(a \wedge b) = f(a) \wedge f(b)}$$

**For example, in our formulation of  
dominators:**

$$\mathbf{Out = f_b(In) = In \cup \{b\}}$$

**where**

$$\mathbf{In = \bigcap_{p \in \text{Pred}(b)} \text{Out}(p)}$$

**In this case,  $\wedge = \cap$ .**

**Now  $f_b(S_1 \cap S_2) = (S_1 \cap S_2) \cup \{b\}$**

**Also,  $f_b(S_1) \cap f_b(S_2) =$**

**$(S_1 \cup \{b\}) \cap (S_2 \cup \{b\}) =$**

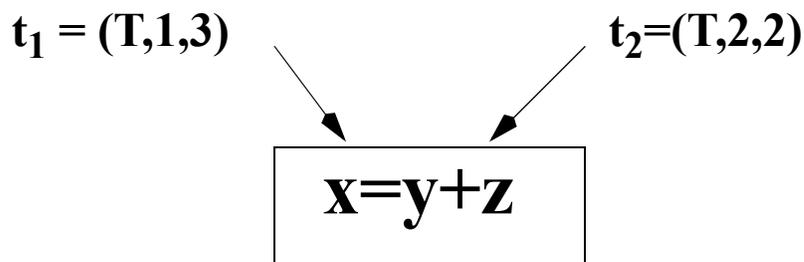
**$(S_1 \cap S_2) \cup \{b\}$**

**So dominators are distributive.**

# Not all Data Flow Problems are Distributive

Constant propagation is *not* distributive.

Consider the following (with variables (x,y,z)):



Now  $f(t) = t'$  where

$t'(y) = t(y)$ ,  $t'(z) = t(z)$ ,

$t'(x) = \text{if } t(y) = \perp \text{ or } t(z) = \perp$

then  $\perp$

elseif  $t(y) = T$  or  $t(z) = T$

then  $T$

else  $t(y) + t(z)$

**Now  $f(t_1 \wedge t_2) = f(T, \perp, \perp) = (\perp, \perp, \perp)$**

**$f(t_1) = (4, 1, 3)$**

**$f(t_2) = (4, 2, 2)$**

**$f(t_1) \wedge f(t_2) = (4, \perp, \perp) \geq (\perp, \perp, \perp)$**

# Why does it Matter if a Data Flow Problem isn't Distributive?

Consider actual program execution paths from  $b_0$  to (say)  $b_k$ .

One path might be  $b_0, b_{i_1}, b_{i_2}, \dots, b_{i_n}$  where  $b_{i_n} = b_k$ .

At  $b_k$  the Data Flow information we want is

$$f_{i_n}(\dots f_{i_2}(f_{i_1}(f_0(T)))) \equiv f(b_0, b_1, \dots, b_{i_n})$$

On a different path to  $b_k$ , say  $b_0, b_{j_1}, b_{j_2}, \dots, b_{j_m}$ , where  $b_{j_m} = b_k$

the Data Flow result we get is

$$f_{j_m}(\dots f_{j_2}(f_{j_1}(f_0(T)))) \equiv f(b_0, b_{j_1}, \dots, b_{j_m}).$$

**Since we can't know at compile time which path will be taken, we must *combine* all possible paths:**

$$\bigwedge_{p \in \text{all paths to } b_k} f(p)$$

**This is the *meet over all paths* (MOP) solution. It is the *best possible static* solution. (Why?)**

**As we shall see, the meet over all paths solution can be computed efficiently, using standard Data Flow techniques, if the problem is Distributive.**

**Other, non-distributive problems (like Constant Propagation) can't be solved as precisely.**

**Explicitly computing and meeting all paths is prohibitively expensive.**

# Conditional Constant Propagation

We can extend our Constant Propagation Analysis to determine that some paths in a CFG aren't executable. This is *Conditional Constant Propagation*.

Consider

```
i = 1;  
if (i > 0)  
    j = 1;  
else j = 2;
```

Conditional Constant Propagation can determine that the else part of the if is unreachable, and hence j must be 1.

**The idea behind Conditional Constant Propagation is simple. Initially, we mark all edges out of conditionals as “not reachable.”**

**Starting at  $b_0$ , we propagate constant information *only* along edges considered reachable.**

**When a boolean expression  $b(v_1, v_2, \dots)$  controls a conditional branch, we evaluate  $b(v_1, v_2, \dots)$  using the  $t(v)$  mapping that identifies the “constant status” of variables.**

**If  $t(v_i) = T$  for any  $v_i$ , we consider all out edges unreachable (for now).**

**Otherwise, we evaluate  $b(v_1, v_2, \dots)$  using  $t(v)$ , getting true, false or  $\perp$ .**

**Note that the short-circuit properties of boolean operators may yield true or false even if  $t(v_i)=\perp$  for some  $v_i$ .**

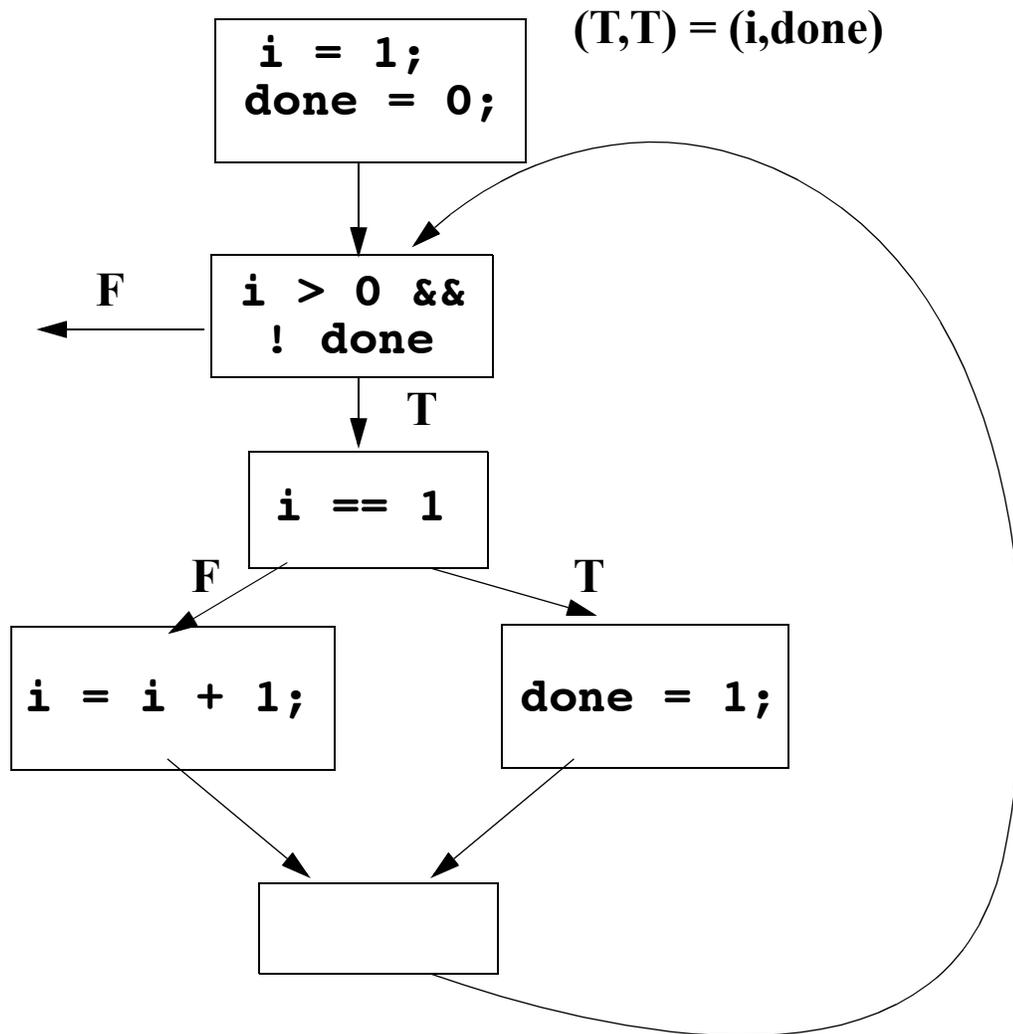
**If  $b(v_1, v_2, \dots)$  is true or false, we mark only one out edge as reachable.**

**Otherwise, if  $b(v_1, v_2, \dots)$  evaluates to  $\perp$ , we mark all out edges as reachable.**

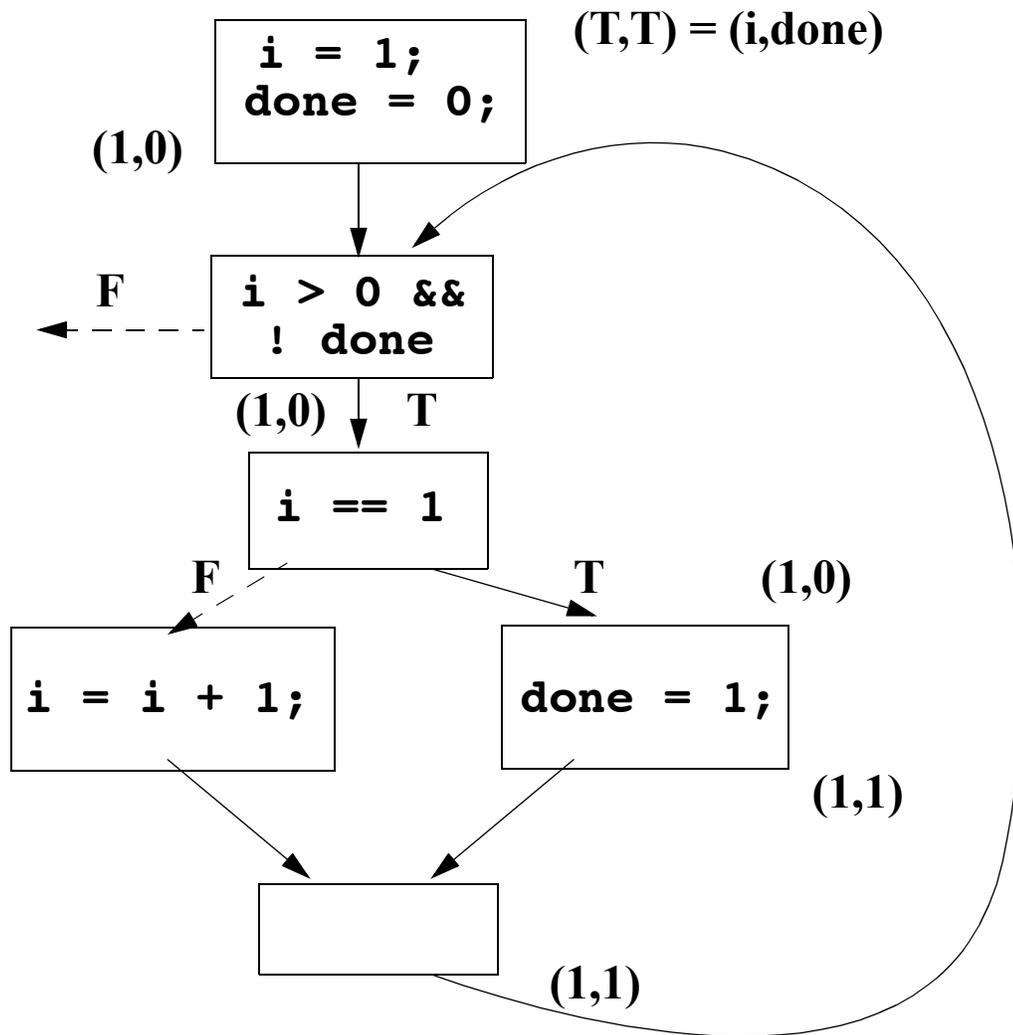
**We propagate constant information only along reachable edges.**

# Example

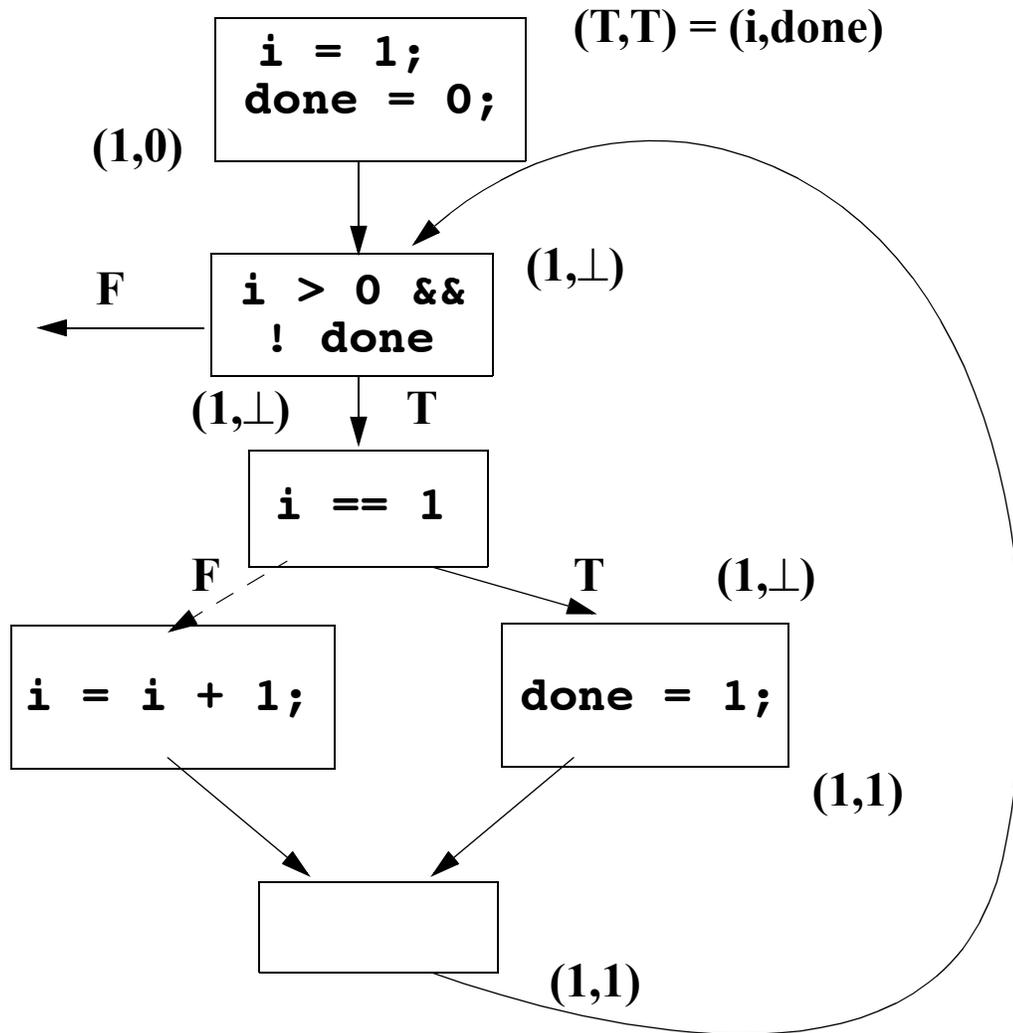
```
i = 1;
done = 0;
while ( i > 0 && ! done) {
    if (i == 1)
        done = 1;
    else i = i + 1; }
```



# Pass 1:



# Pass 2:



# **Iterative Solution of Data Flow Problems**

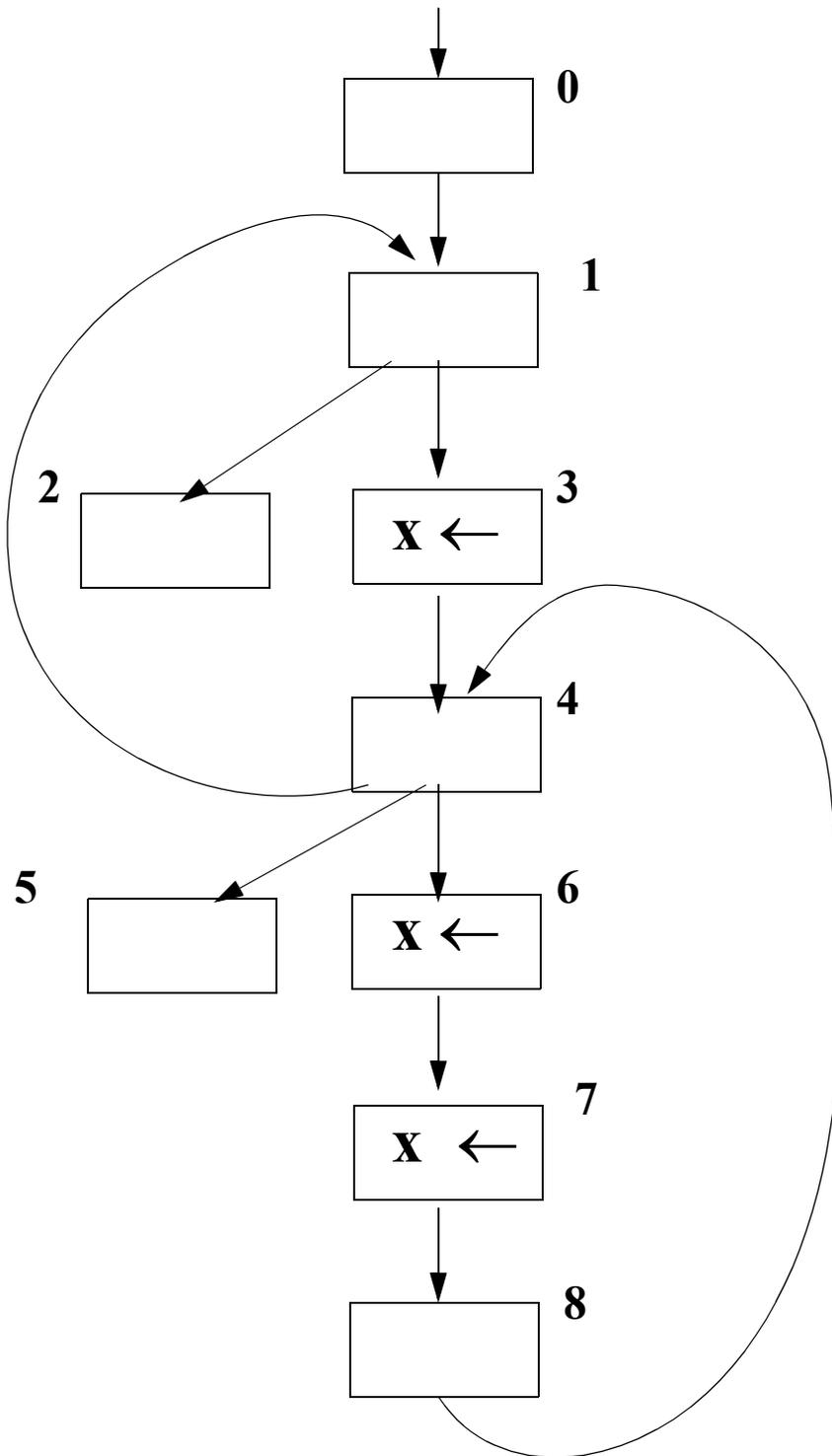
**This algorithm will use DFO numbering to determine the order in which blocks are visited for evaluation. We iterate over the nodes until convergence.**

```

EvalDF{
  For (all n  $\in$  CFG) {
    soln(n) = T
    ReEval(n) = true }
  Repeat
    LoopAgain = false
    For (all n  $\in$  CFG in DFO order) {
      If (ReEval(n)) {
        ReEval(n) = false
        OldSoln = soln(n)
        In =  $\bigwedge_{p \in \text{Pred}(n)} \text{soln}(p)$ 
        soln(n) =  $f_n(\text{In})$ 
        If (soln(n)  $\neq$  OldSoln) {
          For (all s  $\in$  Succ(n)) {
            ReEval(s) = true
            LoopAgain = LoopAgain OR
              IsBackEdge(n,s)
          } }
        } }
    }
  Until (! LoopAgain)
}

```

# Example: Reaching Defs



**We'll do this as a set-valued problem (though it really is just three bit-valued analyses, since each analysis is independent).**

**L is the power set of Basic Blocks**

**$\wedge$  is set union**

**T is  $\phi$ ;  $\perp$  is the set of all blocks**

**$a \leq b \equiv b \subseteq a$**

**$f_3(\text{in}) = \{3\}$**

**$f_6(\text{in}) = \{6\}$**

**$f_7(\text{in}) = \{7\}$**

**For all other blocks,  $f_b(\text{in}) = \text{in}$**

## We'll track soln and ReEval across multiple passes

	0	1	2	3	4	5	6	7	8	Loop- Again
<b>Initial</b>	$\phi$	<b>true</b>								
	<b>true</b>									
<b>Pass 1</b>	$\phi$	$\phi$	$\phi$	{3}	{3}	{3}	{6}	{7}	{7}	<b>true</b>
	<b>false</b>	<b>true</b>	<b>false</b>	<b>false</b>	<b>true</b>	<b>false</b>	<b>false</b>	<b>false</b>	<b>false</b>	
<b>Pass 2</b>	$\phi$	{3}	{3}	{3}	{3,7}	{3,7}	{6}	{7}	{7}	<b>true</b>
	<b>false</b>	<b>true</b>	<b>false</b>							
<b>Pass 3</b>	$\phi$	{3,7}	{3,7}	{3}	{3,7}	{3,7}	{6}	{7}	{7}	<b>false</b>
	<b>false</b>									

# Properties of Iterative Data Flow Analysis

- **If the height of the lattice (the maximum distance from  $T$  to  $\perp$ ) is finite, then termination is *guaranteed*. Why?**

**Recall that transfer functions are assumed monotone ( $a \leq b \Rightarrow f(a) \leq f(b)$ ). Also,  $\wedge$  has the property that  $a \wedge b \leq a$  and  $a \wedge b \leq b$ .**

**At each iteration, some solution value must change, else we halt. If something changes it must “move down” the lattice (we start at  $T$ ). If the lattice has finite height, each block’s value can change only a bounded number of times. Hence termination is guaranteed.**

- **If the iterative data flow algorithm terminates, a valid solution *must* have been computed. (This is because data flow values flow forward, and any change along a backedge forces another iteration.)**

# How Many Iterations are Needed?

**Can we bound the number of iterations needed to compute a data flow solution?**

**In our example, 3 passes were needed, but why?**

**In an “ideal” CFG, with no loops or backedges, only 1 pass is needed.**

**With backedges, it can take several passes for a value computed in one block to reach a block that depends upon the value.**

**Let  $p$  be the maximum number of backedges in any acyclic path in the CFG.**

**Then  $(p+1)$  passes suffice to propagate a data flow value to any other block that uses it.**

**Recall that any block's value can change only a bounded number of times. In fact, the height of the lattice (maximum distance from top to bottom) is that bound.**

**Thus the maximum number of passes in our iterative data flow evaluator =**

**$(p+1) * \text{Height of Lattice}$**

**In our example,  $p = 2$  and lattice height really was 1 (we did 3 independent bit valued problems).**

**So passes needed =  $(2+1)*1 = 3$ .**

# Rapid Data Flow Frameworks

**We still have the concern that it may take many passes to traverse a solution lattice that has a significant height.**

**Many data flow problems are *rapid*. For rapid data flow problems, extra passes to feed back values along cyclic paths aren't needed.**

**For a data flow problem to be rapid we require that:**

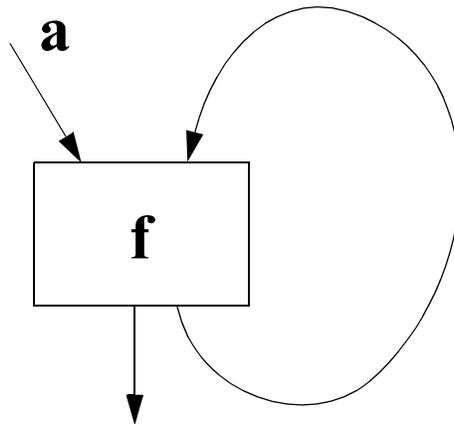
$$(\forall \mathbf{a} \in \mathbf{A})(\forall \mathbf{f} \in \mathbf{F}) \quad \mathbf{a} \wedge \mathbf{f}(\mathbf{T}) \leq \mathbf{f}(\mathbf{a})$$

**This is an odd requirement that states that using  $f(T)$  as a very crude approximation to a value computed by  $f$  is OK when joined using the  $\wedge$  operator. In effect the term “ $a$ ” rather than  $f(T)$  is dominant).**

**(Recall that  $a \wedge f(a) \leq f(a)$  always holds.)**

# How does the Rapid Data Flow Property Help?

**Consider a direct feedback loop (the idea holds for indirect loops too):**



**a is an input from outside the loop.**

**Our concern is how often we'll need to reevaluate f, as new values are computed and fed back into f.**

**Initially, we'll use T to model the value on the backedge.**

**Iteration 1: Input =  $a \wedge T = a$**

**Output =  $f(a)$**

**Iteration 2: Input =  $a \wedge f(a)$**

**Output =  $f(a \wedge f(a))$**

**Iteration 3: Input =  $a \wedge f(a \wedge f(a))$**

**Now we'll exploit the rapid data flow property:  $b \wedge f(T) \leq f(b)$**

**Let  $b \equiv a \wedge f(a)$**

**Then  $a \wedge f(a) \wedge f(T) \leq f(a \wedge f(a))$  (\*)**

**Note that  $x \leq y \Rightarrow a \wedge x \leq a \wedge y$  (\*\*)**

**To prove this, recall that**

**(1)  $p \wedge q = p \Rightarrow p \leq q$**

**(2)  $x \leq y \Rightarrow x \wedge y = x$**

**Thus  $(a \wedge x) \wedge (a \wedge y) = a \wedge (x \wedge y) = (a \wedge x)$   
(by 2)  $\Rightarrow (a \wedge x) \leq (a \wedge y)$  (by 1).**

**From (\*) and (\*\*) we get**

$$\mathbf{a \wedge a \wedge f(a) \wedge f(T) \leq f(a \wedge f(a)) \wedge a \quad (***)}$$

**Now  $\mathbf{a \leq T \Rightarrow f(a) \leq f(T) \Rightarrow}$**

$$\mathbf{f(a) \wedge f(T) = f(a).}$$

**Using this on (\*\*\*) we get**

$$\mathbf{a \wedge f(a) \leq f(a \wedge f(a)) \wedge a}$$

**That is,  $\mathbf{Input_2 \leq Input_3}$**

**Note too that**

$$\mathbf{a \wedge f(a) \leq a \Rightarrow f(a \wedge f(a)) \leq f(a) \Rightarrow}$$

$$\mathbf{a \wedge f(a \wedge f(a)) \leq a \wedge f(a)}$$

**That is,  $\mathbf{Input_3 \leq Input_2}$**

**Thus we conclude  $\mathbf{Input_2 = Input_3}$ ,  
which means we can stop after two  
passes *independent* of lattice height!**

**(One initial visit plus one  
reevaluation via the backedge.)**

# Many Important Data Flow Problems are Rapid

Consider reaching definitions, done as sets. We may have many definitions to the same variable, so the height of the lattice may be large.

$L$  is the power set of Basic Blocks

$\wedge$  is set union

$T$  is  $\phi$ ;  $\perp$  is the set of all blocks

$a \leq b \equiv a \supseteq b$

$f_b(\text{in}) = (\text{In} - \text{Kill}_b) \cup \text{Gen}_b$

where  $\text{Gen}_b$  is the last definition to a variable in  $b$ ,

$\text{Kill}_b$  is all defs to a variable except the last one in  $b$ ,

**Kill<sub>b</sub> is empty if there is no def to a variable in b.**

**The Rapid Data Flow Property is**

$$\mathbf{a \wedge f(T) \leq f(a)}$$

**In terms of Reaching Definitions this is**

$$\mathbf{a \cup f(\phi) \supseteq f(a) \equiv}$$

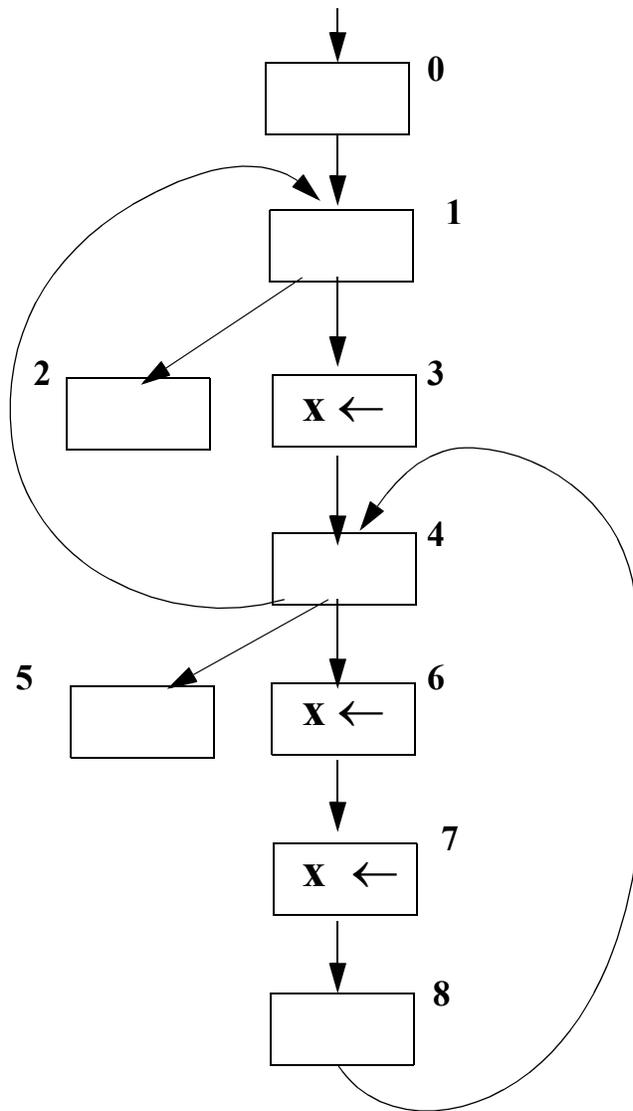
$$\mathbf{a \cup (\phi - Kill) \cup Gen \supseteq (a - Kill) \cup Gen}$$

**Simplifying,**

$$\mathbf{a \cup Gen \supseteq (a - Kill) \cup Gen}$$

**which always holds.**

# Recall



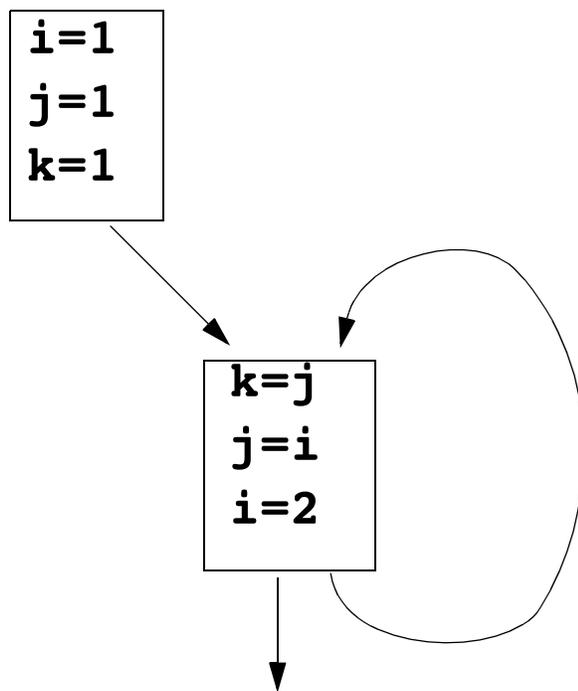
**Here it took two passes to transmit the def in b7 to b1, so we expect 3 passes to evaluate *independent* of the lattice height.**

# Constant Propagation isn't Rapid

We require that

$$\mathbf{a} \wedge \mathbf{f}(\mathbf{T}) \leq \mathbf{f}(\mathbf{a})$$

Consider



**Look at the transfer function for the second (bottom) block.**

**$f(t) = t'$  where**

**$t'(v) = \text{case}(v)\{$**

**$k: t(j);$**

**$j: t(i);$**

**$i: 2; \}$**

**Let  $a = (\perp, 1, 1)$ .**

**$f(T) = (2, T, T)$**

**$a \wedge f(T) = (\perp, 1, 1) \wedge (2, T, T) = (\perp, 1, 1)$**

**$f(a) = f(\perp, 1, 1) = (2, \perp, 1)$ .**

**Now  $(\perp, 1, 1)$  is not  $\leq (2, \perp, 1)$**

**so this problem isn't rapid.**

**Let's follow the iterations:**

**Pass 1: In = (1,1,1)^(T,T,T) = (1,1,1)**

**Out = (2,1,1)**

**Pass 2: In = (1,1,1)^(2,1,1) = ( $\perp$ ,1,1)**

**Out = (2, $\perp$ ,1)**

**Pass 3: In = (1,1,1)^(2, $\perp$ ,1) = ( $\perp$ , $\perp$ ,1)**

**Out = (2, $\perp$ , $\perp$ )**

**This took 3 passes. In general, if we had N variables, we could require N passes, with each pass resolving the constant status of one variable.**

# How Good Is Iterative Data Flow Analysis?

**A single execution of a program will follow some path**

$$\mathbf{b_0, b_{i_1}, b_{i_2}, \dots, b_{i_n}.$$

**The Data Flow solution along this path is**

$$\mathbf{f_{i_n} (\dots f_{i_2} (f_{i_1} (f_0(T)))) \equiv f(b_0, b_1, \dots, b_{i_n})}$$

**The best possible static data flow solution at some block  $b$  is computed over all possible paths from  $b_0$  to  $b$ .**

**Let  $P_b =$  The set of all paths from  $b_0$  to  $b$ .**

$$\mathbf{MOP(b) = \bigwedge_{p \in P_b} f(p)}$$

**Any particular path  $p_i$  from  $b_0$  to  $b$  is included in  $P_b$ .**

**Thus  $\text{MOP}(b) \wedge f(p_i) = \text{MOP}(b) \leq f(p_i)$ .**

**This means  $\text{MOP}(b)$  is *always* a safe approximation to the “true” solution  $f(p_i)$ .**

**If we have the distributive property for transfer functions,**

$$\mathbf{f(a \wedge b) = f(a) \wedge f(b)}$$

**then our iterative algorithm *always* computes the MOP solution, the best static solution possible.**

**To prove this, note that for trivial path of length 1, containing only the start block,  $b_0$ , the algorithm computes  $f_0(T)$  which is  $\text{MOP}(b_0)$  (trivially).**

**Now assume that the iterative algorithm for paths of length  $n$  or less to block  $c$  *does* compute  $\text{MOP}(c)$ .**

**We'll show that for paths to block  $b$  of length  $n+1$ ,  $\text{MOP}(b)$  is computed.**

**Let  $P$  be the set of all paths to  $b$  of length  $n+1$  or less.**

**The paths in  $P$  end with  $b$ .**

**$\text{MOP}(b) = f_b(f(P_1)) \wedge f_b(f(P_2)) \wedge \dots$**

**where  $P_1, P_2, \dots$  are the prefixes (of length  $n$  or less) of paths in  $P$  with  $b$  removed.**

**Using the distributive property,**

**$f_b(f(P_1)) \wedge f_b(f(P_2)) \wedge \dots =$   
 $f_b(f(P_1) \wedge f(P_2) \wedge \dots).$**

**But note that  $f(P_1) \wedge f(P_2) \wedge \dots$  is just the input to  $f_b$  in our iterative algorithm, which then applies  $f_b$ .**

**Thus  $\text{MOP}(b)$  for paths of length  $n+1$  is computed.**

**For data flow problems that aren't distributive (like constant propagation), the iterative solution is  $\leq$  the MOP solution.**

**This means that the solution is a safe approximation, but perhaps not as “sharp” as we might wish.**

# Reading Assignment

**Read “An Efficient Method of  
Computing Static Single  
Assignment Form.”  
(Linked from the class Web page.)**

# Exploiting Structure in Data Flow Analysis

**So far we haven't utilized the fact that CFGs are constructed from standard programming language constructs like IFs, Fors, and Whiles.**

**Instead of iterating across a given CFG, we can isolate, and solve symbolically, subgraphs that correspond to "standard" programming language constructs.**

**We can then progressively simplify the CFG until we reach a single node, or until we reach a CFG structure that matches no standard pattern.**

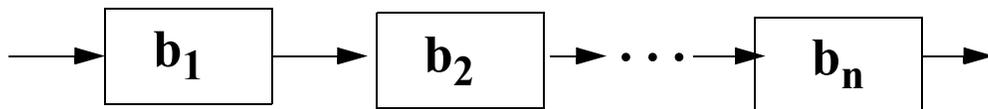
**In the latter case, we can solve the residual graph using our iterative evaluator.**

# Three Program-Building Operations

1. **Sequential Execution (“;”)**
2. **Conditional Execution (If, Switch)**
3. **Iterative Execution  
(While, For, Repeat)**

# Sequential Execution

**We can reduce a sequential “chain” of basic blocks:**



**into a single composite block:**



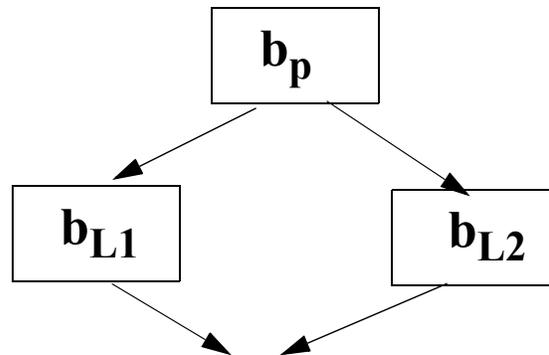
**The transfer function of  $b_{seq}$  is**

$$f_{seq} = f_n \circ f_{n-1} \circ \dots \circ f_1$$

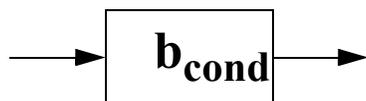
**where  $\circ$  is functional composition.**

# Conditional Execution

Given the basic blocks:



we create a single composite block:



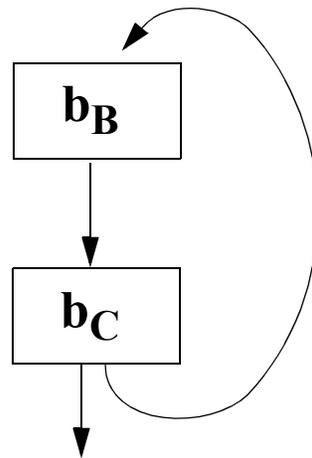
The transfer function of  $b_{\text{cond}}$  is

$$f_{\text{cond}} = f_{L1} \circ f_p \wedge f_{L2} \circ f_p$$

# Iterative Execution

## Repeat Loop

Given the basic blocks:



we create a single composite block:



Here  $b_B$  is the loop body, and  $b_C$  is the loop control.

**If the loop iterates once, the transfer function is  $f_C \circ f_B$ .**

**If the loop iterates twice, the transfer function is  $(f_C \circ f_B) \circ (f_C \circ f_B)$ .**

**Considering all paths, the transfer function is  $(f_C \circ f_B) \wedge (f_C \circ f_B)^2 \wedge \dots$**

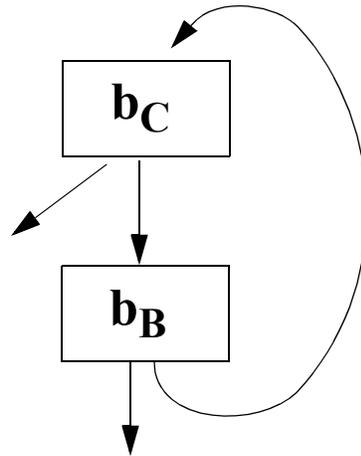
**Define  $\text{fix } f \equiv f \wedge f^2 \wedge f^3 \wedge \dots$**

**The transfer function of repeat is then**

$$f_{\text{repeat}} = \text{fix}(f_C \circ f_B)$$

# While Loop.

Given the basic blocks:



we create a single composite block:



Here again  $b_B$  is the loop body, and  $b_C$  is the loop control.

The loop always executes  $b_C$  at least once, and always executes  $b_C$  as the last block before exiting.

**The transfer function of a while is therefore**

$$\mathbf{f}_{\text{while}} = \mathbf{f}_C \wedge \mathbf{fix}(\mathbf{f}_C \circ \mathbf{f}_B) \circ \mathbf{f}_C$$

# Evaluating Fixed Points

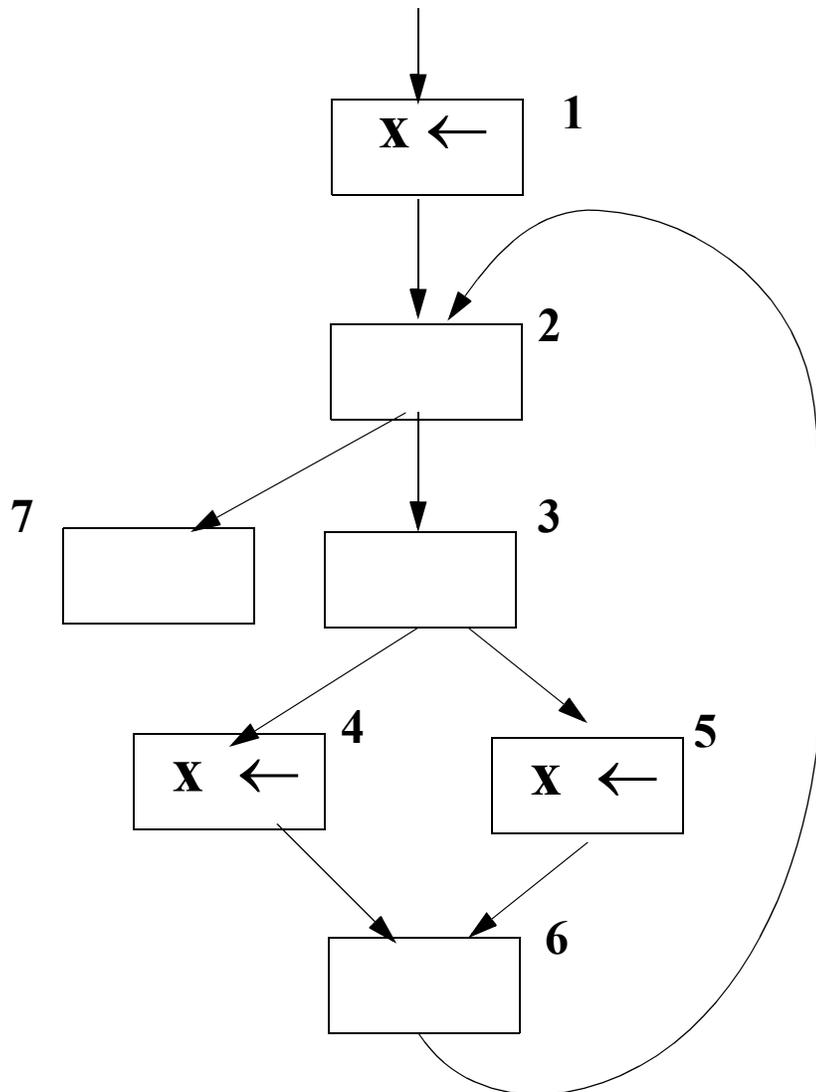
**For lattices of height  $H$ , and monotone transfer functions, `fix f` needs to look at no more than  $H$  terms.**

**In practice, we can give `fix f` an operational definition, suitable for implementation:**

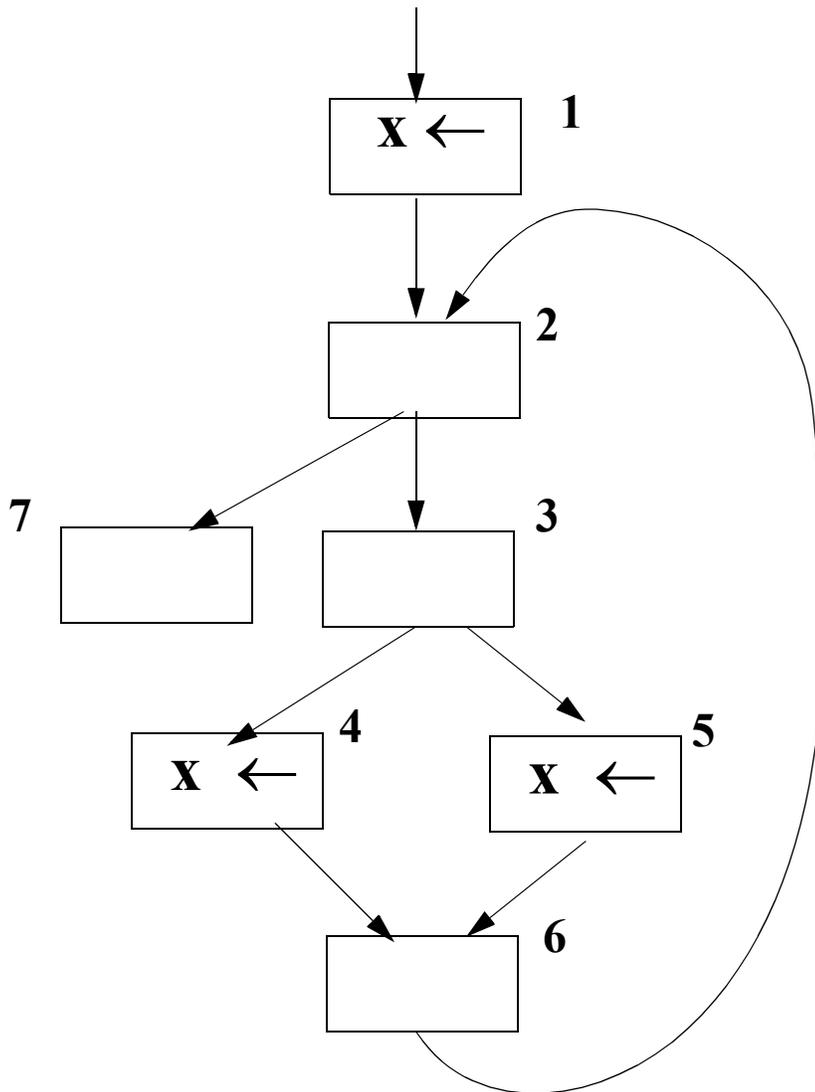
**Evaluate**

```
(fix f)(x) {  
    prev = soln = f(x);  
    while (prev  $\neq$  new = f(prev)){  
        prev = new;  
        soln = soln  $\wedge$  new;  
    }  
    return soln;  
}
```

# Example—Reaching Definitions



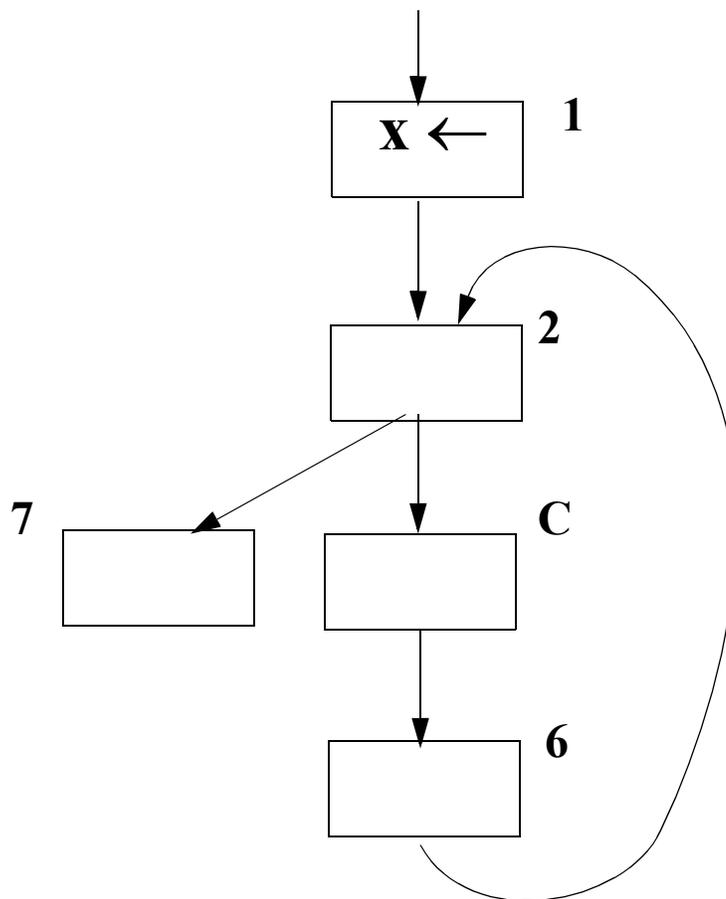
**The transfer functions are either constant-valued ( $f_1=\{b1\}$ ,  $f_4=\{b4\}$ ,  $f_5=\{b5\}$ ) or identity functions ( $f_2=f_3=f_6=f_7=Id$ ).**



**First we isolate and reduce the conditional:**

$$f_C = f_4 \circ f_3 \wedge f_5 \circ f_3 = \{b_4\} \circ \text{Id} \cup \{b_5\} \circ \text{Id} = \{b_4, b_5\}$$

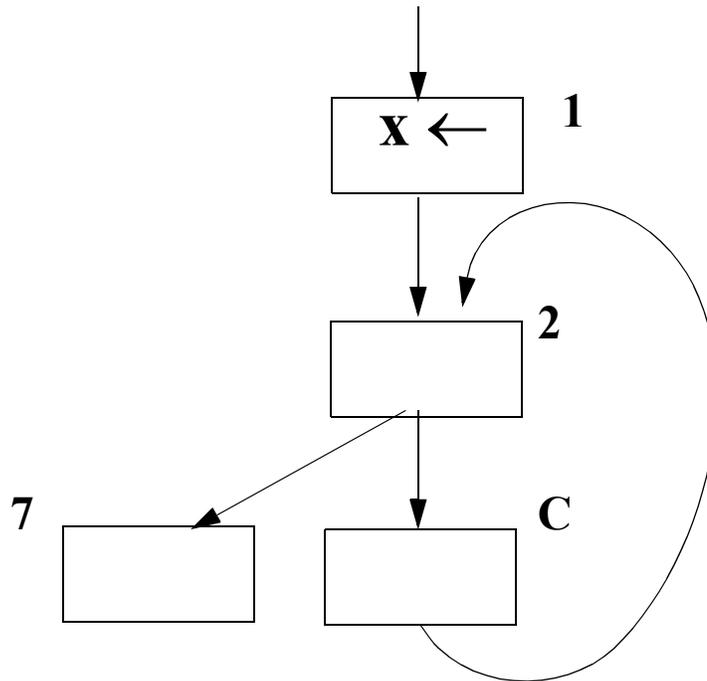
**Substituting, we get**



**We can combine  $b_C$  and  $b_6$ , to get a block equivalent to  $b_C$ . That is,**

$$\mathbf{f_6 \circ f_C = Id \circ f_C = f_C}$$

**We now have**



**We isolate and reduce the while loop formed by  $b_2$  and  $b_C$ , creating  $b_W$ .**

**The transfer function is**

$$f_W = f_2 \wedge (\text{fix}(f_2 \circ f_C)) \circ f_2 =$$

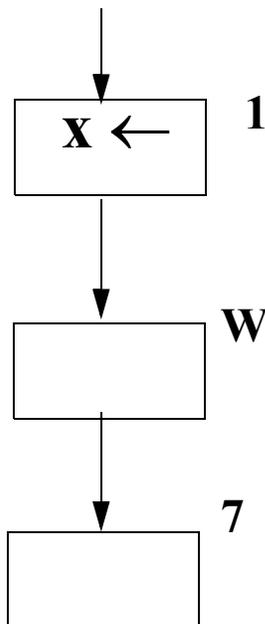
$$\text{Id} \cup (\text{fix}(\text{Id} \circ f_C)) \circ \text{Id} =$$

$$\text{Id} \cup (\text{fix}(f_C)) =$$

$$\text{Id} \cup (f_C \wedge f_C^2 \wedge f_C^3 \wedge \dots) =$$

$$\text{Id} \cup \{b_4, b_5\}$$

**We now have**



**We compose these three sequential blocks to get the whole solution,  $f_P$ .**

$$f_P = \text{Id} \circ (\text{Id} \cup \{b_4, b_5\}) \circ \{b_1\} = \{b_1, b_4, b_5\}.$$

**These are the definitions that reach the end of the program.**

**We can expand subgraphs to get the solutions at interior blocks.**

**Thus at the beginning of the while, the solution is  $\{b1\}$ .**

**At the head if the If, the solution is**

**$(Id \cup (Id \circ f_C \circ Id) \cup$**

**$(Id \circ f_C \circ Id \circ f_C \circ Id) \cup \dots) \circ (\{b1\})$**

**$= \{b1\} \cup \{b4,b5\} \cup \{b4,b5\} \cup \dots =$**

**$\{b1,b4,b5\}$**

**At the head of the then part of the If, the solution is  $Id(\{b1,b4,b5\}) = \{b1,b4,b5\}$ .**

# Static Single Assignment Form

Many of the complexities of optimization and code generation arise from the fact that a given variable may be assigned to in *many* different places.

Thus reaching definition analysis gives us the *set* of assignments that *may* reach a given use of a variable.

Live range analysis must track *all* assignments that may reach a use of a variable and merge them into the same live range.

Available expression analysis must look at *all* places a variable may be assigned to and decide if any kill an already computed expression.

# What If

**each variable is assigned to in only one place?**

**(Much like a named constant).**

**Then for a given use, we can find a single *unique* definition point.**

**But this seems *impossible* for most programs—or is it?**

**In *Static Single Assignment (SSA)* Form each assignment to a variable,  $v$ , is changed into a unique assignment to new variable,  $v_i$ .**

**If variable  $v$  has  $n$  assignments to it throughout the program, then (at least)  $n$  new variables,  $v_1$  to  $v_n$ , are created to replace  $v$ . All uses of  $v$  are replaced by a use of some  $v_i$ .**

# Phi Functions

**Control flow can't be predicted in advance, so we can't always know which definition of a variable reached a particular use.**

**To handle this uncertainty, we create *phi functions*.**

**As illustrated below, if  $v_i$  and  $v_j$  both reach the top of the same block, we add the assignment**

$$v_k \leftarrow \phi(v_i, v_j)$$

**to the top of the block.**

**Within the block, all uses of  $v$  become uses of  $v_k$  (until the next assignment to  $v$ ).**

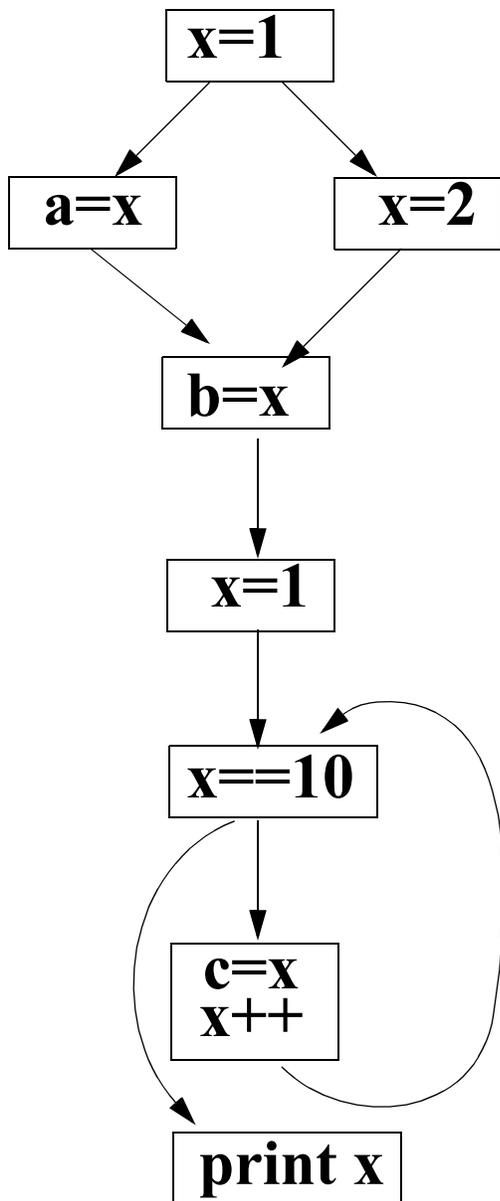
# What does $\phi(v_i, v_j)$ Mean?

**One way to read  $\phi(v_i, v_j)$  is that if control reaches the phi function via the path on which  $v_i$  is defined,  $\phi$  “selects”  $v_i$ ; otherwise it “selects”  $v_j$ .**

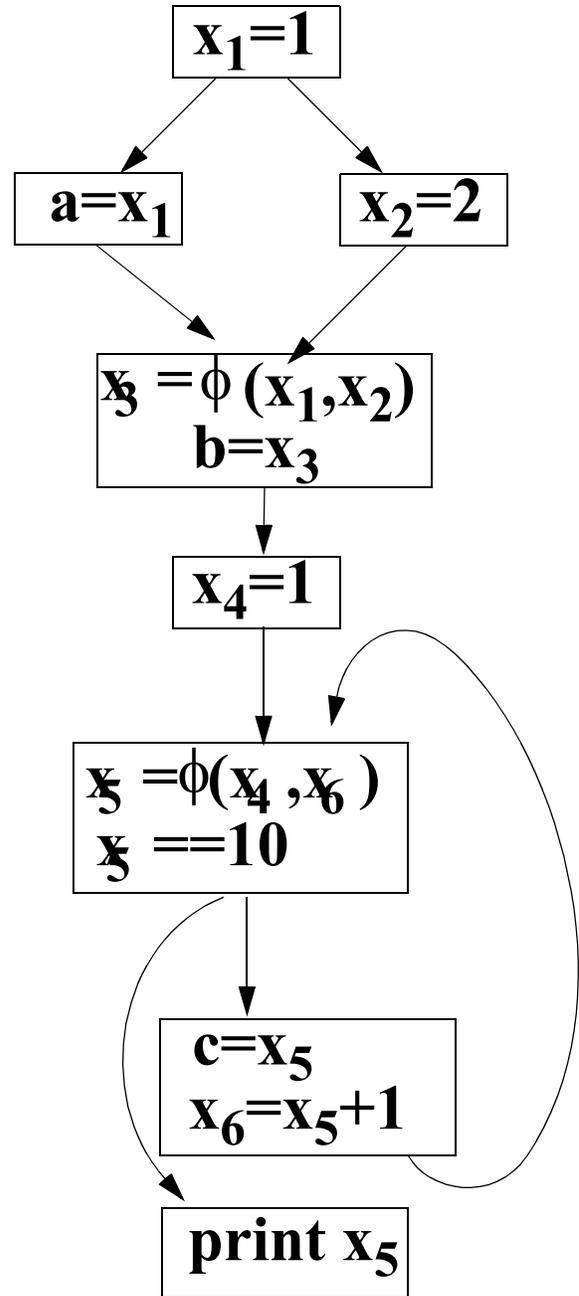
**Phi functions may take more than 2 arguments if more than 2 definitions might reach the same block.**

**Through phi functions we have simple links to all the places where  $v$  receives a value, directly or indirectly.**

# Example



Original CFG



CFG in SSA Form

**In SSA form computing live ranges is almost trivial. For each  $x_i$  include all  $x_j$  variables involved in phi functions that define  $x_i$ .**

**Initially, assume  $x_1$  to  $x_6$  (in our example) are independent. We then union into equivalence classes  $x_i$  values involved in the same phi function or assignment.**

**Thus  $x_1$  to  $x_3$  are unioned together (forming a live range). Similarly,  $x_4$  to  $x_6$  are unioned to form a live range.**

# Constant Propagation in SSA

**In SSA form, constant propagation is simplified since values flow directly from assignments to uses, and phi functions represent natural “meet points” where values are combined (into a constant or  $\perp$ ).**

**Even conditional constant propagation fits in. As long as a path is considered unreachable, it variables are set to T (and therefore ignored at phi functions, which meet values together).**

## Example

```

i=6
j=1
k=1
repeat
  if (i==6)
    k=0
  else
    i=i+1
  i=i+k
  j=j+1
until (i==j)

```

```

i1=6
j1=1
k1=1
repeat
  i2=ϕ(i1, i5)
  j2=ϕ(j1, j3)
  k2=ϕ(k1, k4)
  if (i2==6)
    k3=0
  else
    i3=i2+1
  i4=ϕ(i2, i3)
  k4=ϕ(k3, k2)
  i5=i4+k4
  j3=j2+1
until (i5==j3)

```

	$i_1$	$j_1$	$k_1$	$i_2$	$j_2$	$k_2$	$k_3$	$i_3$	$i_4$	$k_4$	$i_5$	$j_3$
<b>Pass1</b>	6	1	1	$6 \wedge$ T	$1 \wedge$ T	$1 \wedge$ T	0	T	$6 \wedge$ T	0	6	2
<b>Pass2</b>	6	1	1	$6 \wedge 6$	$\perp$	$\perp$	0	T	6	0	6	$\perp$

**We have determined that  $i=6$   
everywhere.**

# Putting Programs into SSA Form

**Assume we have the CFG for a program, which we want to put into SSA form. We must:**

- Rename all definitions and uses of variables**
- Decide where to add phi functions**

**Renaming variable definitions is trivial—each assignment is to a new, unique variable.**

**After phi functions are added (at the heads of selected basic blocks), only one variable definition (the most recent in the block) can reach any use. Thus renaming uses of variables is easy.**

# Placing Phi Functions

**Let  $b$  be a block with a definition to some variable,  $v$ . If  $b$  contains more than one definition to  $v$ , the last (or most recent) applies.**

**What is the first basic block following  $b$  where some other definition to  $v$  *as well as*  $b$ 's definition can reach?**

**In blocks dominated by  $b$ ,  $b$ 's definition *must* have been executed, though other later definitions may have overwritten  $b$ 's definition.**

# Domination Frontiers (Again)

Recall that the **Domination Frontier** of a block  $b$ , is defined as

$$\text{DF}(N) = \{Z \mid M \rightarrow Z \ \& \ (N \text{ dom } M) \ \& \ \neg(N \text{ sdom } Z)\}$$

The **Dominance Frontier** of a basic block  $N$ ,  $\text{DF}(N)$ , is the set of all blocks that are immediate successors to blocks dominated by  $N$ , but which aren't themselves strictly dominated by  $N$ .

Assume that an initial assignment to all variables occurs in  $b_0$  (possibly of some special “uninitialized value.”)

**We will need to place a phi function at the start of all blocks in  $b$ 's Domination Frontier.**

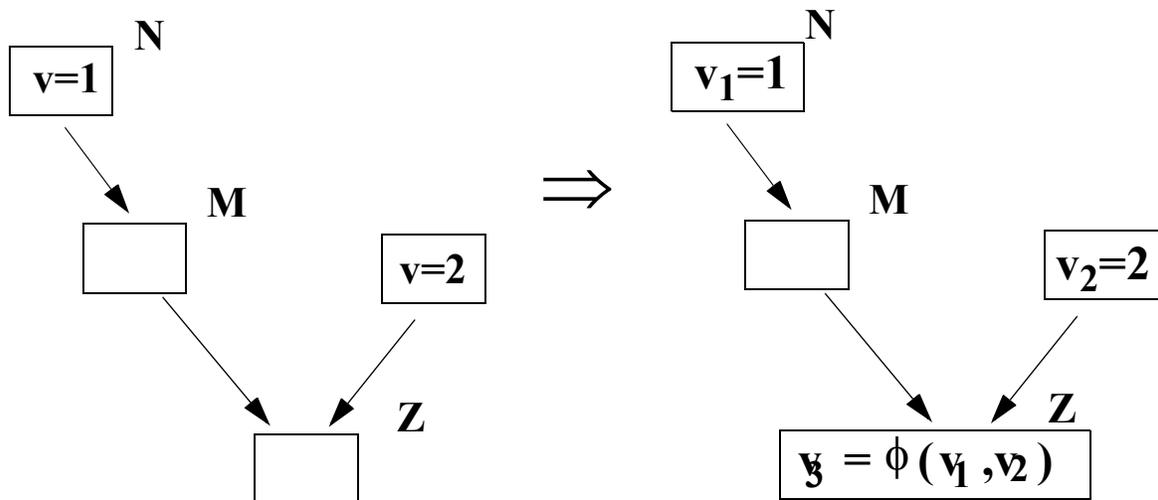
**The phi functions will join the definition to  $v$  that occurred in  $b$  (or in a block dominated by  $b$ ) with definitions occurring on paths that don't include  $b$ .**

**After phi functions are added to blocks in  $DF(b)$ , the domination frontier of blocks with newly added phi's will need to be computed (since phi functions imply assignment to a new  $v_i$  variable).**

# Examples of How Domination Frontiers Guide Phi Placement

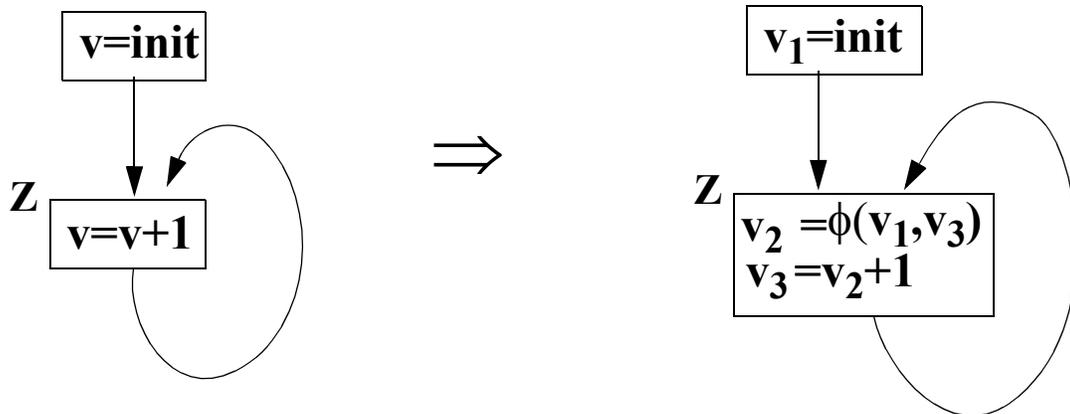
$$\text{DF}(N) = \{Z \mid M \rightarrow Z \ \& \ (N \text{ dom } M) \ \& \ \neg(N \text{ sdom } Z)\}$$

Simple Case:



Here,  $(N \text{ dom } M)$  but  $\neg(N \text{ sdom } Z)$ ,  
so a phi function is needed in  $Z$ .

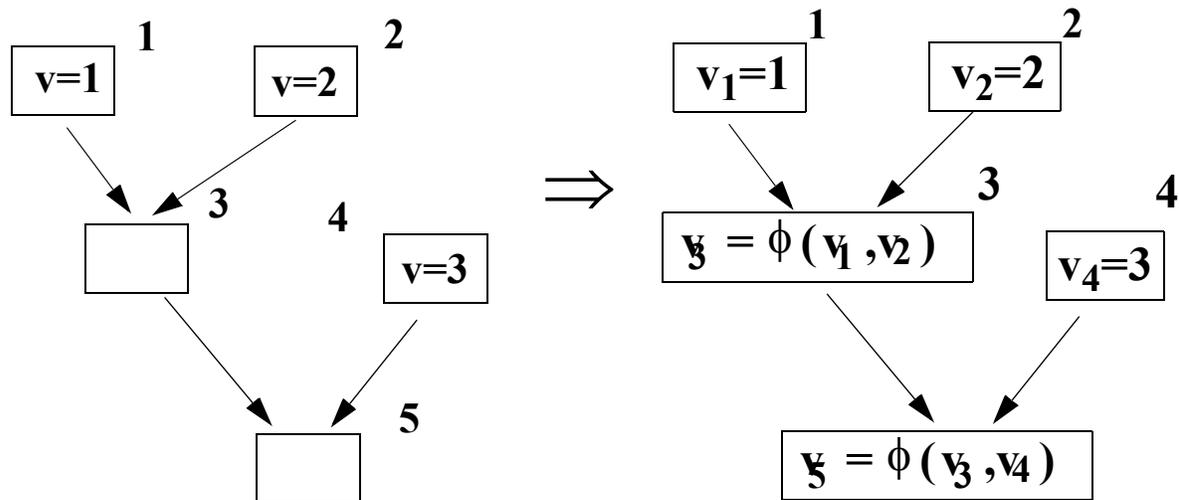
# Loop:



Here, let  $M = Z = N$ .  $M \rightarrow Z$ ,  
( $N \text{ dom } M$ ) but  $\neg(N \text{ sdom } Z)$ ,  
so a phi function *is* needed in  $Z$ .

$DF(N) =$   
 $\{Z \mid M \rightarrow Z \ \& \ (N \text{ dom } M) \ \& \ \neg(N \text{ sdom } Z)\}$

# Sometimes Phi's must be Placed Iteratively



**Now,  $DF(b1) = \{b3\}$ , so we add a phi function in  $b3$ . This adds an assignment into  $b3$ . We then look at  $DF(b3) = \{b5\}$ , so another phi function must be added to  $b5$ .**

# Phi Placement Algorithm

**To decide what blocks require a phi function to join a definition to a variable  $v$  in block  $b$ :**

**1. Compute  $D_1 = DF(b)$ .**

**Place Phi functions at the head of all members of  $D_1$ .**

**2. Compute  $D_2 = DF(D_1)$ .**

**Place Phi functions at the head of all members of  $D_2 - D_1$ .**

**3. Compute  $D_3 = DF(D_2)$ .**

**Place Phi functions at the head of all members of  $D_3 - D_2 - D_1$ .**

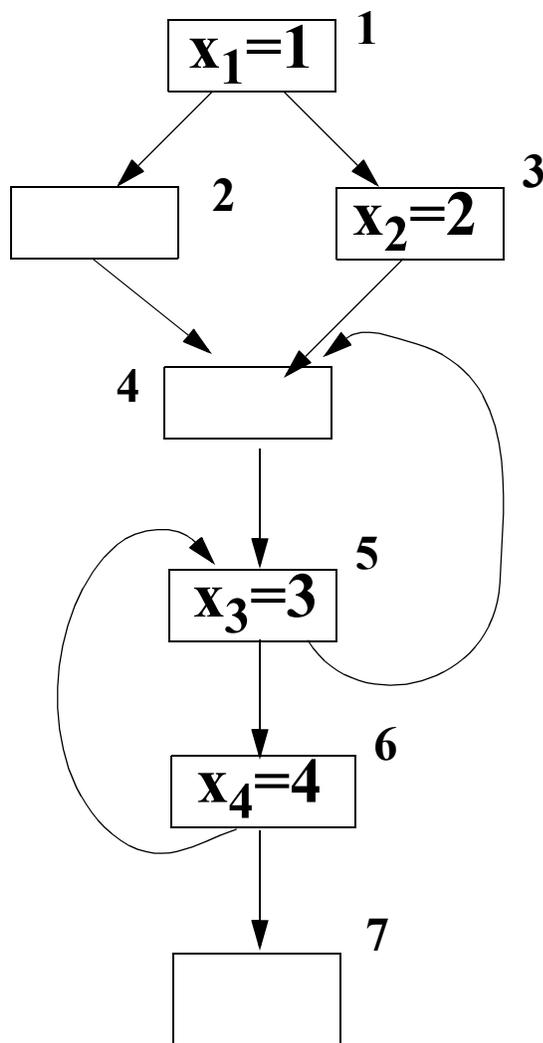
**4. Repeat until no additional Phi functions can be added.**

```

PlacePhi{
  For (each variable  $v \in$  program) {
    For (each block  $b \in$  CFG ) {
      PhiInserted( $b$ ) = false
      Added( $b$ ) = false }
    List =  $\phi$ 
    For (each  $b \in$  CFG that assigns to  $V$  ) {
      Added( $b$ ) = true
      List = List U { $b$ } }
    While (List  $\neq$   $\phi$ ) {
      Remove any  $b$  from List
      For (each  $d \in$  DF( $b$ )) {
        If (! PhiInserted( $d$ )) {
          Add a Phi Function to  $d$ 
          PhiInserted( $d$ ) = true
          If (! Added( $d$ )) {
            Added( $d$ ) = true
            List = List U { $d$ }
          }
        }
      }
    }
  }
}

```

# Example



Initially, List={1,3,5,6}

Process 1:  $DF(1) = \phi$

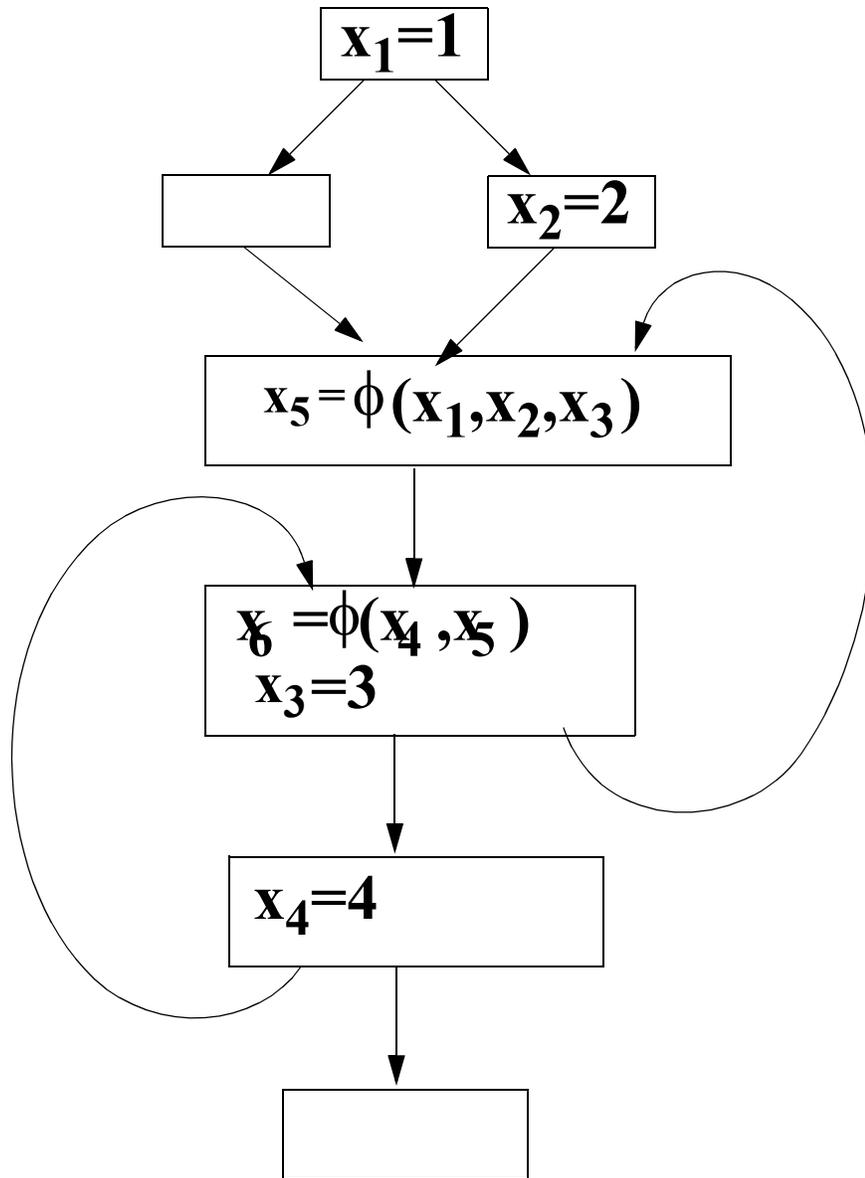
Process 3:  $DF(3) = 4$ ,  
so add 4 to List and  
add phi fct to 4.

Process 5:  $DF(5) = \{4,5\}$   
so add phi fct to 5.

Process 5:  $DF(6) = \{5\}$

Process 4:  $DF(4) = \{4\}$

We will add Phi's into blocks 4 and 5. The arity of each phi is the number of in-arcs to its block. To find the args to a phi, follow each arc "backwards" to the sole reaching def on that path.



# SSA and Value Numbering

**We already know how to do available expression analysis to determine if a previous computation of an expression can be reused.**

**A limitation of this analysis is that it can't recognize that two expressions that aren't syntactically identical may actually still be equivalent.**

**For example, given**

$$\mathbf{t1 = a + b}$$

$$\mathbf{c = a}$$

$$\mathbf{t2 = c + b}$$

**Available expression analysis won't recognize that  $t1$  and  $t2$  must be equivalent, since it doesn't track the fact that  $a = c$  at  $t2$ .**

# Value Numbering

**An early expression analysis technique called *value numbering* worked only at the level of basic blocks. The analysis was in terms of “values” rather than variable or temporary names.**

**Each non-trivial (non-copy) computation is given a number, called its *value number*.**

**Two expressions, using the same operators and operands with the same value numbers, must be equivalent.**

**For example,**

$$\mathbf{t1 = a + b}$$

$$\mathbf{c = a}$$

$$\mathbf{t2 = c + b}$$

**is analyzed as**

$$\mathbf{v1 = a}$$

$$\mathbf{v2 = b}$$

$$\mathbf{t1 = v1 + v2}$$

$$\mathbf{c = v1}$$

$$\mathbf{t2 = v1 + v2}$$

**Clearly  $t_2$  is equivalent to  $t_1$  (and hence need not be computed).**

**In contrast, given**

$$\mathbf{t1 = a + b}$$

$$\mathbf{a = 2}$$

$$\mathbf{t2 = a + b}$$

**the analysis creates**

$$\mathbf{v1 = a}$$

$$\mathbf{v2 = b}$$

$$\mathbf{t1 = v1 + v2}$$

$$\mathbf{v3 = 2}$$

$$\mathbf{t2 = v3 + v2}$$

**Clearly  $t_2$  is not equivalent to  $t_1$   
(and hence will need to be  
recomputed).**

# Extending Value Numbering to Entire CFGs

The problem with a global version of value numbering is how to reconcile values produced on different flow paths. But this is exactly what SSA is designed to do!

In particular, we know that an ordinary assignment

$$x = y$$

does *not* imply that all references to  $x$  can be replaced by  $y$  after the assignment. That is, an assignment is *not* an assertion of value equivalence.

*But,*

**in SSA form**

$$\mathbf{x}_i = \mathbf{y}_j$$

*does* mean the two values are *always* equivalent after the assignment. If  $\mathbf{y}_j$  reaches a use of  $\mathbf{x}_i$ , that use of  $\mathbf{x}_i$  *can* be replaced with  $\mathbf{y}_j$ .

**Thus in SSA form, an assignment *is* an assertion of value equivalence.**

**We will assume that simple variable to variable copies are removed by substituting equivalent SSA names.**

**This alone is enough to recognize some simple value equivalences.**

**As we saw,**

$$\mathbf{t_1 = a_1 + b_1}$$

$$\mathbf{c_1 = a_1}$$

$$\mathbf{t_2 = c_1 + b_1}$$

**becomes**

$$\mathbf{t_1 = a_1 + b_1}$$

$$\mathbf{t_2 = a_1 + b_1}$$

# Partitioning SSA Variables

**Initially, all SSA variables will be partitioned by the *form* of the expression assigned to them.**

**Expressions involving different constants or operators won't (in general) be equivalent, even if their operands happen to be equivalent.**

**Thus**

$$\mathbf{v_1 = 2 \text{ and } w_1 = a_2 + 1}$$

**are always considered inequivalent.**

**But,**

$$\mathbf{v_3 = a_1 + b_2 \text{ and } w_1 = d_1 + e_2}$$

**may *possibly* be equivalent since both involve the same operator.**

**Phi functions are potentially equivalent only if they are in the same basic block.**

**All variables are initially considered equivalent (since they all initially are considered uninitialized until explicit initialization).**

**After SSA variables are grouped by assignment form, groups are split.**

**If  $a_i \text{ op } b_j$  and  $c_k \text{ op } d_l$  are in the same group (because they both have the same operator,  $\text{op}$ ) and  $a_i \neq c_k$  or  $b_j \neq d_l$  then we split the two expressions apart into different groups.**

**We continue splitting based on operand inequivalence, until no more splits are possible. Values still grouped are equivalent.**

# Example

```
if (...) {
  a1=0
  if (...)
    b1=0
  else {
    a2=x0
    b2=x0 }
  a3=φ(a1, a2)
  b3=φ(b1, b2)
  c2=*a3
  d2=*b3 }
else {
  b4=10 }
a5=φ(a0, a3)
b5=φ(b3, b4)
c3=*a5
d3=*b5
e3=*a5
```

Initial Groupings:

$G_1=[a_0, b_0, c_0, d_0, e_0, x_0]$

$G_2=[a_1=0, b_1=0]$

$G_3=[a_2=x_0, b_2=x_0]$

$G_4=[b_4=10]$

$G_5=[a_3=\phi(a_1, a_2),$   
 $b_3=\phi(b_1, b_2)]$

$G_6=[a_5=\phi(a_0, a_3),$   
 $b_5=\phi(b_3, b_4)]$

$G_7=[c_2=*a_3,$   
 $d_2=*b_3,$   
 $d_3=*b_5,$   
 $c_3=*a_5,$   
 $e_3=*a_5]$

**Now  $b_4$  isn't equivalent to anything, so split  $a_5$  and  $b_5$ . In  $G_7$  split operands  $b_3$ ,  $a_5$  and  $b_5$ . We have**

```

if (...) {
  a1=0
  if (...)
    b1=0
  else {
    a2=x0
    b2=x0 }
  a3= $\phi$ (a1, a2)
  b3= $\phi$ (b1, b2)
  c2=*a3
  d2=*b3 }
else {
  b4=10 }
a5= $\phi$ (a0, a3)
b5= $\phi$ (b3, b4)
c3=*a5
d3=*b5
e3=*a5

```

## Final Groupings:

$G_1 = [a_0, b_0, c_0, d_0, e_0, x_0]$

$G_2 = [a_1=0, b_1=0]$

$G_3 = [a_2=x_0, b_2=x_0]$

$G_4 = [b_4=10]$

$G_5 = [a_3=\phi(a_1, a_2),$   
 $b_3=\phi(b_1, b_2)]$

$G_{6a} = [a_5=\phi(a_0, a_3)]$

$G_{6b} = [b_5=\phi(b_3, b_4)]$

$G_{7a} = [c_2=*a_3,$   
 $d_2=*b_3]$

$G_{7b} = [d_3=*b_5]$

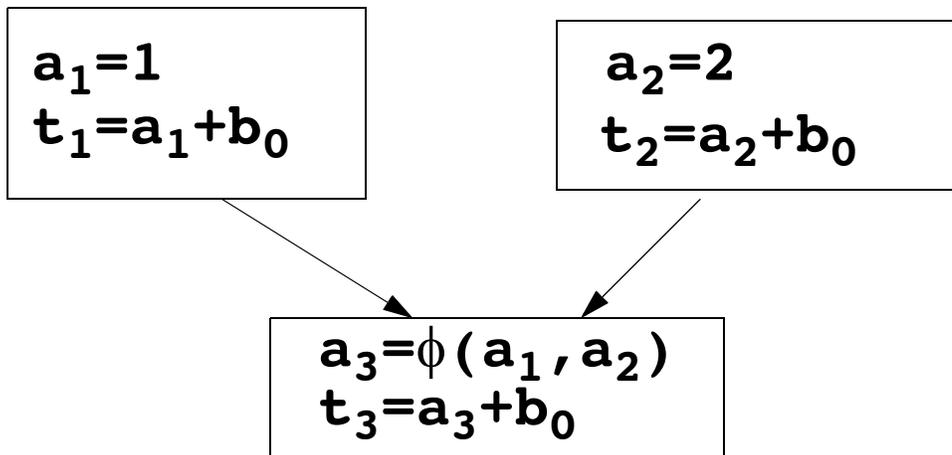
$G_{7c} = [c_3=*a_5,$   
 $e_3=*a_5]$

**Variable  $e_3$  can use  $c_3$ 's value and  $d_2$  can use  $c_2$ 's value.**

# Limitations of Global Value Numbering

As presented, our global value numbering technique doesn't recognize (or handle) computations of the same expression that produce different values along different paths.

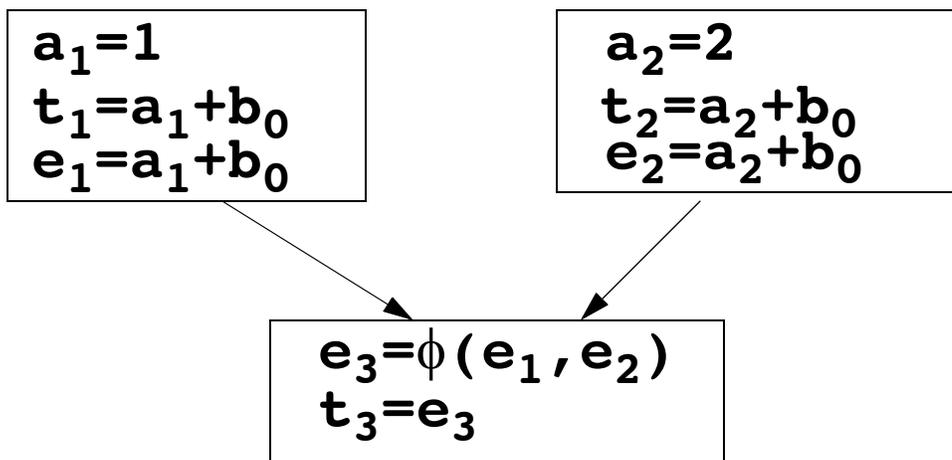
Thus in



variable  $a_3$  isn't equivalent to either  $a_1$  or  $a_2$ .

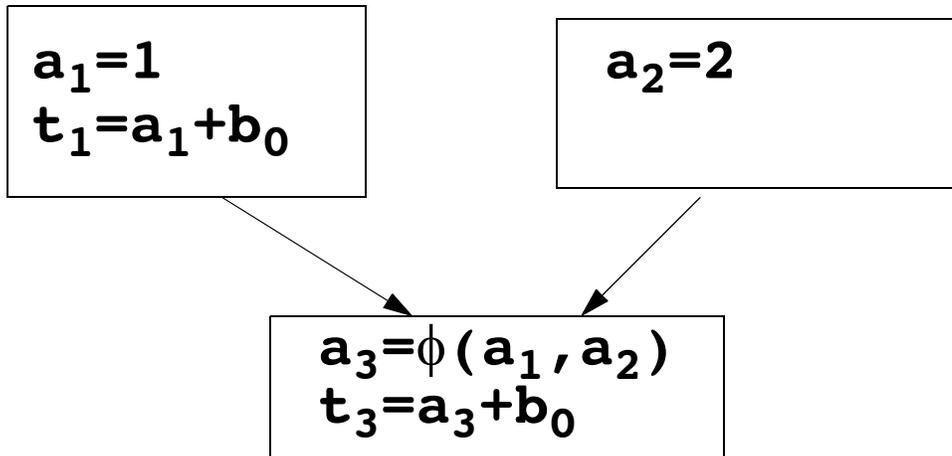
*But,*

**we can still remove a redundant computation of  $a+b$  by moving the computation of  $t_3$  to each of its predecessors:**

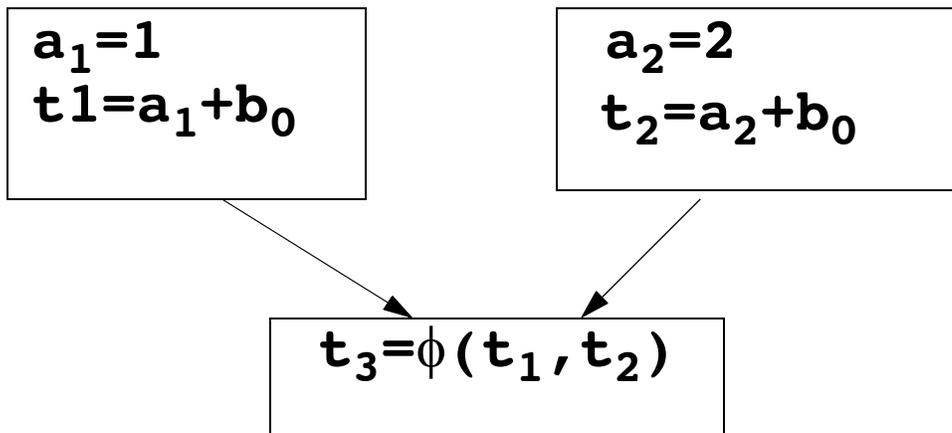


**Now a redundant computation of  $a+b$  is evident in each predecessor block. Note too that this has a nice register targeting effect— $e_1$ ,  $e_2$  and  $e_3$  can be readily mapped to the same live range.**

**The notion of moving expression computations above phi functions also meshes nicely with notion of partial redundancy elimination. Given**



**moving  $a+b$  above the phi produces**



**Now  $a+b$  is computed only once on each path, an improvement.**

# Reading Assignment

- **Read "Pointer Analysis," by Susan Horwitz.  
(Linked from the class Web page.)**

# Points-To Analysis

**All compiler analyses and optimizations are limited by the potential effects of assignments through pointers and references.**

**Thus in C:**

```
b = 1;  
*p = 0;  
print(b);
```

**is 1 or 0 printed?**

**Similarly, in Java:**

```
a[1] = 1;  
b[1] = 0;  
print(a[1]);
```

**is 1 or 0 printed?**

**Points-to** analysis aims to determine what variables or heap objects a pointer or reference may access.

To simplify points-to analysis, a number of reasonable assumptions are commonly made:

- Points to analysis is usually **flow-insensitive**. We don't analyze flow of control within a subprogram, but rather gather points-to information for the subprogram as a whole.

Thus in

```
if (b)
    p = &a;
else p = &c;
```

we conclude **p** may point to either **a** or **c**.

- Points to analysis is usually **context-insensitive** (with respect to calls). This means individual call sites for the same subprogram are not differentiated.

Therefore in

```
*int echo (*int r) {  
    return r; }  
p = echo (&a);  
q = echo (&b);
```

we determine that **r** may point to either **a** or **b** and therefore **p** can point to either **a** or **b**.

- **Heap objects are named by the call site at which they are created. In:**

```
p = new int; //Site 1
```

```
q = new int; //Site2
```

**we know p and q can't interfere since each refers to distinct call site.**

- **Aggregates (arrays, structs, classes) are **collapsed**. Pointers or references to individual components are not distinguished. Given**

```
p = &a[1];
```

```
q = &a[2];
```

**pointers p and q are assumed to interfere.**

**Similarly in**

```
p = Obj.a;
```

```
q = Obj.b;
```

**pointers p and q are assumed to interfere.**

- **Complex pointer expressions are assumed to be simplified prior to points-to analysis.**

**For example,**

```
**p = 1;
```

**is transformed into**

```
temp = *p;
```

```
*temp = 1;
```

# Points-To Representation

There are several ways to represent points-to information. We will use a **points-to graph**, which is concise and easy to understand.

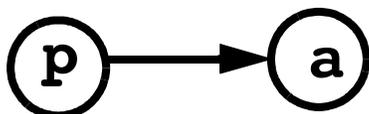
Nodes are pointer variables and “pointed to” locations.

An arc connects a pointer to a location it may potentially reference.

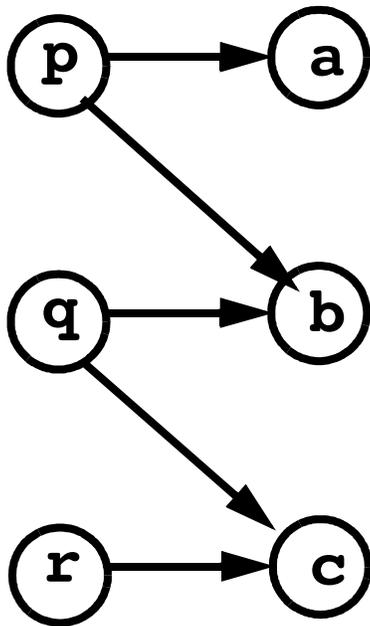
Given

$p = \&a$

we create:



**Therefore in**



**we see  $p$  and  $q$  may both point to  $b$ ,  
but  $p$  and  $r$  can't interfere (since  
their points-to sets are disjoint).**

# Simple Point-To Information

**A primitive points-to analysis can be done using type or “address taken” information.**

**In a type-safe language like Java, a reference to type  $\tau$  can only point to objects of type  $\tau$  (or a subtype of  $\tau$ ).**

**Given**

```
ref1 = new Integer();
```

```
ref2 = new Float();
```

**we trivially know `ref1` and `ref2` can't interfere.**

**Similarly, in C no pointer can access a variable  $v$  unless its address is taken (using the  $\&$  operator). With very little effort we can limit the points-to sets of pointer  $p$  to only those variables of the correct type (excluding casting) whose address has been explicitly taken.**

**In practice both of these observations are too broad to be of much use.**

# Andersen's Algorithm

**An algorithm to build a points-to graph for a C program is presented in:**

**“Program Analysis and Specialization for the C programming Language,” L.O. Andersen, 1994.**

**The algorithm examines statements that create pointers, one by one, in textual order (the algorithm is flow-insensitive). Each statement updates the points-to graph if it can create new points-to relationships.**

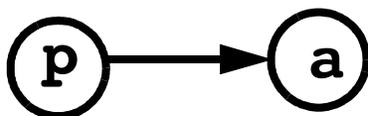
**Six kinds of statements are considered:**

- **$p = \&a;$**
- **$p = q;$**
- **$p = *r;$**
- **$*p = \&a;$**
- **$*p = q;$**
- **$*p = *r;$**

**We will detail the points-to graph updates each of the statements induces.**

**1.  $p = \&a;$**

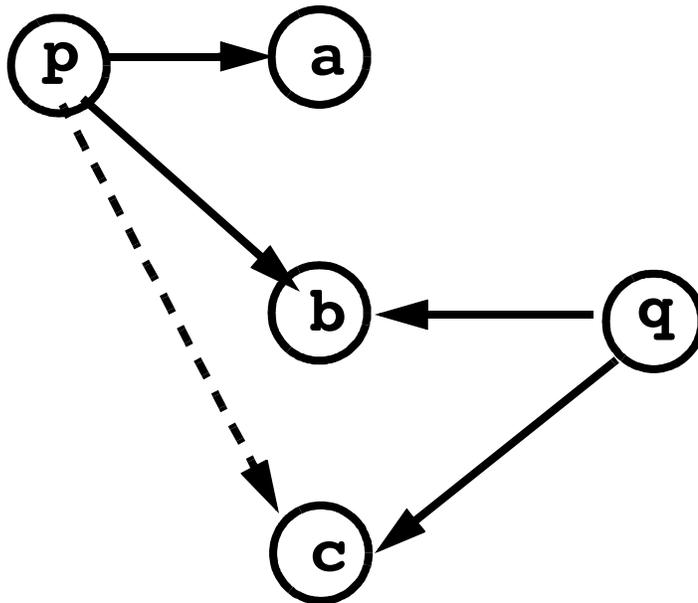
**We add an arc from  $p$  to  $a$ , showing  $p$  can possibly point to  $a$ :**



2.  $p = q;$

We add arcs from  $p$  to everything  $q$  points to. If new arcs from  $q$  are later added, corresponding arcs from  $p$  must also be added (this implies an iterative or worklist algorithm).

For example (the dashed arc is newly added):

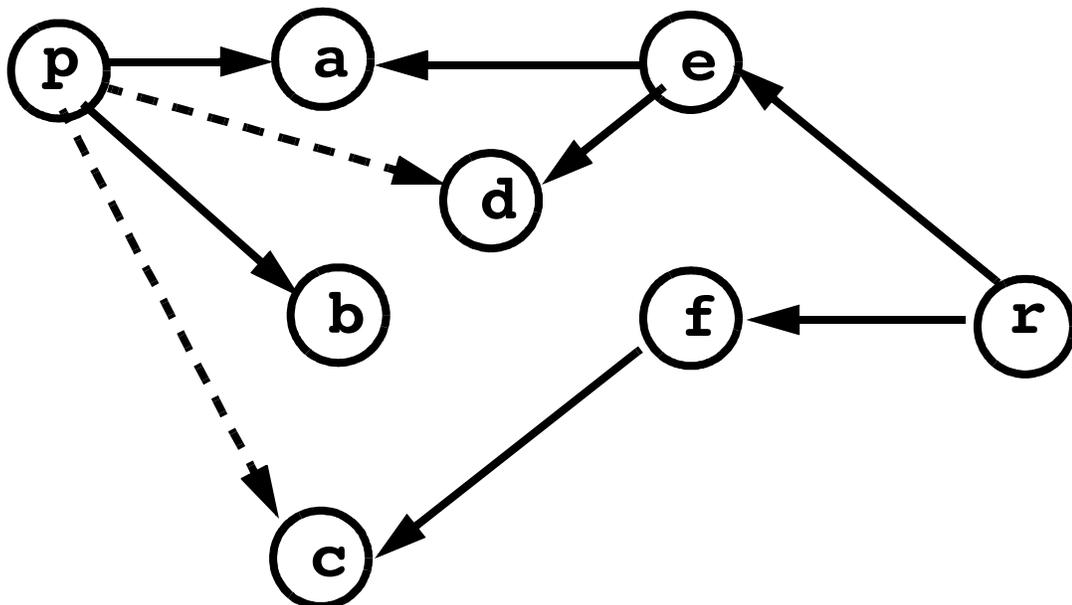


**3.  $p = *r;$**

**Let  $S$  be all the nodes  $r$  points to.**

**Let  $T$  be all the nodes members of  $S$  point to. We add arcs from  $p$  to all nodes in  $T$ . If later pointer assignments increase  $S$  or  $T$ , new arcs from  $p$  must also be added (this again implies an iterative or worklist algorithm).**

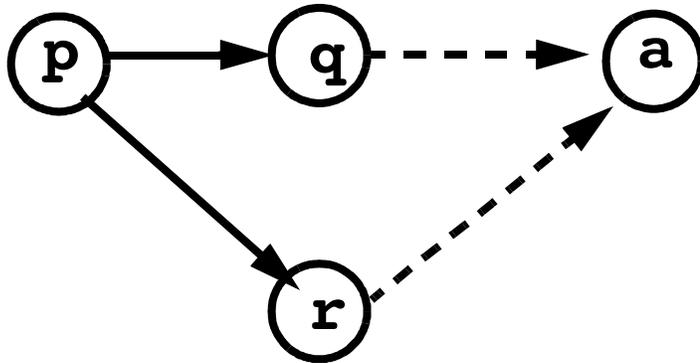
**For example (dashed arcs are newly added):**



4. `*p = &a;`

**Add an arc to a from all nodes p points to. If new arcs from p are later added, new arcs to a must be added (this implies an iterative or worklist algorithm).**

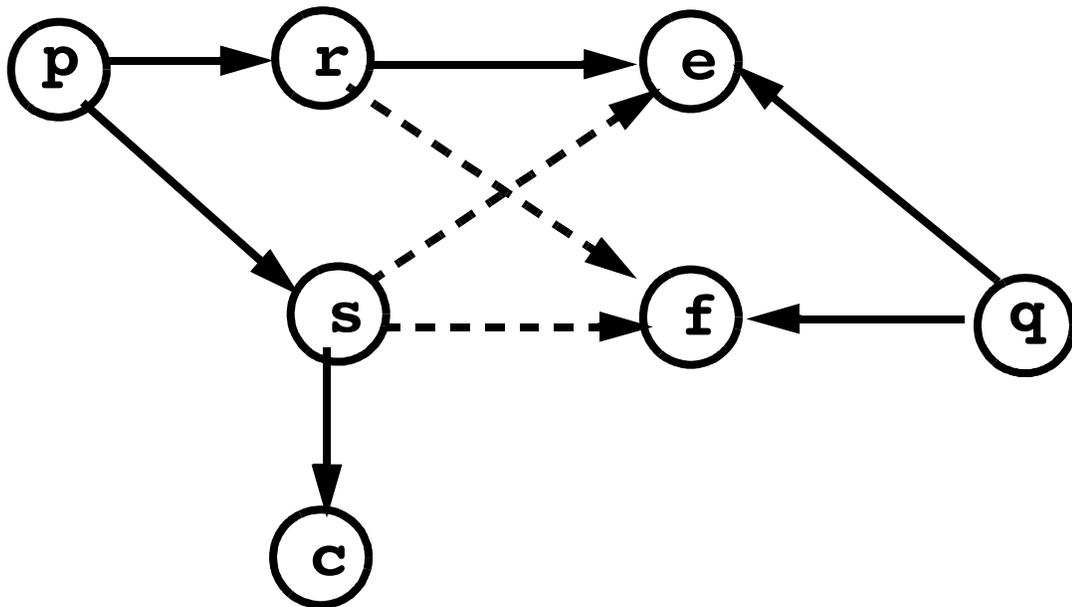
**For example (dashed arcs are newly added):**



5.  $*p = q;$

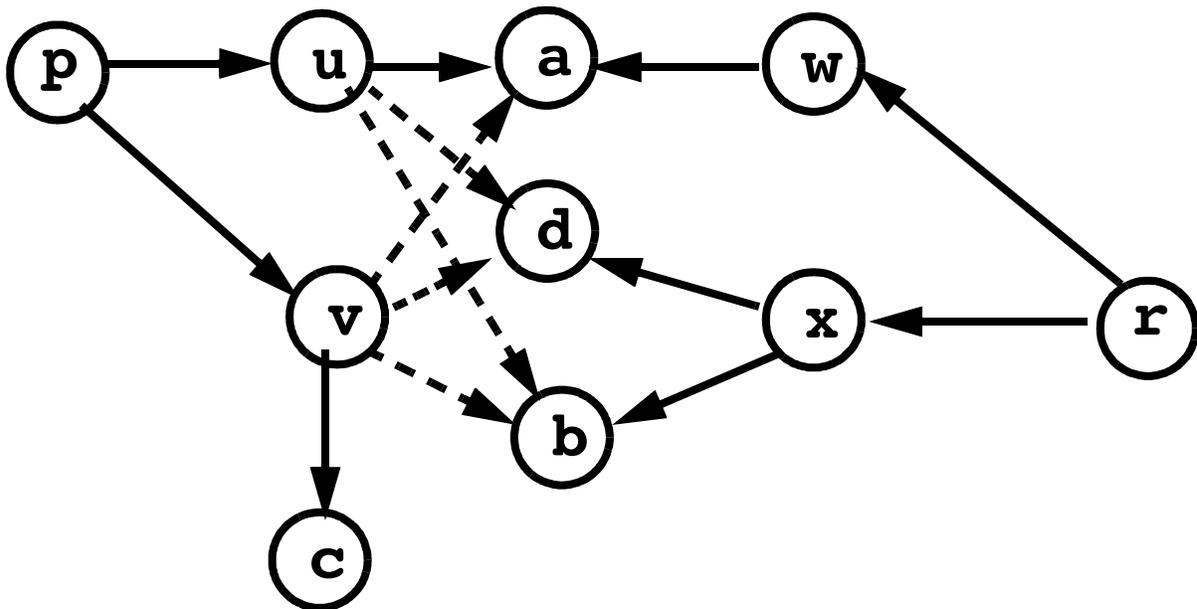
Nodes pointed to by  $p$  must be linked to all nodes pointed to by  $q$ . If later pointer assignments add arcs from  $p$  or  $q$ , this assignment must be revisited (this again implies an iterative or worklist algorithm).

For example (dashed arcs are newly added):



6.  $*p = *r;$

Let  $S$  be all the nodes  $r$  points to. Let  $T$  be all the nodes members of  $S$  point to. We add arcs from all nodes  $p$  points to to all nodes in  $T$ . If later pointer assignments increase  $S$  or  $T$  or link new nodes to  $p$ , this assignment must be revisited (this again implies an iterative or worklist algorithm). For example (dashed arcs are newly added):



## Example

Consider the following pointer manipulations:

```
p1 = &a;
```

```
p2 = &b;
```

```
p1 = p2;
```

```
r = &p1;
```

```
*r = &c;
```

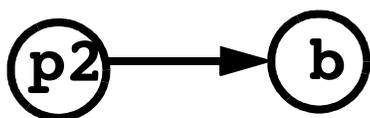
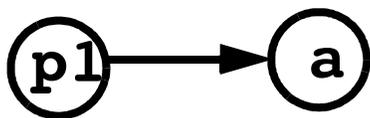
```
p3 = *r;
```

```
p2 = &d;
```

We start with:

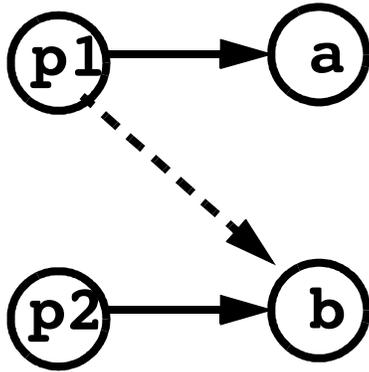
```
p1 = &a;
```

```
p2 = &b;
```



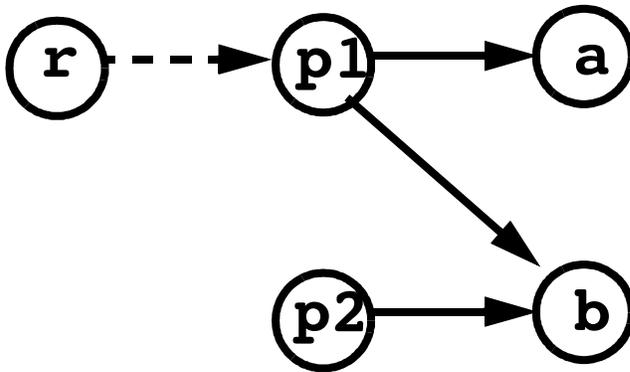
**Next:**

```
p1 = p2;
```



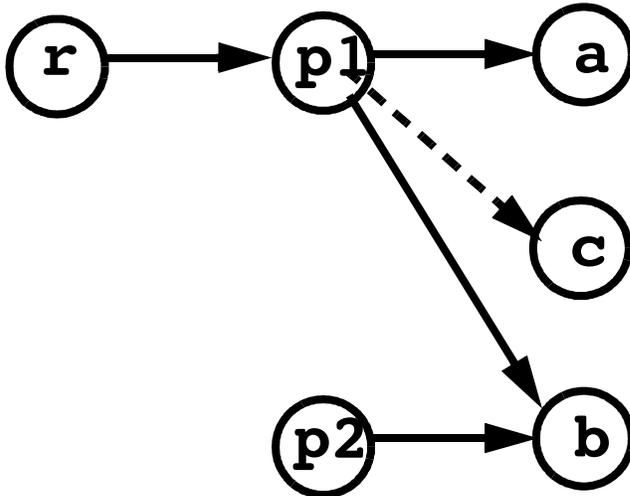
**Then:**

```
r = &p1;
```



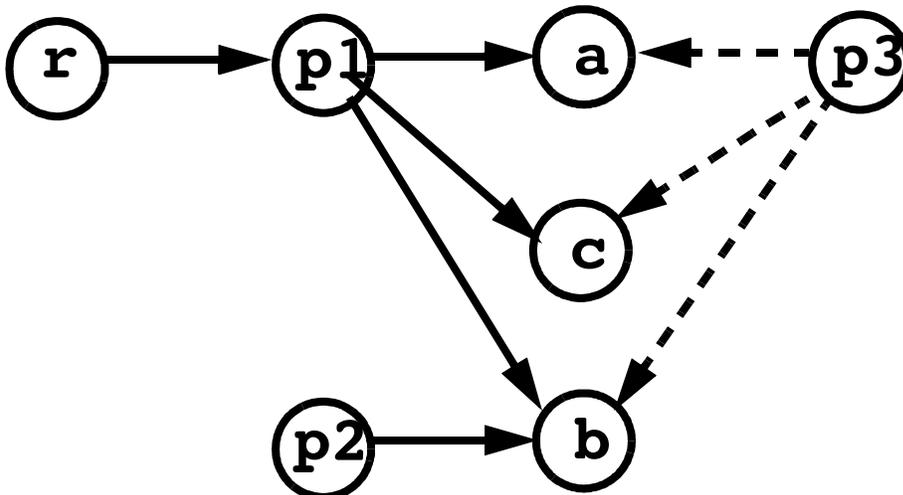
**Next:**

**\*r = &c;**



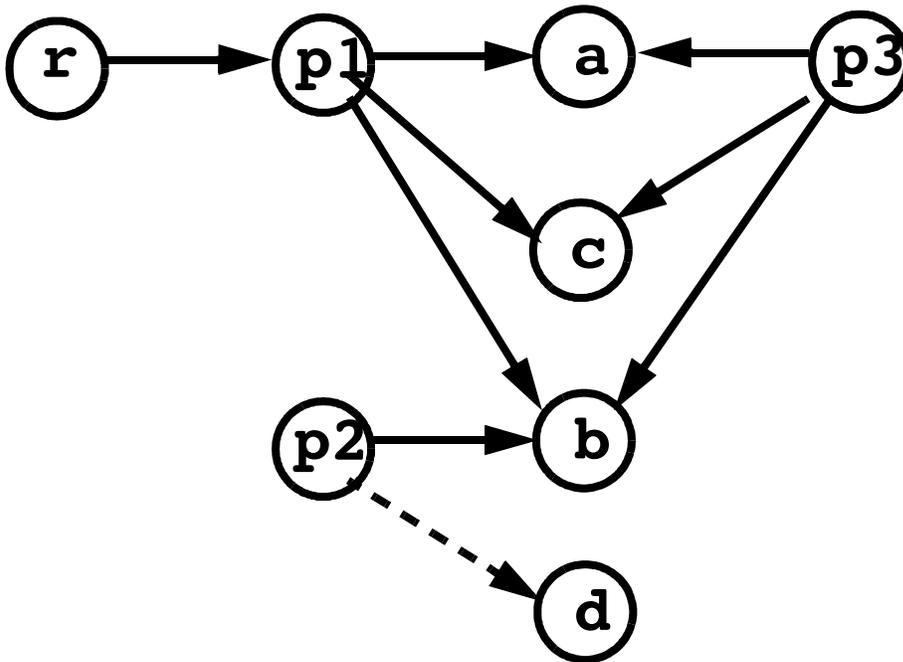
**Then:**

**p3 = \*r;**



**Finally:**

**p2 = &d;**



**But we aren't quite done yet. This algorithm is **flow-insensitive**, so we must consider other execution orders (and iterative re-execution). If we make another pass through the assignments, we see that the**



**Calls are handled by treating pointer parameters and pointer returns as assignments, done at the points of call and return.**

**Subprogram bodies are effectively inlined to capture the points-to relations they induce.**

**Given**

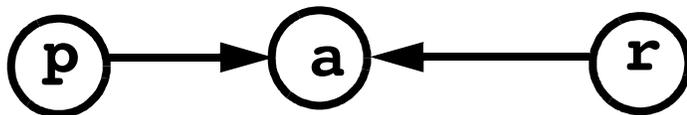
```
*int echo (*int r) {  
    return r; }  
p = echo (&a);
```

**we see the implicit assignments**

```
r = &a;
```

```
p = r;
```

**and add the following points-to information:**



**As an optimization, libraries can be pre-analyzed to determine the points-to relations they induce. Most may use (read) pointers but don't create any new points-to relations visible outside their bodies. Call to such library routines can be ignored as far as the caller's points-to graph is concerned.**

# Performance of Andersen's Algorithm

Experience has shown that Andersen's Algorithm gives useful points-to data and is far superior to the naive address-taken approach.

Interestingly, experiments show that making the technique flow-sensitive or calling context-sensitive doesn't improve results very much on typical benchmarks.

**But** execution time for moderate to large programs can be a problem.

Careful analysis shows that Andersen's Algorithm can require  $O(n^3)$  time (where  $n$  is the number of nodes in the points-to graph).

**The reason for this larger-than-expected analysis time is that a statement like**

**$p = *q;$**

**can force the algorithm to visit  $n^2$  nodes ( $q$  may point to  $n$  nodes and each of these nodes may point to  $n$  nodes). The number of pointer statements analyzed can be  $O(n)$ , leading to an  $O(n^3)$  execution time.**

# Steensgaard's Algorithm

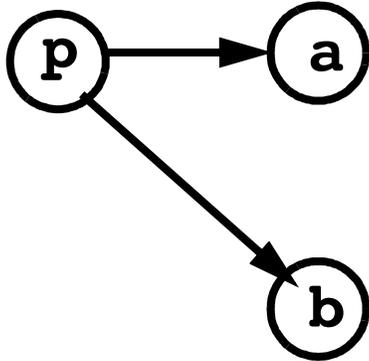
It would be useful to have a reasonably accurate points-to analysis that runs in essentially linear time so that **really** large programs could be handled.

This is what Steensgaard's Algorithm offers.

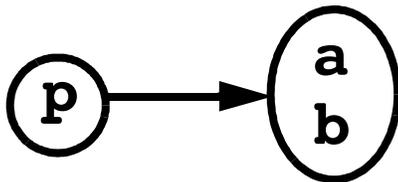
(Points-to Analysis in Almost Linear Time, B. Steensgaard, 1996 Principles of Programming Languages Conference.)

Steensgaard's Algorithm is essentially Andersen's Algorithm, simplified by **merging** nodes **a** and **b** if any pointer can reference both.

**That is, in Andersen's Algorithm we might have**

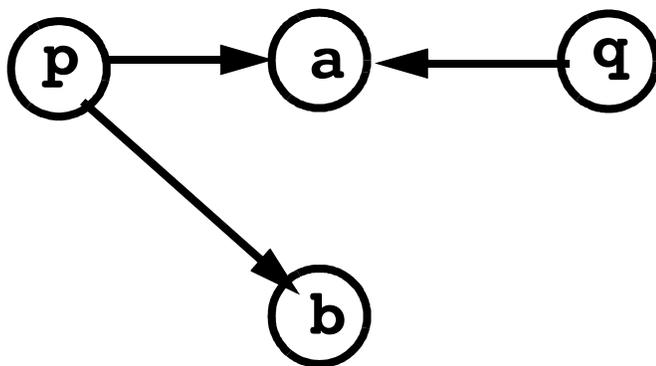


**In Steensgaard's Algorithm we would instead have**

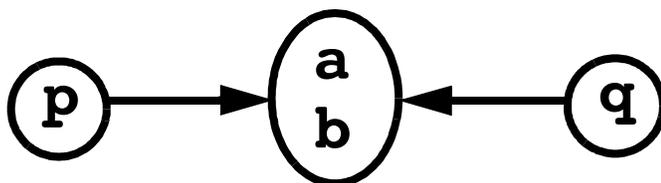


**In effect any two locations that might be pointed to by the same pointer are placed in a single equivalence class.**

**Steensgaard's Algorithm is sometimes less accurate than Andersen's Algorithm. For example, the following points-to graph, created by Andersen's Algorithm, shows that  $p$  may point to  $a$  or  $b$  whereas  $q$  may only point to  $a$ :**



**In Steensgaard's Algorithm we get**



**incorrectly showing that if  $p$  may point to  $a$  or  $b$  then so may  $q$ .**

**But now statements like**

**`p = *q;`**

**can't force the algorithm to visit  $n^2$  nodes, because multiple nodes referenced by the same pointer are always merged. Using the fast union-find algorithm, we can get an execution time of  $O(n \alpha(n))$  which is essentially linear in  $n$ . Now very large programs can be analyzed, and without too much of a loss in precision.**

# Andersen vs. Steensgaard in Practice

- **Horwitz and Shapiro examined 61 C programs, ranging in size from 300 to 24,300 lines.**
- **As expected, Steensgaard is less precise: On average points-to sets are 4 times bigger; at worst 15 times bigger.**
- **As expected, Andersen is slower. On average 1.5 times slower: at worst 31 times slower.**
- **Both are much better than the naive “address taken” approach.**
- **Bottom line: Use Andersen for small programs, use Steensgaard (or something else) for large programs.**

# Reading Assignment

- **Read "Fast and Accurate Flow-Insensitive Points-To Analysis," by Shapiro and Horwitz.  
(Linked from the class Web page.)**

# **The Horwitz-Shapiro Approach**

**It would be nice to have a points-to analysis that is parameterizable, ranging between the accuracy of Andersen and the speed of Steensgaard.**

**Horwitz and Shapiro (Fast and Accurate Flow-Insensitive Points-To Analysis, 1997 Principles of Programming Languages Conference) present a technique intermediate to those proposed by Andersen and Steensgaard.**

**Horwitz and Shapiro suggest each node in the points-to graph be limited to out degree  $k$ , where  $1 \leq k \leq n$ .**

**If  $k = 1$  then they have Steensgaard's approach.**

**If  $k = n$  ( $n$  is number of nodes in points to graph), then they have Andersen's approach.**

**Their worst case run-time is**

**$O(k^2 n)$ , which is not much worse than Steensgaard if  $k$  is kept reasonably small.**

**To use their approach assign each variable that may be pointed to to one of  $k$  categories.**

**Now if  $p$  may point to  $x$  and  $p$  may also point to  $y$ , we merge  $x$  and  $y$  **only if** they both are in the same category.**

**If  $x$  and  $y$  are in different categories, they **aren't** merged, leading to more accurate points-to estimates.**

# Example

```
p1 = &a;
```

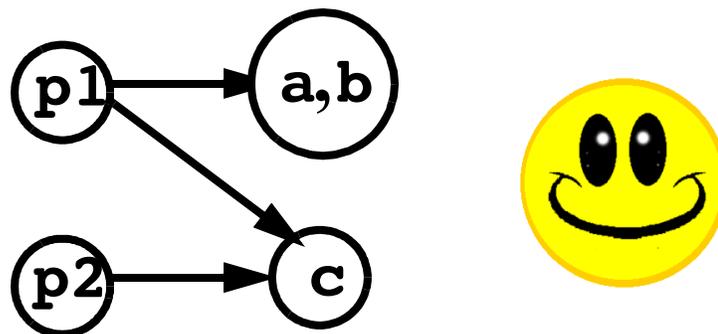
```
p1 = &b;
```

```
p1 = &c;
```

```
p2 = &c;
```

Say we have  $k = 2$  and place **a** and **b** in category 1 and **c** in category 2.

We then build:

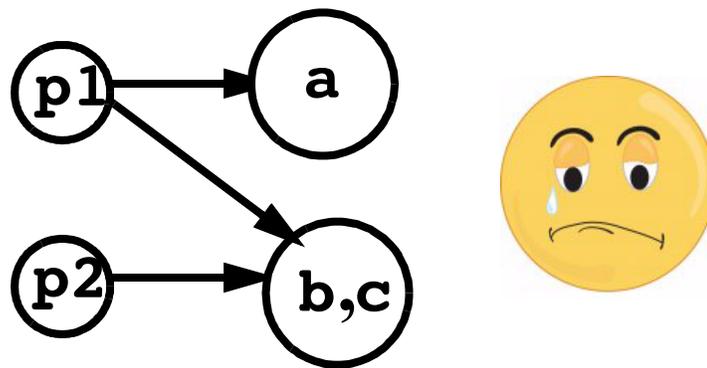


This points-to graph is just as accurate as that built by Andersen's approach.

**But...**

**What if we chose to place *a* in category 1 and *b* and *c* in category 2.**

**We now have:**



**This graph is inexact, since it tells us *p2* may point to *b*, which is false.**

**(Steensgaard would have been worse still, incorrectly telling us *p2* may point to *a* as well as *b* and *c*).**

# Another Good Idea

What if we ran Shapiro and Horwitz's points-to analysis **twice**, each with different category assignments?

Each run may produce a different points-to graph. One may say  $p_2$  points to  $b$  whereas the other says it does not.

Which do we believe?

Neither analysis misses a genuine points-to relation. Rather, merging of nodes sometimes creates false points-to information.

So we will believe  $p_2$  may point to  $b$  only if **all** runs say so. This means multiple runs may “filter out” false points-to relations due to merging.

# How Many Runs are Needed?

# How are Categories to be Set?

We want to assign categories so that during at least one run, any pair of pointed-to variables are in **different** categories.

This guarantees that if all the runs tell us  $p$  may point to  $a$  and  $b$ , it is not just because  $a$  and  $b$  always happened to be assigned the same category.

To force different category assignments for each pair of variables, we assign each pointed-to variable an index and write that index in base  $k$  (the number of categories chosen).

**For example, if we had variables a, b, c and d, and chose  $k = 2$ , we'd use the following binary indices:**

**a 00**

**b 01**

**c 10**

**d 11**

**Note that the number of base  $k$  digits needed to represent indices from 0 to  $n-1$  is just  $\text{ceiling}(\log_k n)$ .**

**This number is just the number of runs we need!**

## **Why?**

**In the first run, we'll use the right most digit in a variable's index as its category.**

**In the next run, we'll use the second digit from the right, then the third digit from the right, ...**

**Any two distinct variables have different index values, so they must differ in at least digit position.**

**Returning to our example,**

**a 00**

**b 01**

**c 10**

**d 11**

**On run #1 we give a and c category 0 and b and d category 1.**

**On run #2, a and b get category 0 and c and d get category 1.**

**So using just 2 runs in this simple case, we eliminate much of the inaccuracy Steensgaard's merging introduces.**

**Run time is now  $O(\log_k(n) k^2 n)$ .**

# **How Well does this Approach Work?**

**On 25 tests, using 3 categories, Horwitz & Shapiro points-to sets on average are 2.67 larger than those of Andersen (Steensgaard's are 4.75 larger).**

**This approach is slower than Steensgaard but on larger programs it is 7 to 25 times faster than Andersen.**

# **How Well do Points-to Analyses Work in Real Data Flow Problems?**

**In “Which Pointer Analysis Should I Use,” Hind and Pioli survey the effectiveness of a number of points-to analyses in actual data flow analyses (mod/ref, liveness, reaching defs, interprocedural constant propagation).**

**Their conclusions are essentially the same across all these analyses:**

- Steensgaard’s analysis is significantly more precise than address-taken analysis and not significantly slower.**
-

- **Andersen's analysis produces modest, but consistent, improvements over Steensgaard's analysis.**
- **Both context-sensitive points-to analysis and flow-sensitive points-to analysis give **little** improvement over Andersen's analysis.**

# Reading Assignment

- Section 13.3 of *Crafting a Compiler*

# **“On the Fly” Local Register Allocation**

**Allocate registers as needed during code generation.**

**Partition registers into 3 classes.**

- **Allocatable**

**Explicitly allocated and freed; used to hold a variable, literal or temporary.**

**On SPARC: Local registers & unused In registers.**

- **Reserved**

**Reserved for specific purposes by OS or software conventions.**

**On SPARC: %fp, %sp, return address register, argument registers, return value register.**

- **Work**

**Volatile**—used in short code sequences that need to use a register.

**On SPARC: %g1 to %g4, unused out registers.**

## **Register Targeting**

**Allow “end user” of a value to state a register preference in AST or IR.**

*or*

**Use Peephole Optimization to eliminate unnecessary register moves.**

*or*

**Use *preferencing* in a graph coloring register allocator.**

# Register Tracking

**Improve upon standard `getReg/`  
`freeReg` allocator by *tracking*  
(remembering) register contents.**

**Remember the value(s) currently  
held within a register; store  
information in a *Register Association*  
*List*.**

**Mark each value as *Saved* (in  
memory) or *Unsaved* (in memory).**

**Each value in a register has a *Cost*.  
This is the cost (in instructions) to  
restore the value to a register.**

**The cost of allocating a register is the sum of the costs of the values it holds.**

$$\text{Cost}(\text{register}) = \sum_{\text{values} \in \text{register}} \text{cost}(\text{values})$$

**When we allocate a register, we will choose the *cheapest* one.**

**If 2 registers have the same cost, we choose that register whose values have the *most distant* next use.**

**(Why most distant?)**

# Costs for the SPARC

- 0**    **Dead Value**
- 1**    **Saved Local Variable**
- 1**    **Small Literal Value (13 bits)**
- 2**    **Saved Global Variable**
- 2**    **Large Literal Value (32 bits)**
- 2**    **Unsaved Local Variable**
- 4**    **Unsaved Global Variable**

# Register Tracking Allocator

```
reg getReg() {
    if (  $\exists$  r  $\in$  regSet and cost(r) == 0)
        choose(r)
    else {
        c = 1;
        while(true) {
            if (  $\exists$  r  $\in$  regSet and cost(r) == c){
                choose r with cost(r) == c and
                most distant next use of
                associated values;
                break;
            }
            c++;
        }
        Save contents of r as necessary;
    }
    return r;
}
```

- **Once a value becomes dead, it may be purged from the register association list without any saves.**
- **Values no longer used, but unsaved, can be purged (and saved) at *zero* cost.**
- **Assignments of a register to a simple variable may be *delayed*—just add the variable to the Register’s Association List entry as unsaved.**

**The assignment may be done later or made *unnecessary* (by a later assignment to the variable)**

- **At the end of a basic block all unsaved values are stored into memory.**

# Example

```
int a,b,c,d; // Globals
a = 5;
b = a + d;
c = b - 7;
b = 10;
```

# Naive Code

```
mov    5,%10
st     %10,[a]
ld     [a],%10
ld     [d],%11
add    %10,%11,%11
st     %11,[b]
ld     [b],%11
sub    %11,7,%11
st     %11,[c]
mov    10,%11
st     %11,[b]
```

**18 instructions are needed (memory references take 2 instructions)**

# With Register Tracking

Instruction Generated	%10	%11
<code>mov 5,%10</code>	5(S)	
<code>! Defer assignment to a</code>	5(S), a(U)	
<code>ld [d], %11</code>	5(S), a(U)	d(S)
<code>!d unused after next inst</code>		
<code>add %10,%11,%11</code>	5(S), a(U)	b(U)
<code>!b is dead after next inst</code>		
<code>sub %11,7,%11</code>	5(S), a(U)	c(U)
<code>! %11 has lower cost</code>		
<code>st %11, [c]</code>	5(S), a(U)	
<code>mov 10, %11</code>	5(S), a(U)	b(U), 10(S)
<code>! save unsaved values</code>		
<code>st %10, [a]</code>		b(U), 10(S)
<code>st %11,[b]</code>		

**12 instructions (rather than 18)**

# Pointers, Arrays and Reference Parameters

**When an array, reference parameter or pointed-to variable is read, all unsaved register values that might be aliased must be *stored*.**

**When an array, reference parameter or pointed-to variable is written, all unsaved register values that might be aliased must be *stored*, then *cleared* from the register association list.**

**Thus if  $a[3]$  is in a register and  $a[i]$  is assigned to,  $a[3]$  must be stored (if unsaved) and removed from the association list.**

# Optimal Expression Tree Translation—Sethi-Ullman Algorithm

**Reference:** R. Sethi & J. D. Ullman, “The generation of optimal code for arithmetic expressions,” *Journal of the ACM*, 1970.

**Goal:** Translate an expression tree using the *fewest* possible registers.

**Approach:** Mark each tree node,  $N$ , with an *Estimate* of the minimum number of registers needed to translate the tree rooted by  $N$ .

Let  $RN(N)$  denote the **Register Needs** of node  $N$ .

**In a Load/Store architecture  
(ignoring immediate operands):**

**$RN(\text{leaf}) = 1$**

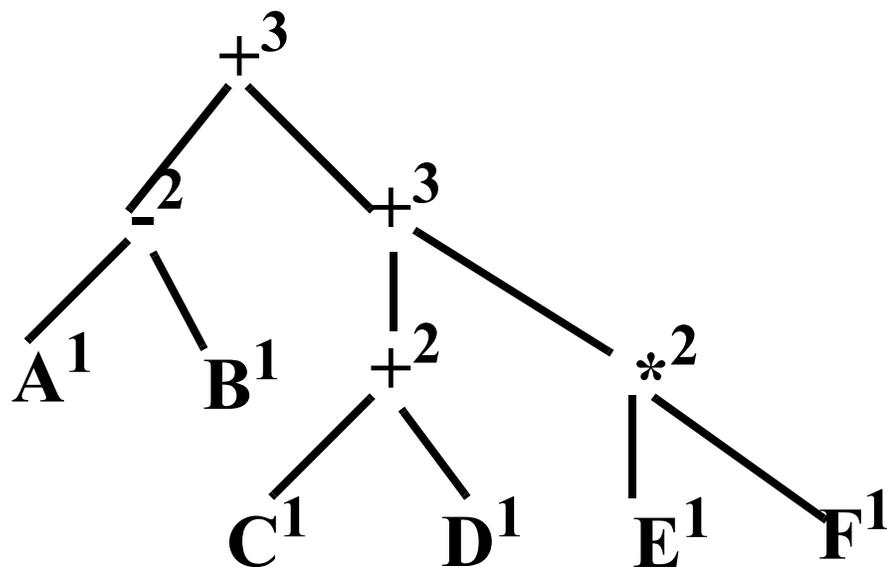
**$RN(\text{Op}) =$**

**If  $RN(\text{Left}) = RN(\text{Right})$**

**Then  $RN(\text{Left}) + 1$**

**Else  $\text{Max}(RN(\text{Left}),$   
 $RN(\text{Right}))$**

**Example:**



# Key Insight of SU Algorithm

**Translate subtree that needs more registers *first*.**

**Why?**

**After translating one subtree, we'll need a register to hold its value.**

**If we translate the more complex subtree first, we'll still have enough registers to translate the less complex expression (without *spilling* register values into memory).**

# Specification of SU Algorithm

**TreeCG(tree \*T, regList RL);**

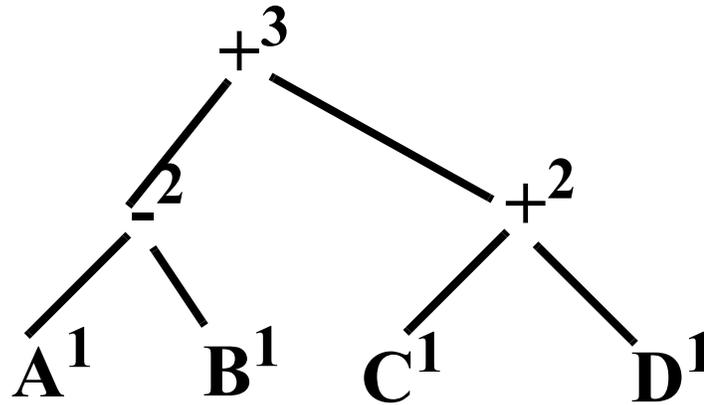
## **Operation:**

- **Translate expression tree T using only registers in RL.**
- **RL must contain at least 2 registers.**
- **Result of T will be computed into head(RL).**

# Summary of SU Algorithm

```
if T is a node (variable or literal)
    load T into R1 = head(RL)
else (T is a binary operator)
    Let R1 = head(RL)
    Let R2 = second(RL)
    if RN(T.left) >= Size(RL) and
       RN(T.right) >= Size(RL)
        (A spill is unavoidable)
        TreeCG(T.left, RL)
        Store R1 into a memory temp
        TreeCG(T.right, RL)
        Load memory temp into R2
        Generate (OP R2,R1,R1)
    elsif RN(T.left) >= RN(T.right)
        TreeCG(T.left, RL)
        TreeCG(T.right, tail(RL))
        Generate (OP R1,R2,R1)
    else
        TreeCG(T.right, RL)
        TreeCG(T.left, tail(RL))
        Generate (OP R2,R1,R1)
```

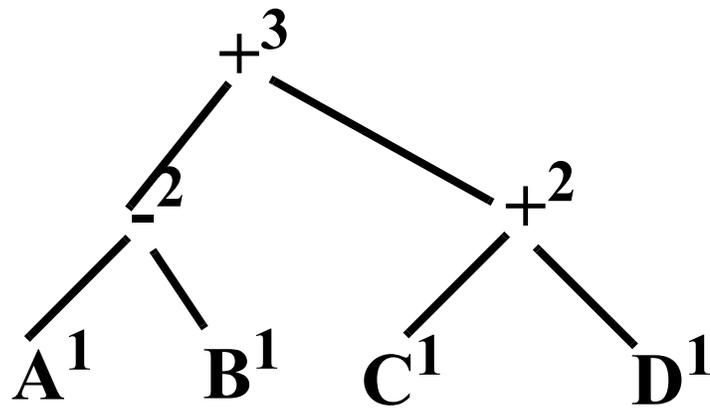
# Example (with Spilling)



**Assume only 2 Registers;**

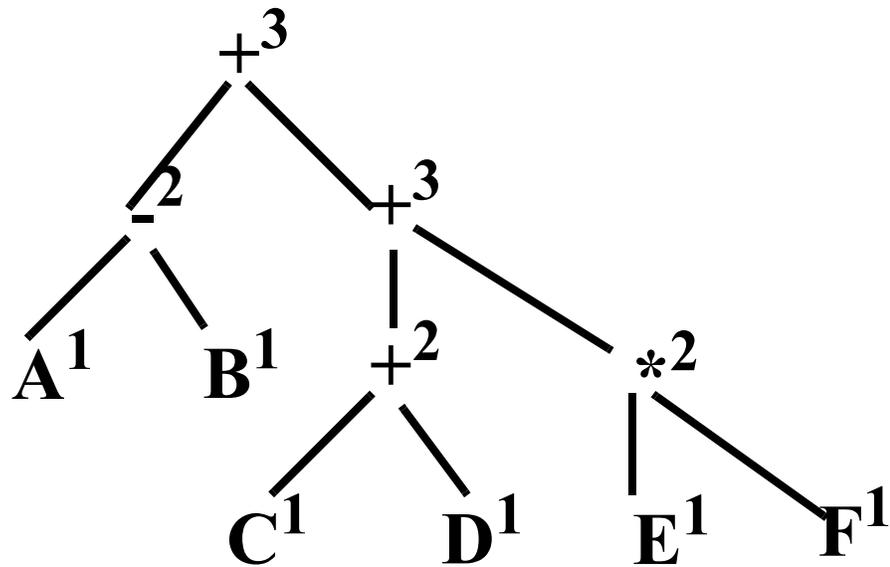
**RL = [%10,%11]**

**We Translate the left subtree first (using 2 registers), store its result into memory, translate the right subtree, reload the left subtree's value, then do the final operation.**



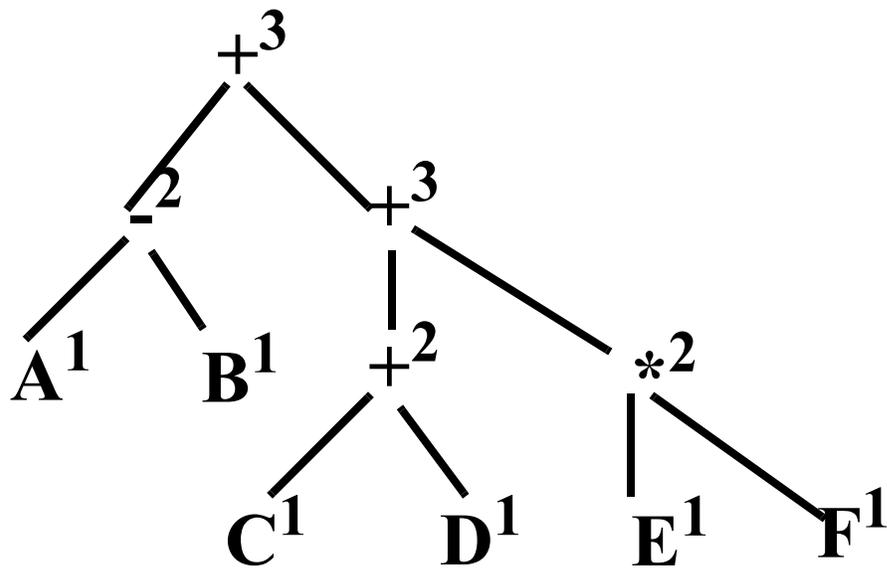
```
ld  [A], %10
ld  [B], %11
sub  %10,%11,%10
st  %10, [temp]
ld  [C], %10
ld  [D], %11
add  %10,%11,%10
ld  [temp], %11
add  %11,%10,%10
```

# Larger Example



**Assume 3 Registers;  
RL = [%10,%11,%12]**

**Since right subtree is more complex,  
it is translated first.**



```

ld [C], %10
ld [D], %11
add %10,%11,%10
ld [E], %11
ld [F], %12
mul %11,%12,%11
add %10,%11,%10
ld [A], %11
ld [B], %12
sub %11,%12,%11
add %11,%10,%10
  
```

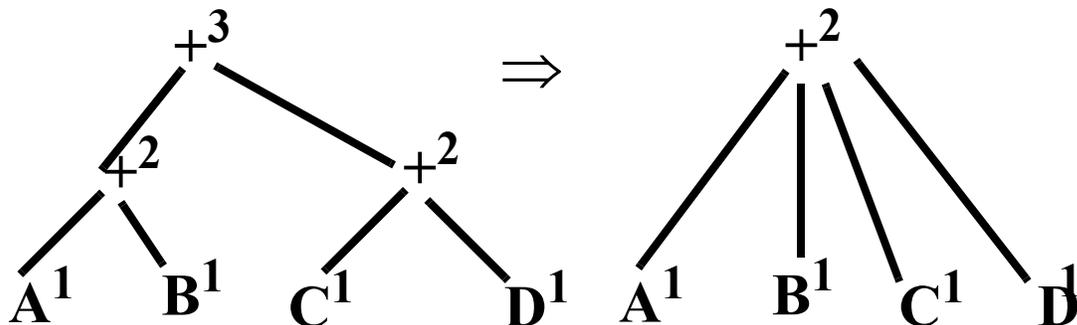
# Refinements & Improvements

- Register needs rules can be modified to model various architectural features.

For example, Immediate operands, that need not be loaded into registers, can be modeled by the following rule:

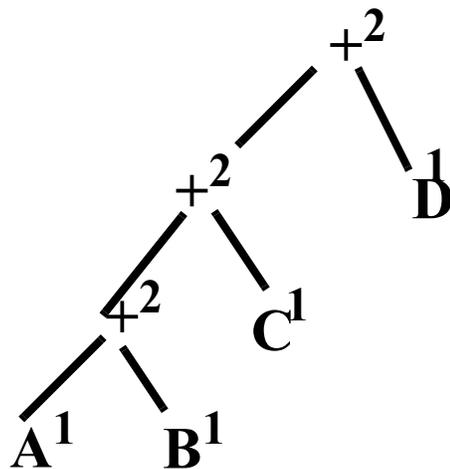
**$RN(\text{literal}) = 0$  if literal may be used as an immediate operand**

- Commutativity & Associativity of operands may be exploited:



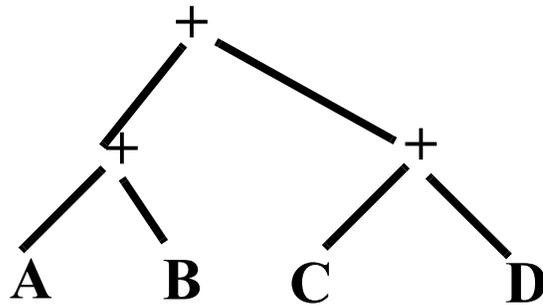
# Is Minimizing Register Use Always Wise?

SU minimizes the *number* of registers used but at the *cost* of reduced ILP.



Since only 2 registers are used, there is little possibility of parallel evaluation.

**When more registers are used, there is often more potential for parallel evaluation:**



**Here as many as *four* registers may be used to increase parallelism.**

# Optimal Translation for DAGs is Much Harder

If variables or expression values may be *shared* and *reused*, optimal code generation becomes NP-Complete.

**Example:**  $a + b * (c + d) + a * (c + d)$

We must decide how long to hold each value in a register. Best orderings may “skip” between subexpressions

**Reference:** R. Sethi, “Complete Register Allocation Problems,” *SIAM Journal of Computing*, 1975.

# Scheduling Expression Trees

**Reference: S. Kurlander, T. Proebsting and C. Fischer,  
“Efficient Instruction Scheduling  
for Delayed-Load Architectures,”  
*ACM Transactions on Programming  
Languages and Systems*, 1995.  
(Linked from class Web page)**

**The Sethi-Ullman Algorithm  
minimizes register usage, without  
regard to code scheduling.**

**On machines with *Delayed Loads*,  
we also want to avoid stalls.**

# What is a Delayed Load?

**Most pipelined processors require a delay of one or more instructions between a load of register R and the first use of R.**

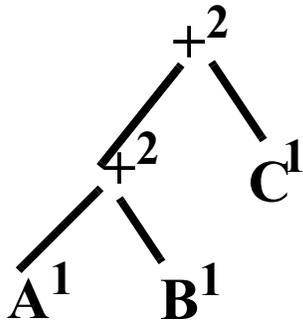
**If a register is used “too soon,” the processor may stall execution until the register value becomes available.**

```
ld    [a], %r1  
add   %r1, 1, %r1 ← Stall!
```

**We try to place an instruction that doesn't use register R immediately after a load of R.**

**This allows useful work instead of a wasteful stall.**

**The Sethi-Ullman Algorithm generates code that will stall:**



```
ld  [A], %10
ld  [B], %11 ← Stall!
add %10,%11,%10
ld  [C], %11 ← Stall!
add %10,%11,%10
```

**In fact, if we use the fewest possible registers, stalls are *Unavoidable!***

# Why?

**Loads increase the number of registers in use.**

**Binary operations decrease the number of registers in use (2 Operands, 1 Result).**

**The load that brings the number of registers in use up to the minimum number needed *must* be followed by an operator that uses the just-loaded value. This implies a stall.**

**We'll need to allocate an *extra register* to allow an independent instruction to fill each delay slot of a load.**

# Extended Register Needs

Abbreviated as *ERN*

**ERN(Identifier) = 2**

**ERN(Literal) = 1**

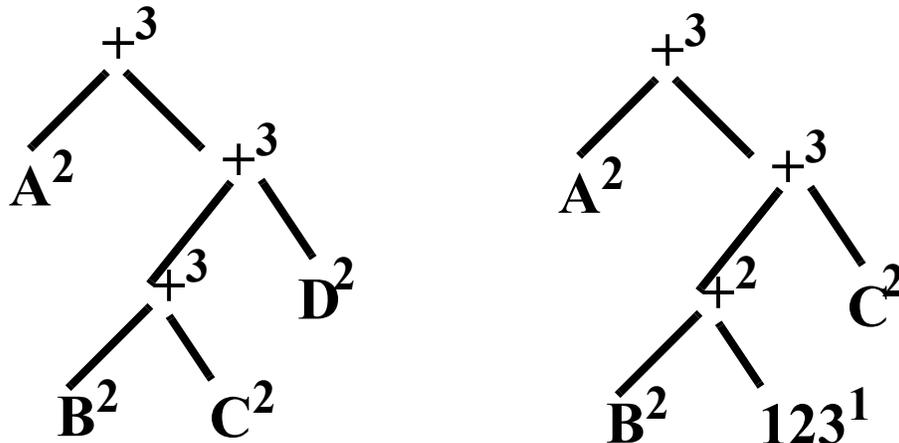
**ERN(Op) =**

**If ERN(Left) = ERN(Right)**

**Then ERN(Left) + 1**

**Else Max(ERN(Left),  
ERN(Right))**

# Example



## Idea of the Algorithm

- 1. Generate instructions in the same order as Sethi-Ullman, but use Pseudo-Registers instead of actual machine registers.**
- 2. Put generated instructions into a “Canonical Order” (as defined below).**
- 3. Map pseudo-registers to actual machine registers.**

# What are Pseudo-Registers?

**They are unique temporary locations, unlimited in number and generated as needed, that are used to model registers prior to register allocation.**

## Canonical Form for Expression Code

**(Assume R registers will be used)**

**Desired instruction ordering:**

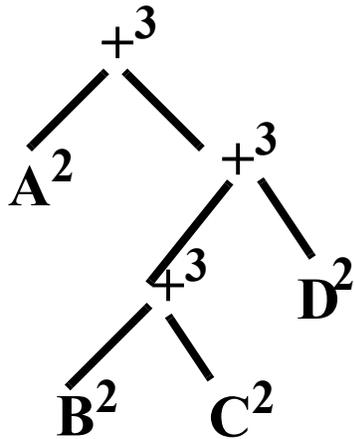
- 1. R load instructions**
- 2. Pairs of Operator/Load instructions**
- 3. Remaining operators**

**This canonical form is obtained by “sliding” load instructions upward (earlier) in the original code ordering.**

**Note that:**

- **Moving loads upward is *always* safe, since each pseudo-register is assigned to only once.**
- **No more than  $R$  registers are ever live.**

# Example



```
ld [B], PR1
ld [C], PR2
add PR1, PR2, PR3
ld [D], PR4
add PR3, PR4, PR5
ld [A], PR6
add PR6, PR5, PR7
```

Let  $R = 3$ , the minimum needed for a delay-free schedule.

Put into Canonical Form:

```
ld [B], PR1
ld [C], PR2
ld [D], PR4
add PR1, PR2, PR3
ld [A], PR6
add PR3, PR4, PR5
add PR6, PR5, PR7
```

(Before Register Assignment)

```
ld [B], %10
ld [C], %11
ld [D], %12
add %10, %11, %10
ld [A], %11
add %10, %12, %10
add %11, %10, %10
```

(After Register Assignment)

No Stalls!

# Does This Algorithm Always Produce a Stall-Free, Minimum Register Schedule?

**Yes—if one exists!**

**For very simple expressions (one or two operands) no stall-free schedule exists.**

**For example: `a=b;`**

```
ld    [b], %10  
st    %10, [a]
```

# **Why Does the Algorithm Avoid Stalls?**

**Previously, certain “critical” loads had to appear just before an operation that used their value.**

**Now, we have an “extra” register. This allows critical loads to move up one or more places, avoiding any stalls.**

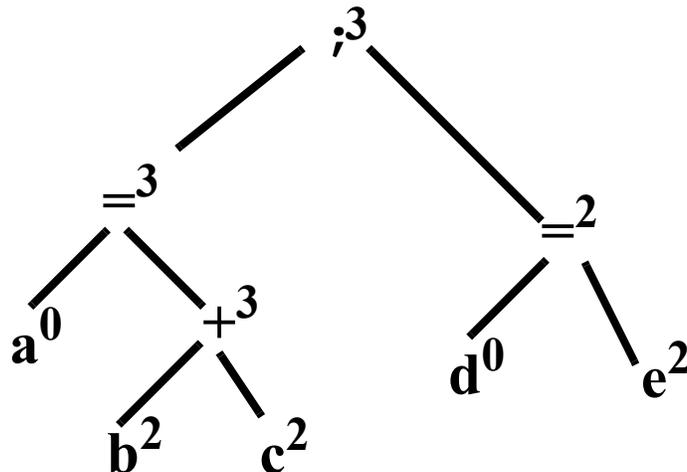
# How Do We Schedule Small Expressions?

**Small expressions (one or two operands) are common. We'd like to avoid stalls when scheduling them.**

**Idea—Blend small expressions together into larger expression trees, using “,” and “;” like binary operators.**

# Example

**a=b+c; d=e;**



```
ld [b], PR1
ld [c], PR2
add PR1,PR2,PR3
st PR3, [a]
ld [e], PR4
st PR4, [d]
```

Original Code

```
ld [b], PR1
ld [c], PR2
ld [e], PR4
add PR1,PR2,PR3
st PR3, [a]
st PR4, [d]
```

In Canonical Form

```
ld [b], %10
ld [c], %11
ld [e], %12
add %10,%11,%10
st %10, [a]
st %12, [d]
```

After Register Assignment

# Global Register Allocation

**Allocate registers across an entire subprogram.**

**A Global Register Allocator must decide:**

- **What values are to be placed in registers?**
- **Which registers are to be used?**
- **For how long is each *Register Candidate* held in a register?**

# Live Ranges

**Rather than simply allocate a value to a fixed register throughout an entire subprogram, we prefer to *split variables into Live Ranges.***

## **What is a Live Range?**

**It is the span of instructions (or basic blocks) from a definition of a variable to all its uses.**

**Different assignments to the same variable may reach distinct & disjoint instructions or basic blocks.**

**If so, the live ranges are *Independent*, and may be assigned *Different* registers.**

# Example

```
a = init();  
for (int i = a+1; i < 1000; i++){  
    b[i] = 0; }  
a = f(i);  
print(a);
```

**The two uses of variable a comprise *Independent* live ranges.**

**Each can be allocated separately.**

**If we insisted on allocating variable a to a fixed register for the whole subprogram, it would *conflict* with the loop body, greatly reducing its chances of successful allocation.**

# **Granularity of Live Ranges**

**Live ranges can be measured in terms of individual instructions or basic blocks.**

**Individual instructions are more precise but basic blocks are less numerous (reducing the size of sets that need to be computed).**

**We'll use basic blocks to keep examples concise.**

**You can define basic blocks that hold only one instruction, so computation in terms of basic blocks is still fully general.**

# Computation of Live Ranges

**First construct the Control Flow Graph (CFG) of the subprogram.**

**For a Basic Block  $b$  and Variable  $V$ :**

**Let  $\text{DefsIn}(b)$  = the set of basic blocks that contain definitions of  $V$  that reach (may be used in) the beginning of Basic Block  $b$ .**

**Let  $\text{DefsOut}(b)$  = the set of basic blocks that contain definitions of  $V$  that reach (may be used in) the end of Basic Block  $b$ .**

**If a definition of  $V$  reaches  $b$ , then the register that holds the value of that definition must be allocated to  $V$  in block  $b$ .**

**Otherwise, the register that holds the value of that definition may be used for other purposes in  $b$ .**

**The sets  $\text{Preds}$  and  $\text{Succ}$  are derived from the structure of the CFG.**

**They are given as part of the definition of the CFG.**

**DefsIn and DefsOut must be computed, using the following rules:**

**1. If Basic Block  $b$  contains a definition of  $V$  then**

$$\mathbf{DefsOut}(b) = \{b\}$$

**2. If there is no definition to  $V$  in  $b$  then**

$$\mathbf{DefsOut}(b) = \mathbf{DefsIn}(b)$$

**3. For the First Basic Block,  $b_0$ :**

$$\mathbf{DefsIn}(b_0) = \phi$$

**4. For all Other Basic Blocks**

$$\mathbf{DefsIn}(b) = \bigcup_{p \in \text{Preds}(b)} \mathbf{DefsOut}(p)$$

# Liveness Analysis

**Just because a definition reaches a Basic Block,  $b$ , *does not* mean it must be allocated to a register at  $b$ .**

**We also require that the definition be *Live* at  $b$ . If the definition is dead, then it will no longer be used, and register allocation is unnecessary.**

**For a Basic Block  $b$  and Variable  $V$ :**

**LiveIn( $b$ ) = true if  $V$  is Live (will be used before it is redefined) at the beginning of  $b$ .**

**LiveOut( $b$ ) = true if  $V$  is Live (will be used before it is redefined) at the end of  $b$ .**

**LiveIn and LiveOut are computed,  
using the following rules:**

**1. If Basic Block  $b$  has no successors  
then**

**LiveOut( $b$ ) = false**

**2. For all Other Basic Blocks**

$$\text{LiveOut}(b) = \bigvee_{s \in \text{Succ}(b)} \text{LiveIn}(s)$$

**3. LiveIn( $b$ ) =**

**If  $V$  is used before it is defined in  
Basic Block  $b$**

**Then true**

**Elsif  $V$  is defined before it is  
used in Basic Block  $b$**

**Then false**

**Else LiveOut( $b$ )**

# Merging Live Ranges

**It is possible that each Basic Block that contains a definition of  $v$  creates a *distinct* Live Range of  $V$ .**

**$\forall$  Basic Blocks,  $b$ , that contain a definition of  $V$ :**

$$\text{Range}(b) = \{b\} \cup \{k \mid b \in \text{DefsIn}(k) \ \& \ \text{LiveIn}(k)\}$$

**This rule states that the Live Range of a definition to  $V$  in Basic Block  $b$  is  $b$  plus all other Basic Blocks that the definition of  $V$  reaches and in which  $V$  is live.**

**If two Live Ranges overlap (have one of more Basic Blocks in common), they *must* share the same register too. (Why?)**

**Therefore,**

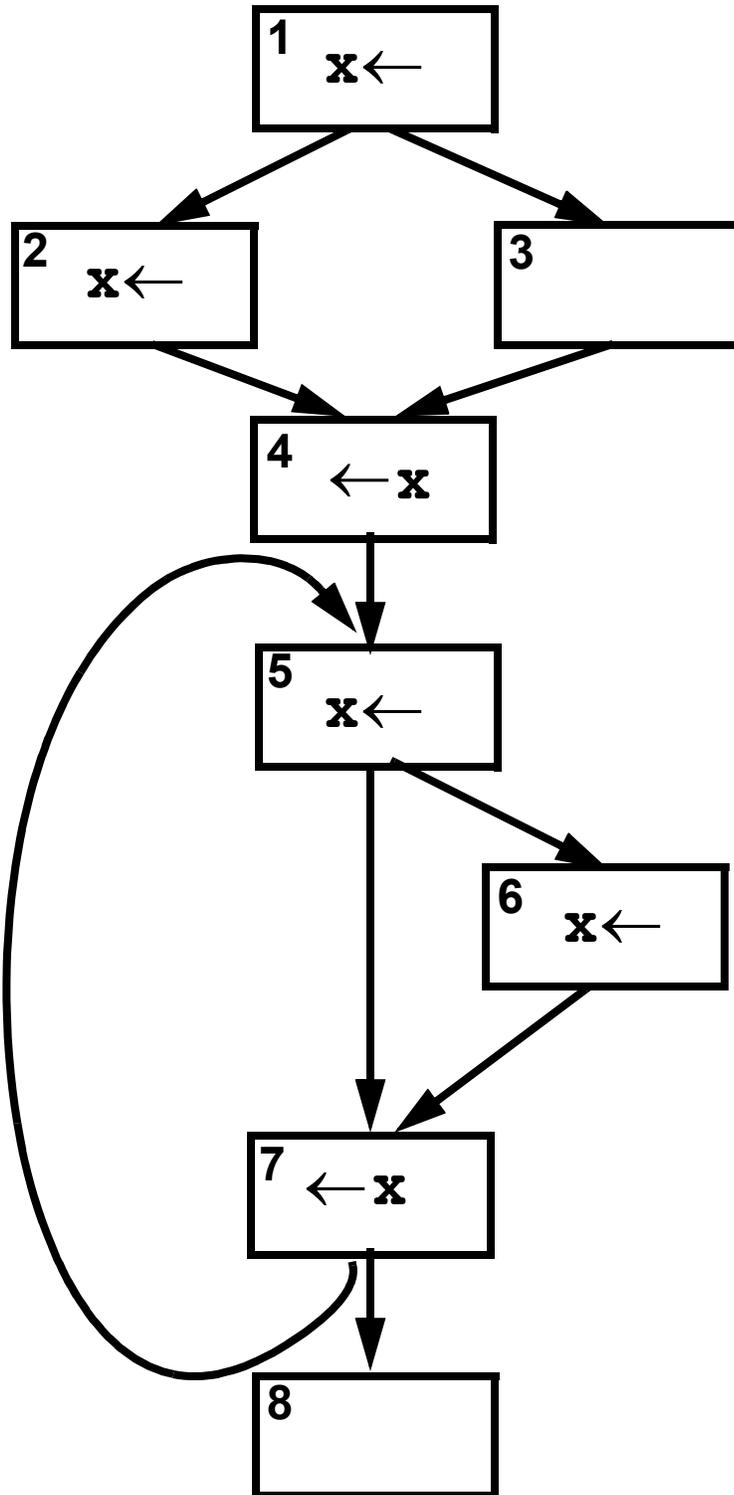
**If  $\text{Range}(b_1) \cap \text{Range}(b_2) \neq \phi$**

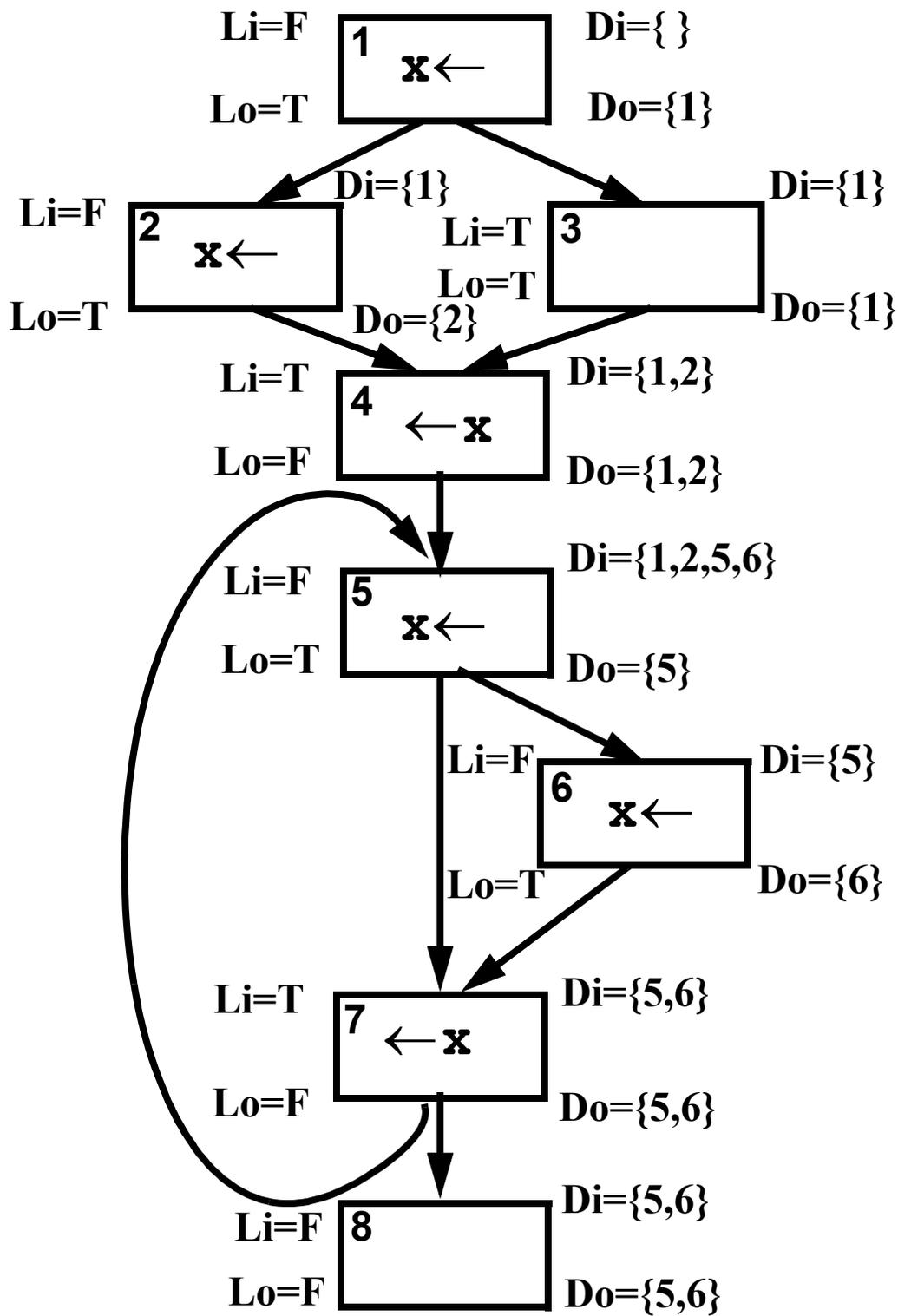
**Then replace**

**$\text{Range}(b_1)$  and  $\text{Range}(b_2)$**

**with  $\text{Range}(b_1) \cup \text{Range}(b_2)$**

# Example





## The Live Ranges we Compute are

$$\text{Range}(1) = \{1\} \cup \{3,4\} = \{1,3,4\}$$

$$\text{Range}(2) = \{2\} \cup \{4\} = \{2,4\}$$

$$\text{Range}(5) = \{5\} \cup \{7\} = \{5,7\}$$

$$\text{Range}(6) = \{6\} \cup \{7\} = \{6,7\}$$

Ranges 1 and 2 overlap, so

$$\text{Range}(1) = \text{Range}(2) = \{1,2,3,4\}$$

Ranges 5 and 6 overlap, so

$$\text{Range}(5) = \text{Range}(6) = \{5,6,7\}$$

# Interference Graph

**An *Interference Graph* represents interferences between Live Ranges.**

**Two Live Ranges *interfere* if they share one or more Basic Blocks in common.**

**Live Ranges that interfere *must* be allocated different registers.**

**In an Interference Graph:**

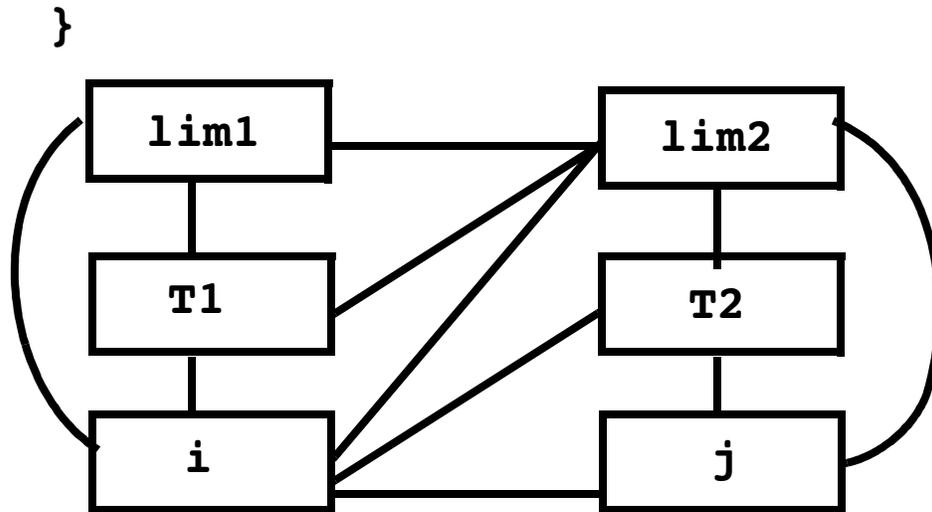
- **Nodes are Live Ranges**
- **An undirected arc connects two Live Ranges if and only if they interfere**

# Example

```
int p(int lim1, int lim2) {
    for (i=0; i<lim1 && A[i]>0;i++){
        for (j=0; j<lim2 && B[j]>0;j++){
            return i+j;
        }
    }
}
```

**We optimize array accesses by placing `&A[0]` and `&B[0]` in temporaries:**

```
int p(int lim1, int lim2) {
    int *T1 = &A[0];
    for (i=0; i<lim1 && *(T1+i)>0;i++){
        int *T2 = &B[0];
        for (j=0; j<lim2 && *(T2+j)>0;j++){
            return i+j;
        }
    }
}
```



## Register Allocation via Graph Coloring

**We model global register allocation as a Coloring Problem on the Interference Graph**

**We wish to use the fewest possible colors (registers) subject to the rule that two connected nodes can't share the same color.**

# **Optimal Graph Coloring is NP-Complete**

## **Reference:**

**“Computers and Intractability,”  
M. Garey and D. Johnson,  
W.H. Freeman, 1979.**

**We’ll use a Heuristic Algorithm  
originally suggested by Chaitin et. al.  
and improved by Briggs et. al.**

## **References:**

**“Register Allocation Via Coloring,”  
G. Chaitin et. al., Computer  
Languages, 1981.**

**“Improvement to Graph Coloring  
Register Allocation,” P. Briggs et.  
al., PLDI, 1989.**

# Coloring Heuristic

**To R-Color a Graph (where R is the number of registers available)**

**1. While any node, n, has  $< R$  neighbors:**

**Remove n from the Graph.**

**Push n onto a Stack.**

**2. If the remaining Graph is non-empty:**

**Compute the Cost of each node.**

**The Cost of a Node (a Live Range) is the number of extra instructions needed if the Node isn't assigned a register, scaled by  $10^{\text{loop\_depth}}$ .**

**Let  $NB(n) =$**

**Number of Neighbors of n.**

**Remove that node n that has the smallest  $\text{Cost}(n)/NB(n)$  value.**

**Push  $n$  onto a Stack.  
Return to Step 1.**

**3. While Stack is non-empty:  
Pop  $n$  from the Stack.**

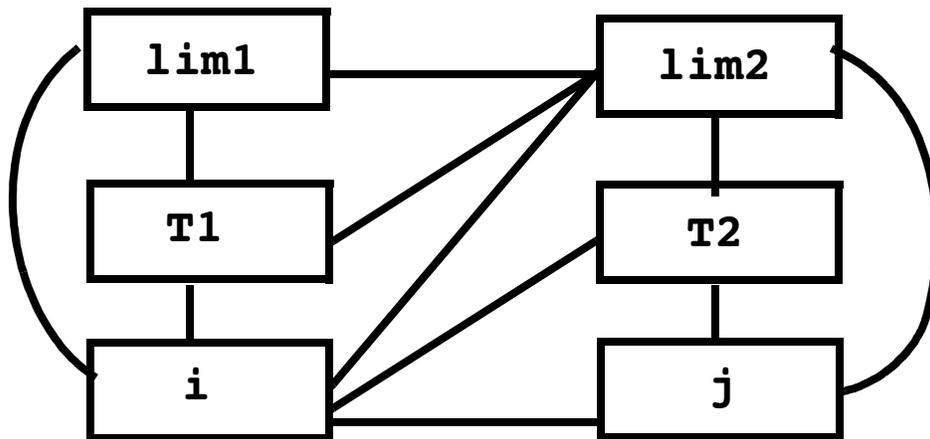
**If  $n$ 's neighbors are assigned fewer  
than  $R$  colors**

**Then assign  $n$  any unassigned  
color**

**Else leave  $n$  uncolored.**

# Example

```
int p(int lim1, int lim2) {  
  int *T1 = &A[0];  
  for (i=0; i<lim1 && *(T1+i)>0;i++){  
  int *T2 = &B[0];  
  for (j=0; j<lim2 && *(T2+j)>0;j++){  
  return i+j;  
  }  
}
```



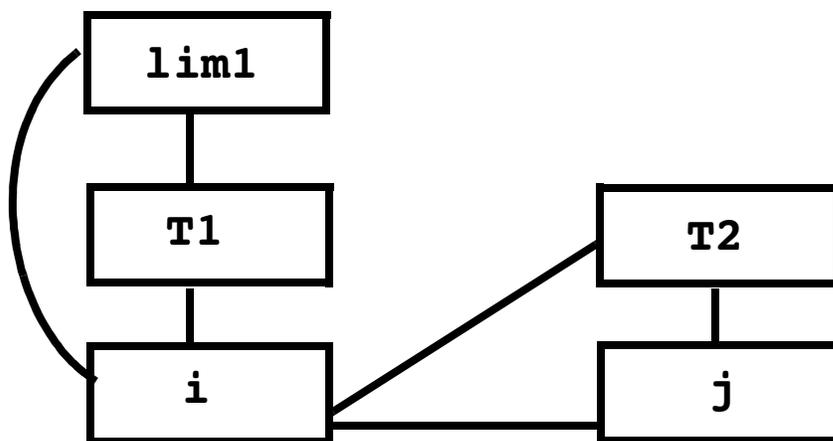
	lim1	lim2	T1	T2	i	j
Cost	11	11	11	11	42	42
Cost/ Neighbors	11/3	11/5	11/3	11/3	42/5	42/3

**Do a 3 coloring**

**Since no node has fewer than 3 neighbors, we remove a node based on the minimum Cost/Neighbors value.**

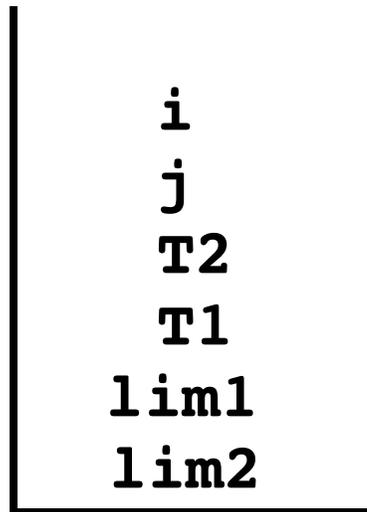
**lim2 is chosen.**

**We now have:**



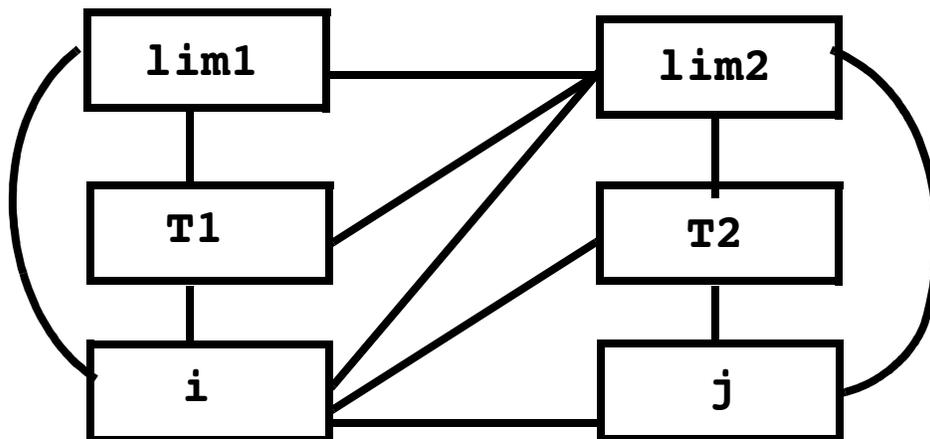
**Remove (say) lim1, then T1, T2, j and i (order is arbitrary).**

**The Stack is:**



**Assuming the colors we have are R1, R2 and R3, the register assignment we choose is**

**`i`:R1, `j`:R2, `T2`:R3, `T1`:R2, `lim1`:R3, `lim2`:spill**



# Color Preferences

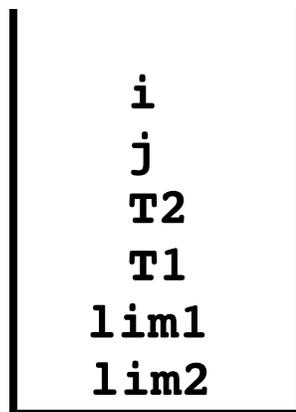
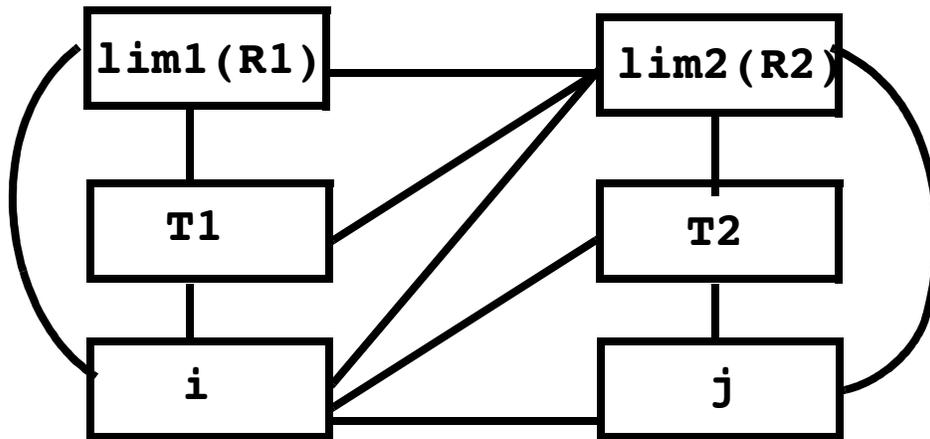
**Sometimes we wish to assign a particular register (color) to a selected Live Range (e.g., a parameter or return value) *if possible*.**

**We can mark a node in the Interference Graph with a *Color Preference*.**

**When we unstack nodes and assign colors, we will avoid choosing color *c* if an uncolored neighbor has indicted a preference for it. If only color *c* is left, we take it (and ignore the preference).**

# Example

**Assume in our previous example that `lim1` has requested register `R1` and `lim2` has requested register `R2` (because these are the registers the parameters are passed in).**



**Now when  $i$ ,  $j$  and  $T1$  are unstacked, they respect  $lim1$ 's and  $lim2$ 's preferences:**

**$i:R3$ ,  $j:R1$ ,  $T2:R2$ ,  $T1:R2$ ,  $lim1:R1$ ,  $lim2:spill$**

# Using Coloring to Optimize Register Moves

A nice “fringe benefit” of allocating registers via coloring is that we can often *optimize away* register to register moves by giving the source and target the *same color*.

Consider

Live in: a,b

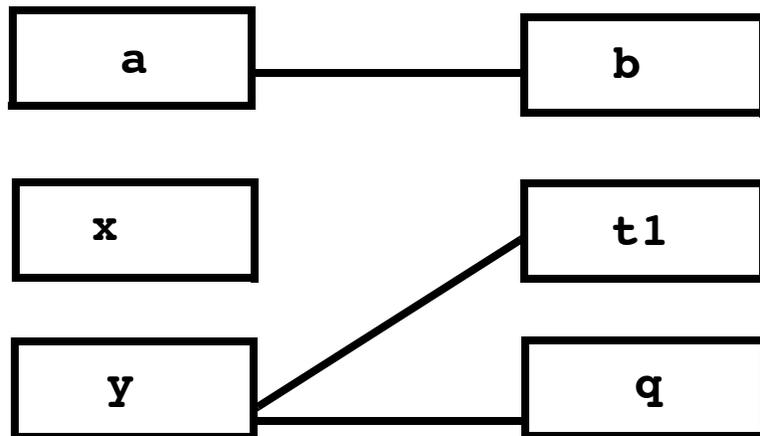
t1 = a + b

x = t1

y = x + 1

q = t1

Live out: y,q



We’d like x, t1 and q to get the same color. How do we “force” this?

## We can “merge” x, t1 and q

Live in: a,b



t1 = a + b

x = t1



y = x + 1

q = t1

Live out: y,q

**together:**

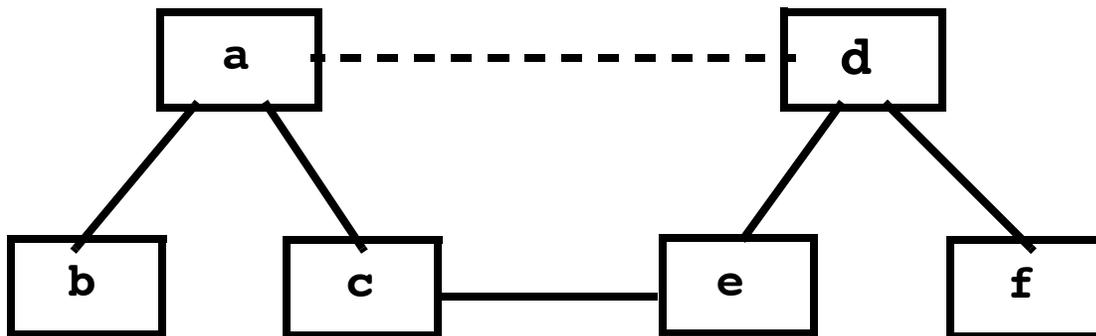
**Now a 2-coloring that optimizes away both register to register moves is trivial.**

# Reckless Coalescing

Originally, Chaitin suggested merging *all* move-related nodes that don't interfere.

This is *reckless*—the merged node may not be colorable!

(Is it worth a spill to save a move??)



This Graph is 2-colorable before the reckless merge, but *not* after.

# Reading Assignment

- **Read George and Appel’s paper, “Iterated Register Coalescing.” (Linked from Class Web page)**
- **Read Larus and Hilfinger’s paper, “Register Allocation in the SPUR Lisp Compiler.”**

# Iterated Coalescing

**This is an intermediate approach, that seeks to be safer than reckless coalescing and more effective than conservative coalescing. It was proposed by George and Appel.**

## **1. Build:**

**Create an Interference Graph, as usual. Mark source-target pairs with a special move-related arc (denoted as a dashed line).**

## **2. Simplify:**

**Remove and stack non-move-related nodes with  $< R$  neighbors.**

## **3. Coalesce:**

**Combine move-related pairs that will have  $< R$  neighbors after coalescing.**

**Repeat steps 2 and 3 until only nodes with  $R$  or more neighbors or move-related nodes remain or the graph is empty.**

#### **4. Freeze:**

**If the Interference Graph is  
non-empty:**

**Then If there exists a move-related  
node with  $< R$  neighbors**

**Then: “Freeze in” the move and  
make the node  
non-move-related.**

**Return to Steps 2 and 3.**

**Else: Use Chaitin’s  
Cost/Neighbors criterion  
to remove and stack  
a node.**

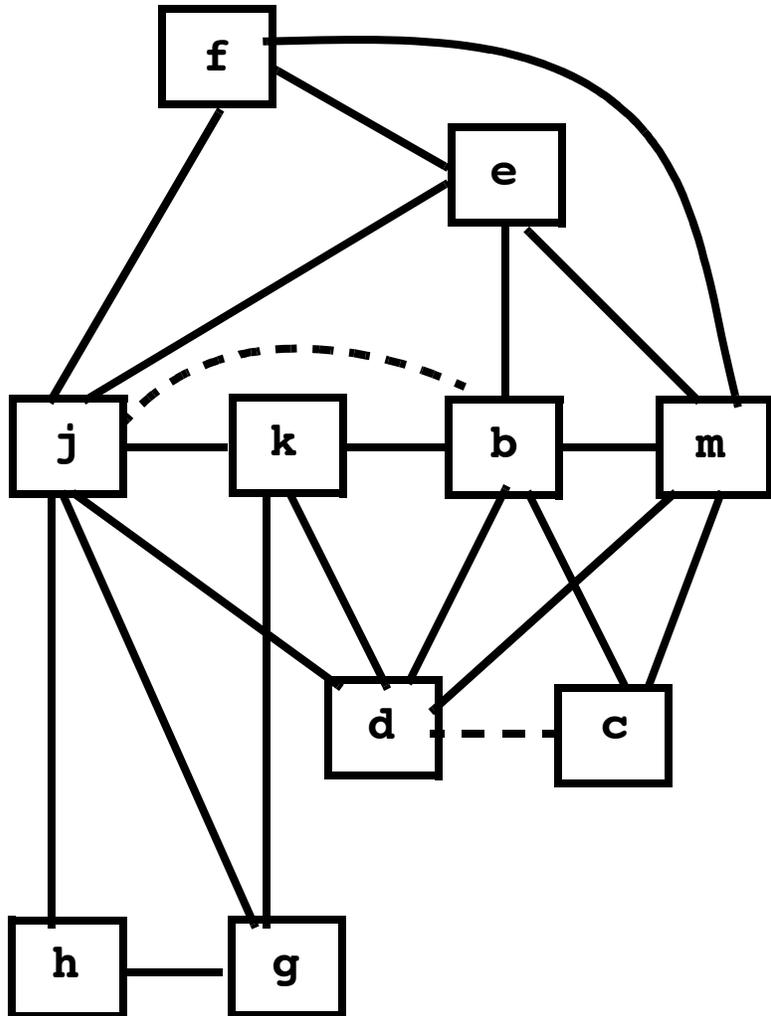
**Return to Steps 2 and 3.**

#### **5. Unstack:**

**Color nodes as they are unstacked  
as per Chaitin and Briggs.**

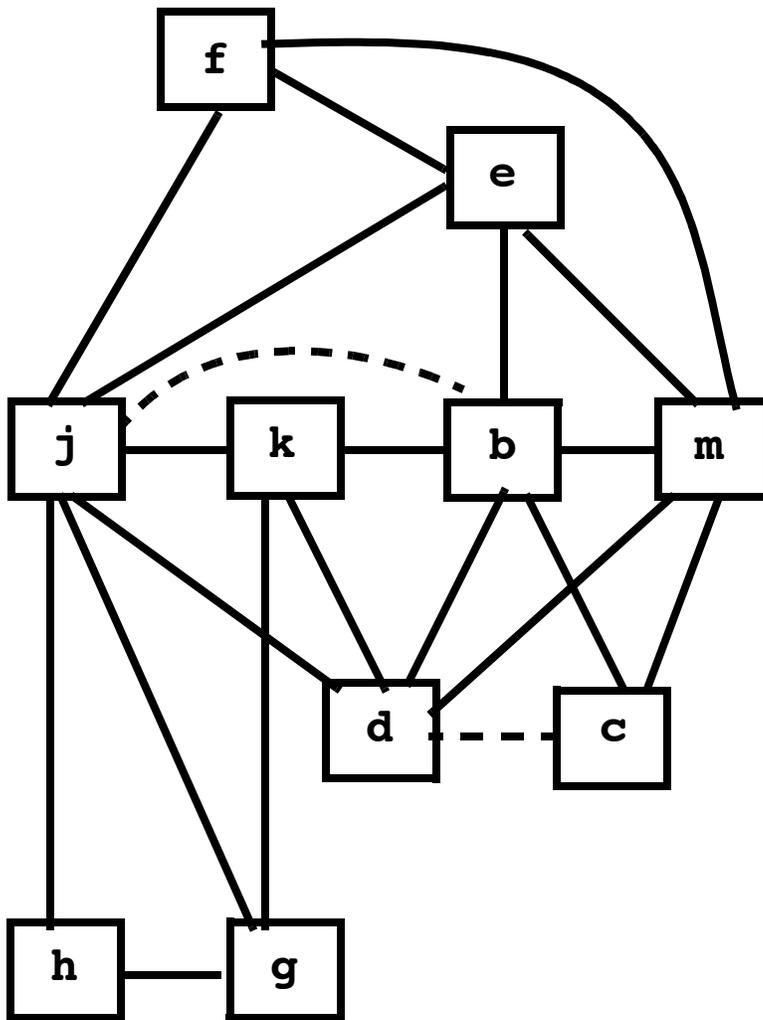
# Example

```
Live in: k, j
g = mem[j+12]
h = k-1
f = g*h
e = mem[j+8]
m = mem[j+16]
b = mem[f]
c = e+8
d = c
k = m+4
j = b
goto d
Live out: d, k, j
```



**Assume we want a 4-coloring.**

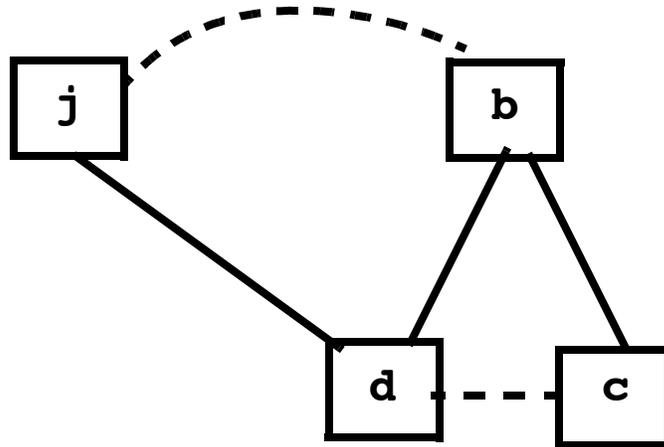
**Note that neither j&b nor d&c can be conservatively colored.**



**We simplify by removing nodes with fewer than 4 neighbors.**

**We remove and stack: g, h, k, f, e, m**

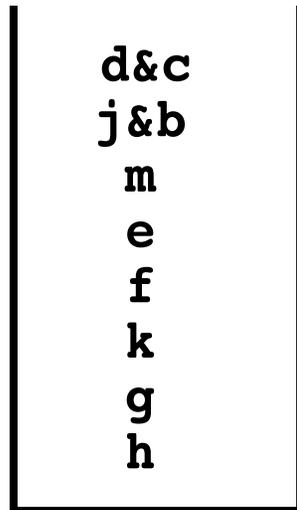
**The remaining Interference Graph is**



**We can now conservatively coalesce the move-related pairs to obtain**



**These remaining nodes can now be removed and stacked.**



**We can now unstack and color:**

**d&c:R1, j&b:R2, m:R3, e:R4, f:R1,  
k:R3, h:R1, g:R4**

**No spills were required and both  
moves were optimized away.**

# Priority-Based Register Allocation

**Alternatives to Chaitin-style register allocation are presented in:**

- **Hennessy and Chow, “The priority-based coloring approach to register allocation,” ACM TOPLAS, October 1990.**
- **Larus and Hilfinger, “Register allocation in the SPUR Lisp compiler,” SIGPLAN symposium on Compiler Construction, 1986.**

**These papers suggest two innovations:**

**1. Use of a *Priority Value* to choose nodes to color in an Interference Graph.**

**A Priority measures  
(Spill cost)/(Size of Live Range)**

**The idea is that small live ranges with a high spill cost are ideal candidates for register allocation. As the size of a live range grows, it becomes less attractive for register allocation (since it “ties up” a register for a larger portion of a program).**

**2. Live Range Splitting**

**Rather than spill an entire live range that can't be colored, the live range is split into two or more smaller live ranges that may be colorable.**

# Large vs. Small Live Ranges

- **A large live range has less spill code. Values are directly read from and written to a register.**  
*But, a large live range is harder to allocate, since it may conflict with many other register candidates.*
- **A small live range is easier to allocate since it competes with fewer register candidates.**  
*But, more spill code is needed to load and save register values across live ranges.*
- **In the limit a live range can shrink to a single definition or use of a register.**  
*But, then we really don't have an effective register allocation at all!*

# Terminology

## In an Interference Graph:

- A node with fewer neighbors than colors is termed *unconstrained*. It is trivial to color.
- A node that is not unconstrained is termed *constrained*. It may need to be split or spilled.

# Reading Assignment

- **Read “Minimum Cost Interprocedural Register Allocation,” by S. Kurlander et al. (linked from class Web page).**
- **Read David Wall’s paper, “Global Register Allocation at Link Time.”**

```

PriorityRegAlloc(proc, regCount) {
  ig ← buildInterferenceGraph(proc)
  unconstrained ←
    { n ∈ nodes(ig) | neighborCount(n) < regCount }
  constrained ←
    { n ∈ nodes(ig) | neighborCount(n) ≥ regCount }

  while( constrained ≠ φ ) {
    for ( c ∈ constrained such that not colorable(c)
          and canSplit(c) ) {
      c1, c2 ← split(c)
      constrained ← constrained - {c}
      if ( neighborCount(c1) < regCount )
        unconstrained ← unconstrained U { c1}
      else constrained ← constrained U {c1}
      if ( neighborCount(c2) < regCount )
        unconstrained ← unconstrained U { c2}
      else constrained ← constrained U {c2}
      for ( d ∈ neighbors(c) such that
            d ∈ unconstrained and
            neighborCount(d) ≥ regCount ){
        unconstrained ← unconstrained - {d}
        constrained ← constrained U {d}
      }
    } // End of both for loops
  }
}

```

```
/* At this point all nodes in constrained are  
colorable or can't be split */
```

```
    Select  $p \in$  constrained such that  
           priority( $p$ ) is maximized
```

```
    if ( colorable( $p$ ) )
```

```
        color( $p$ )
```

```
    else spill( $p$ )
```

```
    } // End of While
```

```
color all nodes  $\in$  unconstrained
```

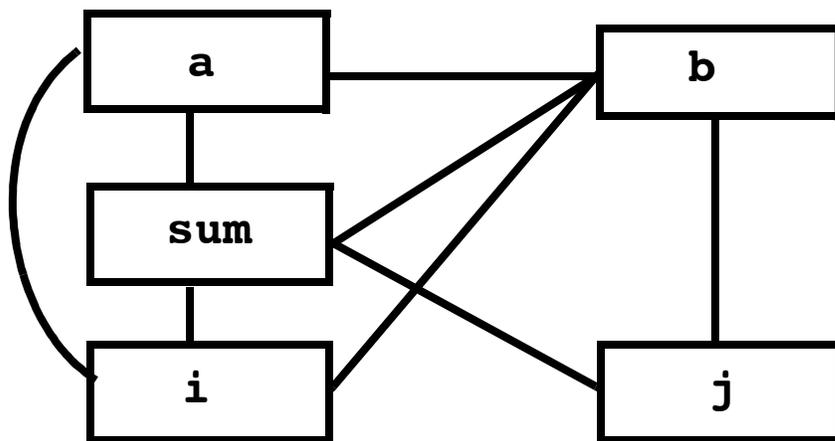
```
}
```

# How to Split a Constrained Node

- **There are many possible partitions of a live range; too many to fully explore.**
- **Heuristics are used instead. One simple heuristic is:**
  - 1. Remove the first basic block (or instruction) of the live range. Put it into a new live range, NR.**
  - 2. Move successor blocks (or instructions) from the original live range into NR, as long as NR remains colorable.**
  - 3. Single Basic Blocks (or instructions) that can't be colored are spilled.**

# Example

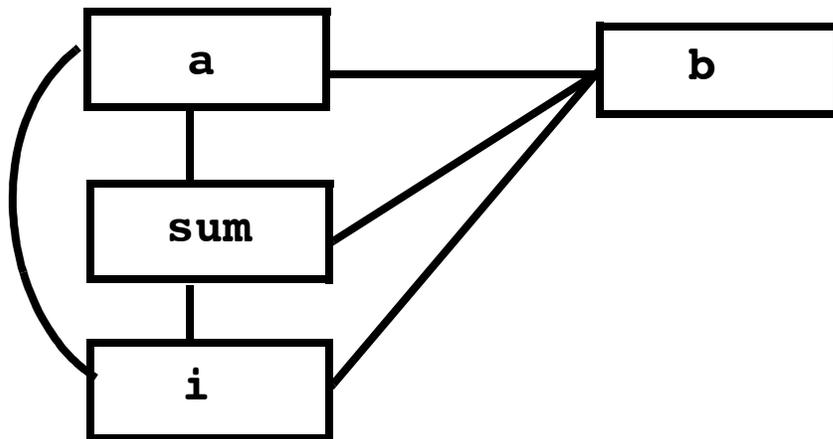
```
int sum(int a[], int b[]) {  
    int sum = 0;  
    for (int i=0; i<1000; i++)  
        sum += a[i];  
    for (int j=0; j<1000; j++)  
        sum += b[j];  
    return sum;  
}
```



**Assume we want a 3-coloring.**

**We first simplify the graph by removing unconstrained nodes (those with  $< 3$  neighbors).**

**Node  $j$  is removed. We now have:**

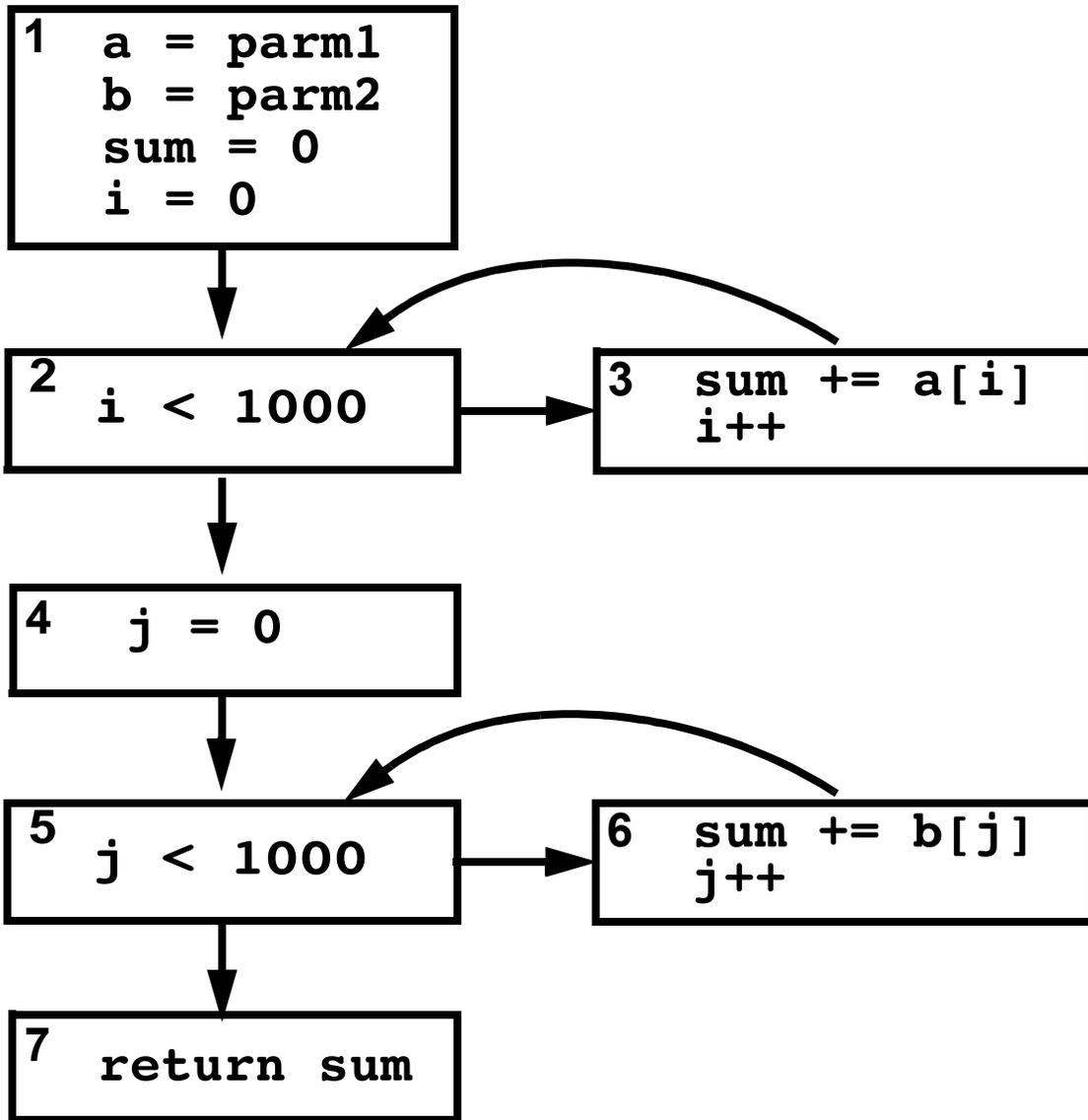


**At this point, each node has 3 neighbors, so either spilling or splitting is necessary.**

**A spill really isn't attractive as each of the 4 register candidates is used within a loop, magnifying the costs of accessing memory.**

# Coloring by Priorities

**We'll color constrained nodes by priority values, with preference given to large priority values.**



	a	b	sum	i
Cost	11	11	42	41
Cost/Size	11/3	11/6	42/7	41/3

**Variables *i*, *sum* and *a* are assigned colors R1, R2 and R3.**

**Variable *b* can't be colored, so we will try to split it. *b*'s live range is blocks 1 to 6, with 1 as *b*'s entry point.**

**Blocks 1 to 3 can't be colored, so *b* is spilled in block 1. However, blocks 4 to 6 form a split live range that can be colored (using R3).**

**We will reload *b* into R3 in block 4, and it will be register-allocated throughout the second loop. The added cost due to the split is minor—a store in block 1 and a reload in block 4.**

# Choice of Spill Heuristics

**We have seen a number of heuristics used to choose the live ranges to be spilled (or colored).**

**These heuristics are typically chosen using one's intuition of what register candidates are most (or least) important. Then a heuristic is tested and "fine tuned" using a variety of test programs.**

**Recently, researchers have suggested using machine learning techniques to automatically determine effective heuristics.**

**In "Meta Optimization: Improving Compiler Heuristics with Machine Learning," Stephenson, Amarasinghe, et al, suggest using *genetic programming* techniques in**

**which priority functions (like choice of spill candidates) are mutated and allowed to “evolve.”**

**Although the approach seems rather random and unfocused, it can be effective. Priority functions *better than* those used in real compilers have been reported, with research still ongoing.**

# Interprocedural Register Allocation

**The goal of register allocation is to keep frequently used values in registers.**

**Ideally, we'd like to go to memory only to access values that may be aliased or pointed to.**

**For example, array elements and heap objects are routinely loaded from and stored to memory each time they are accessed.**

**With alias analysis, optimizations like Scalarization are possible.**

```
for (i=0; i<1000; i++)  
    for (j=0; j<1000; j++)  
        a[i] += i * b[j];
```

**is optimized to**

```
for (i=0; i<1000; i++){  
    int Ai = a[i];  
    for (j=0; j<1000; j++)  
        Ai += i * b[j];  
    a[i] = Ai;  
}
```

# Attacking Call Overhead

- **Even with good global register allocation calls are still a problem.**
- **In general, the caller and callee may use the same registers, requiring saves and restores across calls.**
- **Register windows help, but they are inflexible, forcing all subprograms to use the same number of registers.**
- **We'd prefer a register allocator that is sensitive to the calling structure of a program.**

# Call Graphs

**A *Call Graph* represents the calling structure of a program.**

- **Nodes are subprograms (procedures and functions).**
- **Arcs represent calls between subprograms. An arc from A to B denotes that a call to B appears within A.**
- **For an indirect call (a function parameter or a function pointer) an arc is added to all potential callees.**

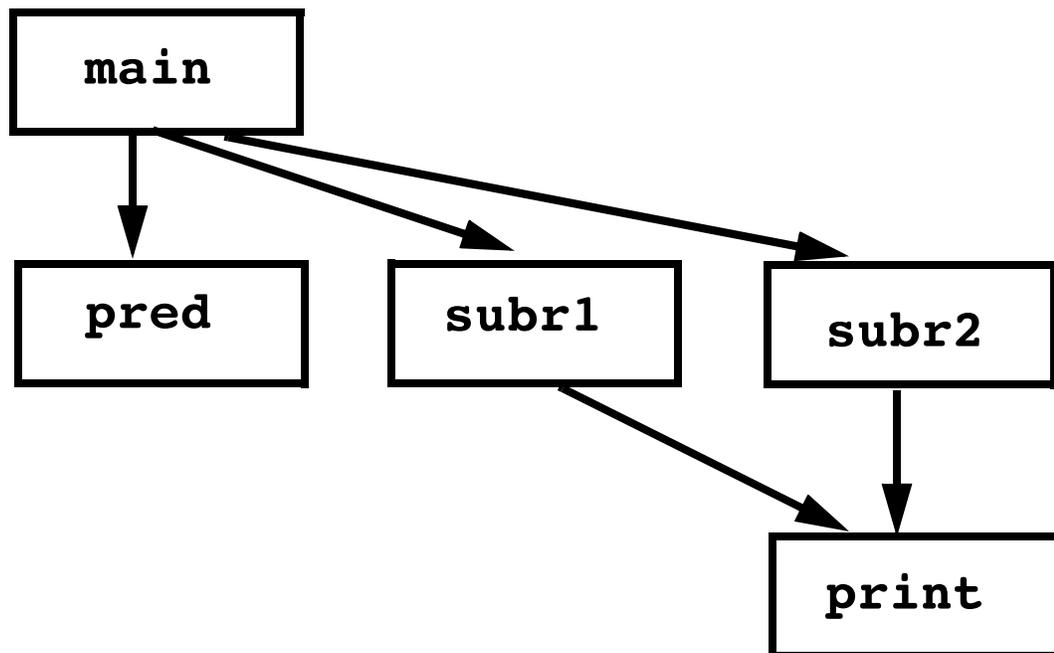
# Example

```
main() {  
    if (pred(a,b))  
        subr1(a)  
    else subr2(b);}
```

```
bool pred(int a, int b){  
    return a==b; }
```

```
subr1(int a){ print(a);}
```

```
subr2(int x){ print(2*x);}
```



# Wall's Interprocedural Register Allocator

**Operates in two phases:**

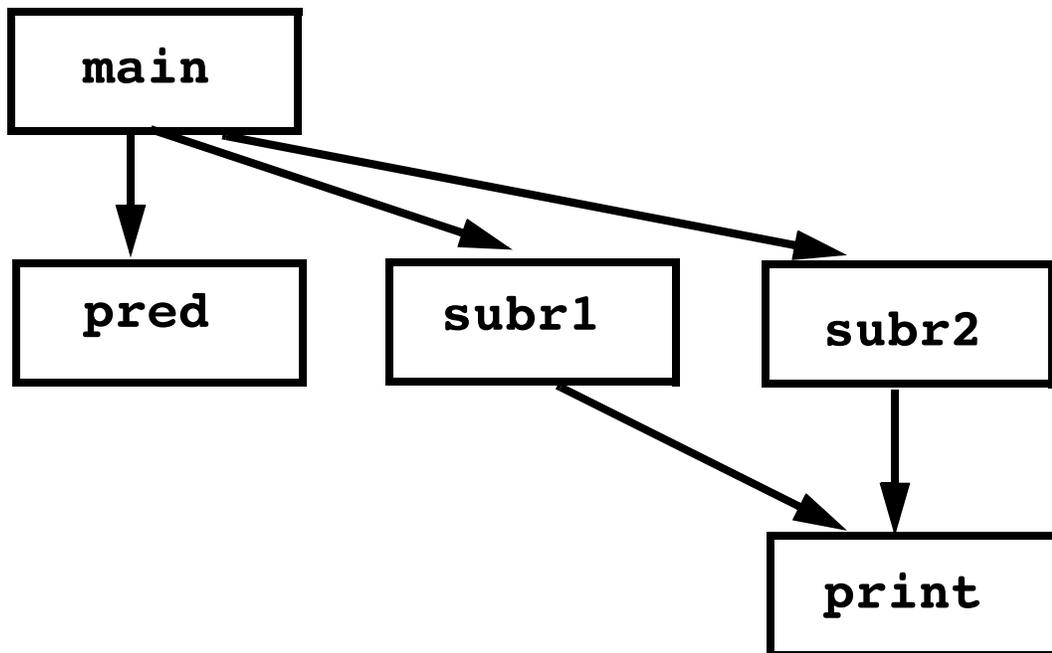
**1. Register candidates are identified at the subprogram level.**

**Each candidate (a single variable *or* a set of non-interfering live ranges) is compiled as if it *won't* get a register. At link-time unnecessary loads and stores are edited away *if* the candidate *is* allocated a register.**

**2. At link-time, when all subprograms are known and available, register candidates are allocated registers.**

# Identifying Interprocedural Register Sharing

If two subprograms are not connected in the call graph, a register candidate in each can share the same register without any saving or restoring across calls.



A register candidate from `pred`, `subr1` and `subr2` can all share one register.

**At the interprocedural level we must answer 2 questions:**

- 1. A local candidate of one subprogram can share a register with candidates of what other subprograms?**
- 2. Which local register candidates will yield the greatest benefit if given a register?**

**Wall designed his allocator for a machine with 52 registers. This is enough to divide all the registers among the subprograms without any saves or restores at call sites.**

**With fewer registers, spills, saves and restores will often be needed (if registers are used aggressively within a subprogram).**

# Restrictions on the Call Graph

**Wall limited calls graphs to DAGs since cycles in a call graph imply recursion, which will force saves and restores (why?)**

## Cost Computations

**Each register candidate is given a per-call cost, based on the number of saves and restores that can be removed, scaled by  $10^{\text{loop\_depth}}$ .**

**This benefit is then multiplied by the *expected* number of calls, obtained by summing the total number of call sites, scaled by loop nesting depth.**

# Grouping Register Candidates

**We now have an estimate of the benefit of allocating a register to a candidate. Call this estimate  $\text{cost}(\text{candidate})$**

**We estimate potential interprocedural sharing of register candidates by assigning each candidate to a *Group*.**

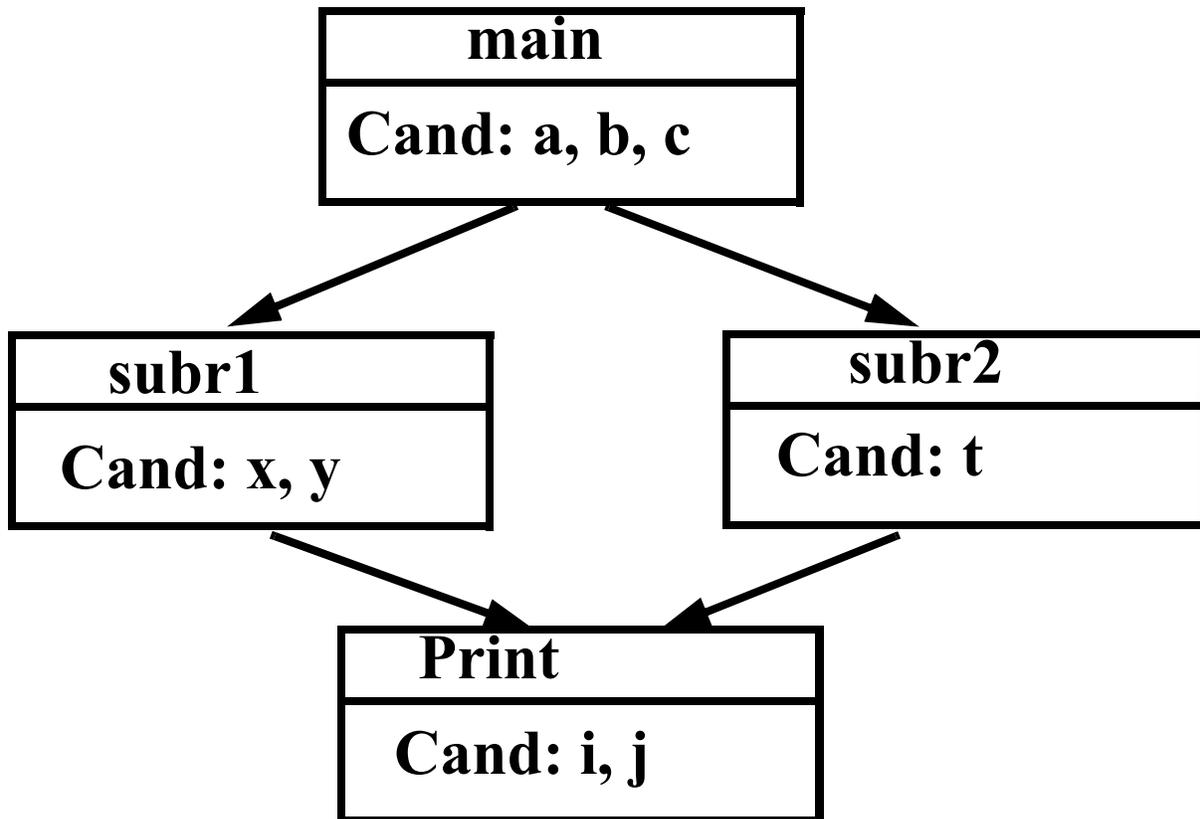
**All candidates within a group can share a register. No two candidates in any subprogram are in the same group.**

**Groups are assigned during a reverse depth-first traversal of the call graph.**

```
AsgGroup(node n) {  
  if (n is a leaf node)  
    grp = 0  
  else { for (each c ∈ children(n)) {  
    AsgGroup(c) }  
    grp =  
    1+ Max (Max group used in c)  
    c ∈ children(n)  
  }  
  for (each r ∈ registerCandidates(n)){  
    assign r to grp  
    grp++ }  
}
```

**Global variables are assigned to a singleton group.**

# Example



**At Print:  $\text{grp}(i)=0$ ,  $\text{grp}(j)=1$**

**At subr1: Max grp used in print is 1  
 $\text{grp}(x)=2$ ,  $\text{grp}(y)=3$**

**At subr2: Max grp used in print is 1  
 $\text{grp}(t)=2$**

**At main: Max grp used in children is 3  
 $\text{grp}(a)=4$ ,  $\text{grp}(b)=5$ ,  $\text{grp}(c)=6$**

**If A calls B (directly or indirectly),  
then none of A's register candidates  
are in the same group as any of B's  
register candidates.**

**This *guarantees* that A and B will  
use different registers.**

**Thus no saves or restores are  
needed across a call from A to B.**

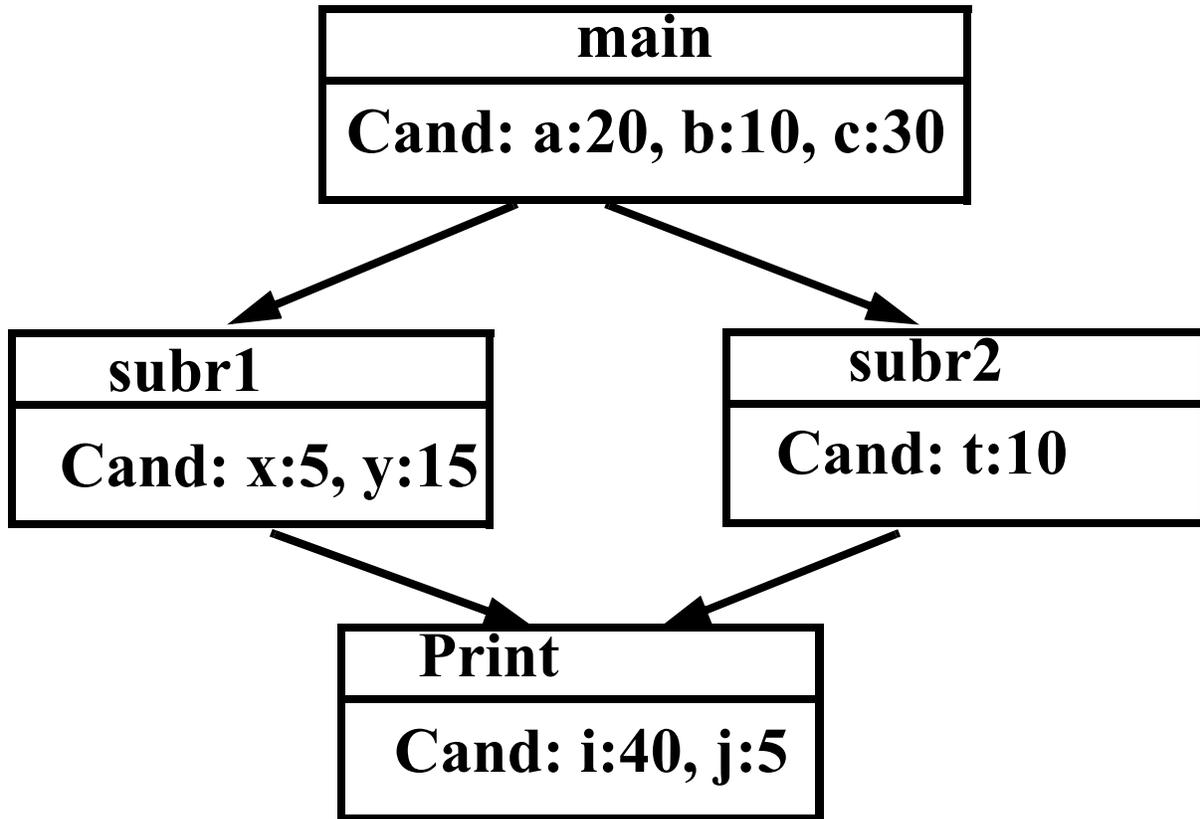
# Assigning Registers to Groups

$$\text{Cost}(\text{group}) = \sum_{\text{candidates} \in \text{group}} \text{cost}(\text{candidates})$$

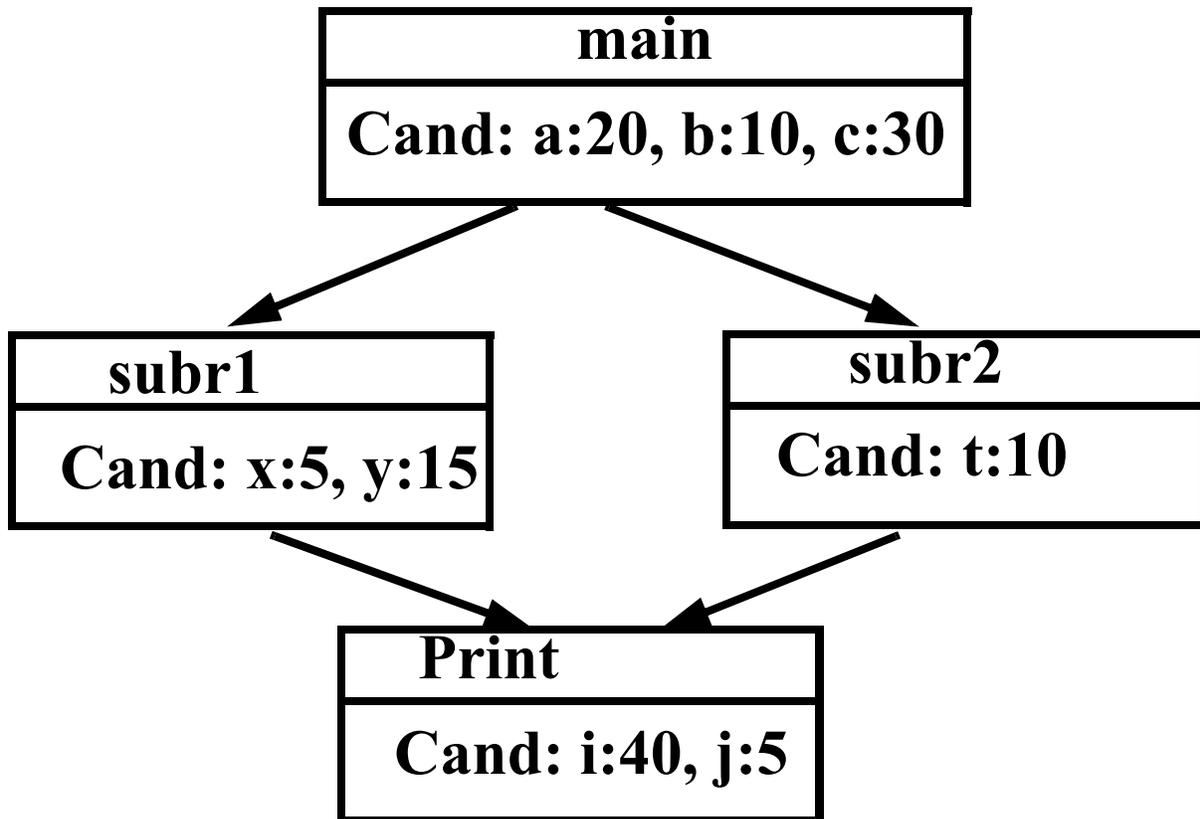
**We assign registers to groups based on the cost of each group, using an “auction.”**

```
for (r=0; r < RegisterCount; r++) {  
    Let G be the group with the  
        greatest cost that has not yet  
        been assigned a register.  
    Assign r to G  
}
```

# Example (3 Registers)



Group	Members	Cost
0	i	40
1	j	5
2	x, t	15
3	y	15
4	a	20
5	b	10
6	c	30



**The 3 registers are given to the groups with the highest weight,**

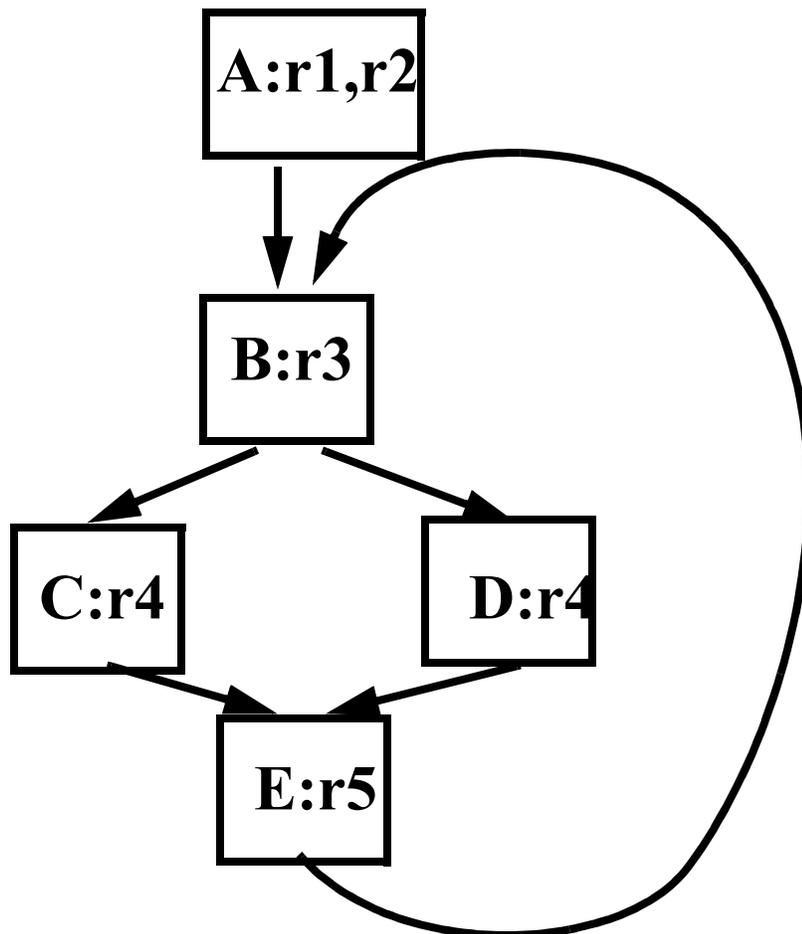
**i (40), c(30), a(20).**

**Is this optimal?**

**No! If y and t are grouped together, y and t (cost=25) get the last register.**

# Recursion

To handle recursion, any call to a subprogram “up” in the call graph must save and restore all registers possibly in use between the caller and callee.



**A call from E to B saves r3 to r5.**

# Performance

**Wall found interprocedural register allocation to be very effective (given 52 Registers!).**

**Speedups of 10-28% were reported. Even with only 8 registers, speedups of 5-20% were observed.**

# Optimal Interprocedural Register Allocation

**Wall's approach to interprocedural register allocation isn't optimal because register candidates aren't grouped to achieve maximum benefit.**

**Moreover, the placement of save and restore code *if needed* isn't considered.**

**These limitations are addressed by Kurlander in "Minimum Cost Interprocedural Register Allocation."**

# Optimal Save-Free Interprocedural Register Allocation

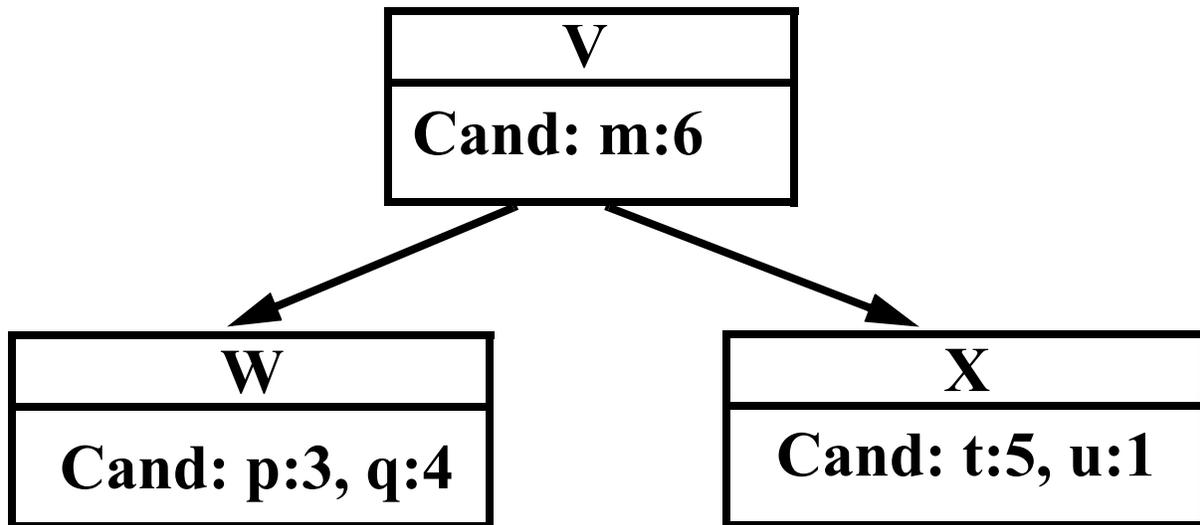
- Allocation is done on a cycle-free call graph.
- Each subprogram has one or more register candidates,  $c_i$ .
- Each register candidate,  $c_i$ , has a cost (or benefit),  $w_i$ , that is the improvement in performance if  $c_i$  is given a register. (This  $w_i$  value is scaled to include nested loops and expected call frequencies.)

# **Interference Between Register Candidates**

**The notion of interference is extended to include interprocedural register candidates:**

- Two Candidates in the same subprogram always interfere. (Local non-interfering variables and values have already been grouped into interprocedural register candidates.)**
- If subprogram P calls subprogram Q (directly or indirectly) then register candidates within P always interfere with register candidates within Q.**

# Example



**The algorithm can group candidate p with either t or u (since they don't interfere). It can also group candidate q with either t or u.**

**If two registers are available, it must “discover” that assigning R1 to q&t, and R2 to m is optimal.**

**Non-interfering register candidates are grouped into registers so as to solve:**

$$\text{Maximize } \sum_k w_j$$
$$c_j \in \bigcup_{i=1} R_i$$

**That is, we wish to group sets of non-interfering register candidates into  $k$  registers such that the overall benefit is maximized.**

**But how do we solve this?**

**Certainly examining all possible groupings will be prohibitively expensive!**

**Kurlander solved this problem by mapping it to a known problem in Integer Programming:  
the Dual Network Flow Problem.**

**Solution techniques for this problem are well known—libraries of standard solution algorithms exist.**

**Moreover, this problem can be solved in *polynomial time*.**

**That is, it is “easier” than optimal global (intraprocedural) register allocation, which is NP-complete!**

# Adding Saves & Restores

**Wall designed his save-free interprocedural allocator for a machine with 52 registers.**

**Most computers have far fewer registers, and hence saving and restoring across calls, *when profitable*, should be allowed.**

**Kurlander's Technique can be extended to include save/restore costs. If the cost of saving and restoring is *less* than the benefit of allocating an extra register, saving is done. Moreover, saving is done where it is *cheapest* (not closest!).**

# Example

```
main() { ... p(); ...}

p() { ...
      for (i=0; i<1000000; i++) {
          q():
      }
}
```

**We first allocate registers in a save-free mode. After all Registers have been allocated, `q` may need additional registers.**

**Most allocators would add save/restore code at `q`'s call site (or `q`'s prologue and epilogue).**

**An optimal allocator will place save/restore code at `p`'s call site, freeing a register that `p` doesn't even want (but that `q` does want!)**

# Extending the Cost Model

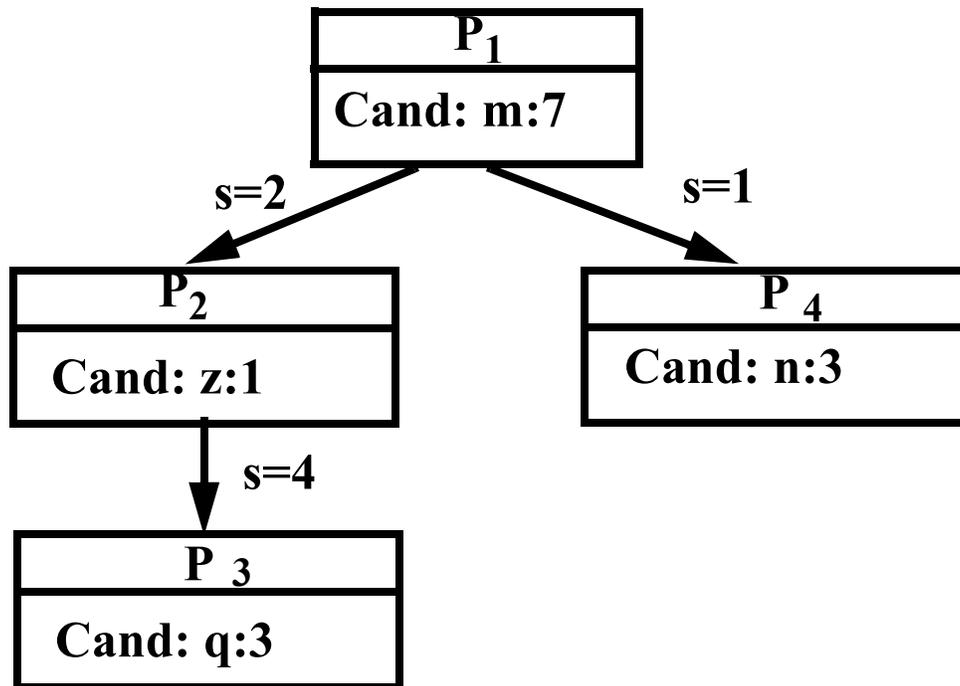
- As before, we group register candidates of different subprograms into registers.
- Now only candidates within the same subprogram automatically interfere.
- Saves are placed on the edges of the call graph.
- We aim to solve

$$\text{Maximize } \sum_k w_j - \sum_{e_m \in \text{Edges}} s_m * \text{Saved}_m$$
$$c_j \in \bigcup_{i=1} R_i$$

where  $s_m$  is the per/register save/restore cost and  $\text{Saved}_m$  is the number of registers saved on edge  $e_m$ .

- **As registers are saved, they may be reused in child subprograms.**
- **This optimization problem can be solved as a Network Dual Flow Problem.**
- **Again, the solution algorithm is *polynomial*.**

# Example (One Register)



**$P_1$  gets R1 save-free for m.**

**A save on  $P_1 \rightarrow P_4$  costs 1 and gives a register to n (net profit =2), so we do it.**

**A save on  $P_1 \rightarrow P_2$  for z costs 2, and yields 1, which isn't profitable.**

**A save on  $P_2 \rightarrow P_3$  for q costs 4, and yields 3, which isn't profitable.**

**A save on  $P_1 \rightarrow P_2$  for q costs 2, and yields 3, which *is* a net gain.**

# Handling Global Variables

- **Wall's technique handled globals by assuming they interfere with all subprograms and all other globals.**
- **Kurlander's approach is incremental (and non-optimal):**

**First, an optimal allocation for  $r$  registers is computed.**

**Next, one register is “stolen” and assigned interprocedurally to the most beneficial global.**

**(Subprograms that don't use the global can save and restore it locally, allowing local reuse).**

**An optimal allocation using  $R-1$  registers is computed. If this solution plus the shared global is more profitable than the  $R$  register**

**solution, the global allocation is “locked in.”**

**Next, another register is “stolen” for a global, leaving R-2 for interprocedural allocation.**

**This process continues until stealing another register for a global isn’t profitable.**

# Why is Optimal Interprocedural Register Allocation Easier than Optimal IntraProcedural Allocation?

This result seems counter-intuitive. How can allocating a whole program be *easier* (computationally) than allocating only one subprogram.

Two observations provide the answer:

- Interprocedural allocation assumes some form of local allocation has occurred (to identify register candidates).
- Interprocedural interference is *transitive* (if A interferes with B and B interferes with C then A interferes with B). But intraprocedural interference *isn't* transitive!

# Reading Assignment

- **Read Section 15.4 (Code Scheduling) of *Crafting a Compiler*.**
- **Read Gibbon’s and Muchnick’s paper, “Efficient Instruction Scheduling for a Pipelined Architecture.”**
- **Read Kerns and Eggers’ paper, “Balanced Scheduling: Instruction Scheduling When Memory Latency is Uncertain.”**

# Code Scheduling

**Modern processors are pipelined.**

**They give the impression that all instructions take unit time by executing instructions in *stages* (steps), as if on an assembly line.**

**Certain instructions though (loads, floating point divides and square roots, delayed branches) take more than one cycle to execute.**

**These instructions may *stall* (halt the processor) or require a nop (null operation) to execute properly.**

**A *Code Scheduling* phase may be needed in a compiler to avoid stalls or eliminate nops.**

# Scheduling Expression DAGs

**After generating code for a DAG or basic block, we may wish to schedule (reorder) instructions to reduce or eliminate stalls.**

***A Postpass Scheduler is run after code selection and register allocation.***

**Postpass schedulers are very general and flexible, since they can be used with code generated by any compiler with any degree of optimization**

***But, since they can't modify register allocations, they can't always avoid stalls.***

# Dependency DAGs

**Obviously, not all reorderings of generated instructions are acceptable.**

**Computation of a register value must precede all uses of that value.**

**A store of a value must precede all loads that might read that value.**

**A *Dependency Dag* reflects essential ordering constraints among instructions:**

- **Nodes are Instructions to be scheduled.**
- **An arc from Instruction *i* to Instruction *j* indicates that *i* must be executed before *j* may be executed.**

# Kinds of Dependencies

We can identify several kinds of dependencies:

- **True Dependence:**

An operation that uses a value has a true dependence (also called a flow dependence) upon an earlier operation that computes the value.

For example:

```
mov  1, %12
add  %12, 1, %12
```

- **Anti Dependence:**

An operation that writes a value has an anti dependence upon an earlier operation that reads the value. For example:

```
add  %12, 1, %10
mov  1, %12
```

- **Output Dependence:**

**An operation that writes a value has a output dependence upon an earlier operation that writes the value. For example:**

```
mov  1, %12  
mov  2, %12
```

**Collectively, true, anti and output dependencies are called data dependencies. Data dependencies constrain the order in which instructions may be executed.**

# Example

**Consider the code that might be generated for**

**$a = ((a+b) + (c*d)) + ((c+d) * d);$**

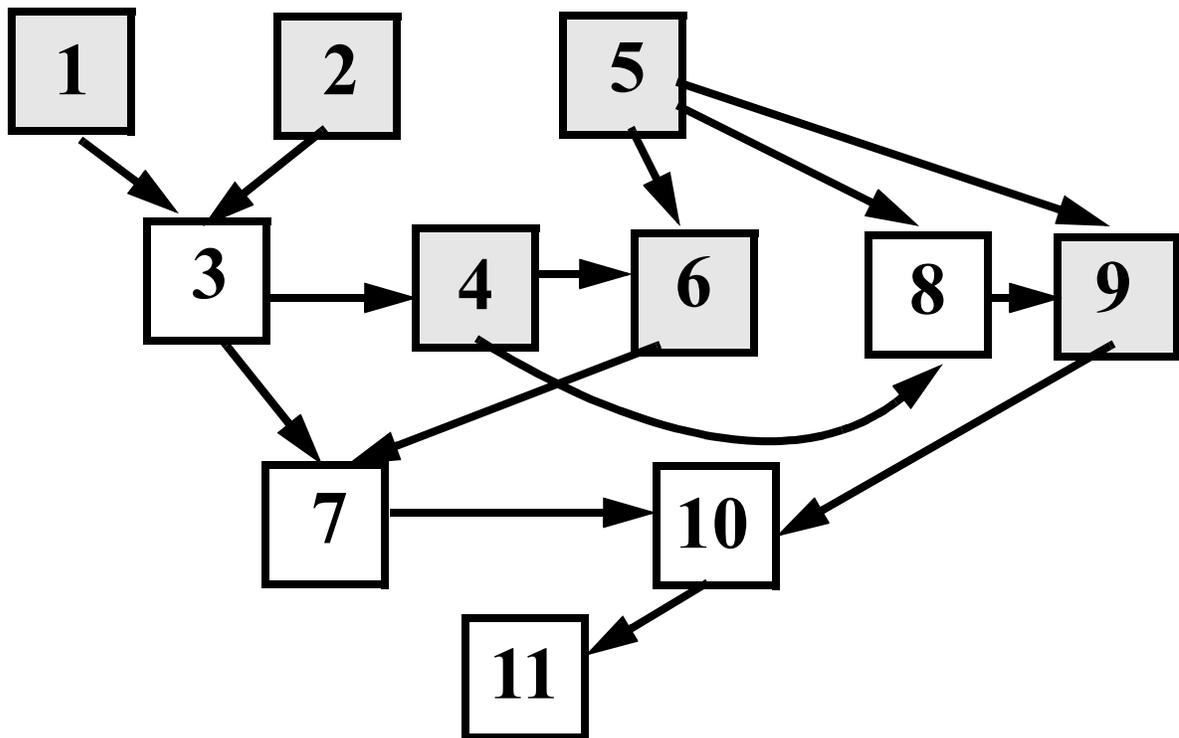
**We'll assume 4 registers, the minimum possible, and we'll reuse already loaded values.**

**Assume a 1 cycle stall between a load and use of the loaded value and a 2 cycle stall between a multiplication and first use of the product.**

```

1. ld    [a], %r1
2. ld    [b], %r2    ← Stall
3. add   %r1,%r2,%r1
4. ld    [c], %r2
5. ld    [d], %r3
6. smul  %r2,%r3,%r4 ← Stall
7. add   %r1,%r4,%r1 ← Stall*2
8. add   %r2,%r3,%r2
9. smul  %r2,%r3,%r2 ← Stall*2
10. add  %r1,%r2,%r1 ← Stall*2
11. st   %r1,[a]    (6 Stalls Total)

```



# Scheduling Requires Topological Traversal

**Any valid code schedule is a *Topological Sort* of the dependency dag.**

**To create a code schedule you**

- (1) Pick any root of the Dag.**
- (2) Remove it from the Dag and schedule it.**
- (3) Iterate!**

**Choosing a *Minimum Delay* schedule is NP-Complete:**

**“Computers and Intractability,”  
M. Garey and D. Johnson,  
W.H. Freeman, 1979.**

# **Dynamically Scheduled (Out of Order) Processors**

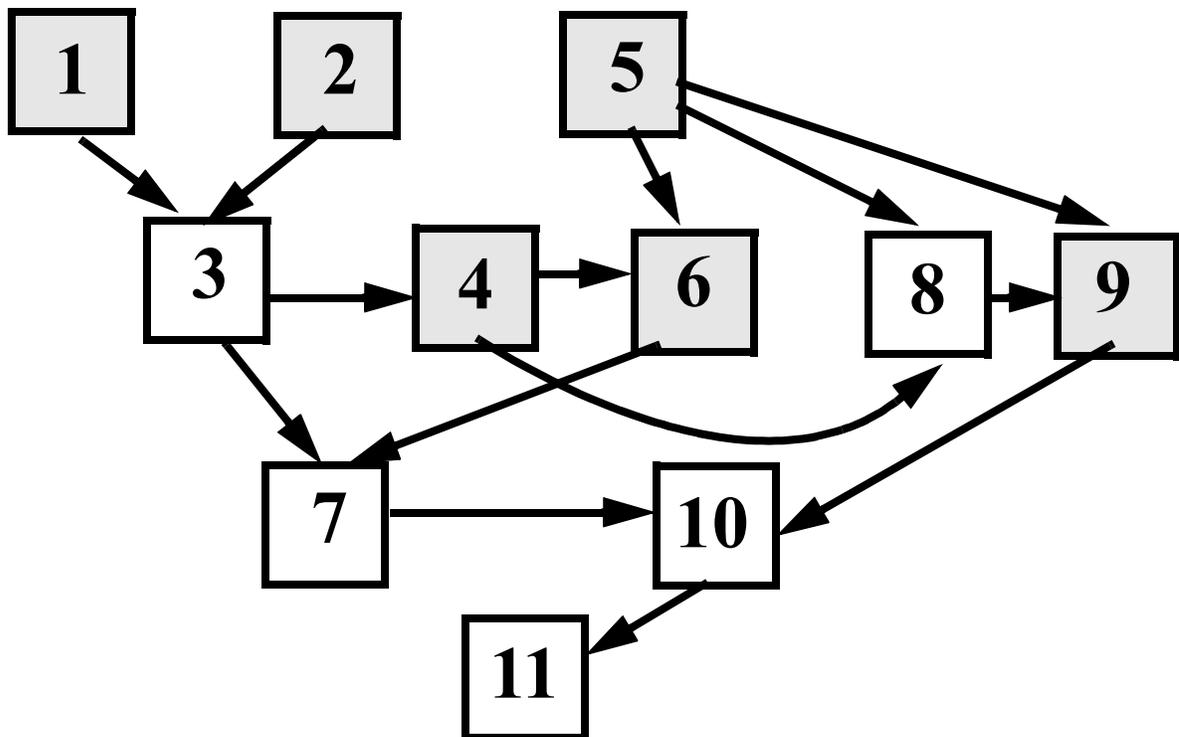
**To avoid stalls, some processors can execute instructions *Out of Program Order*.**

**If an instruction can't execute because a previous instruction it depends upon hasn't completed yet, the instruction can be "held" and a successor instruction executed instead.**

**When needed predecessors have completed, the held instruction is released for execution.**

# Example

1. ld [a], %r1
2. ld [b], %r2
5. ld [d], %r3
3. add %r1, %r2, %r1
4. ld [c], %r2 ← Stall
6. smul %r2, %r3, %r4
8. add %r2, %r3, %r2
9. smul %r2, %r3, %r2
7. add %r1, %r4, %r1 ← Stall
10. add %r1, %r2, %r1
11. st %r1, [a] (2 Stalls Total)



# Limitations of Dynamic Scheduling

- 1. Extra processor complexity.**
- 2. Register renaming (to avoid *False Dependencies*) may be needed.**
- 3. Identifying instructions to be delayed takes time.**
- 4. Instructions “late” in the program can’t be started earlier.**

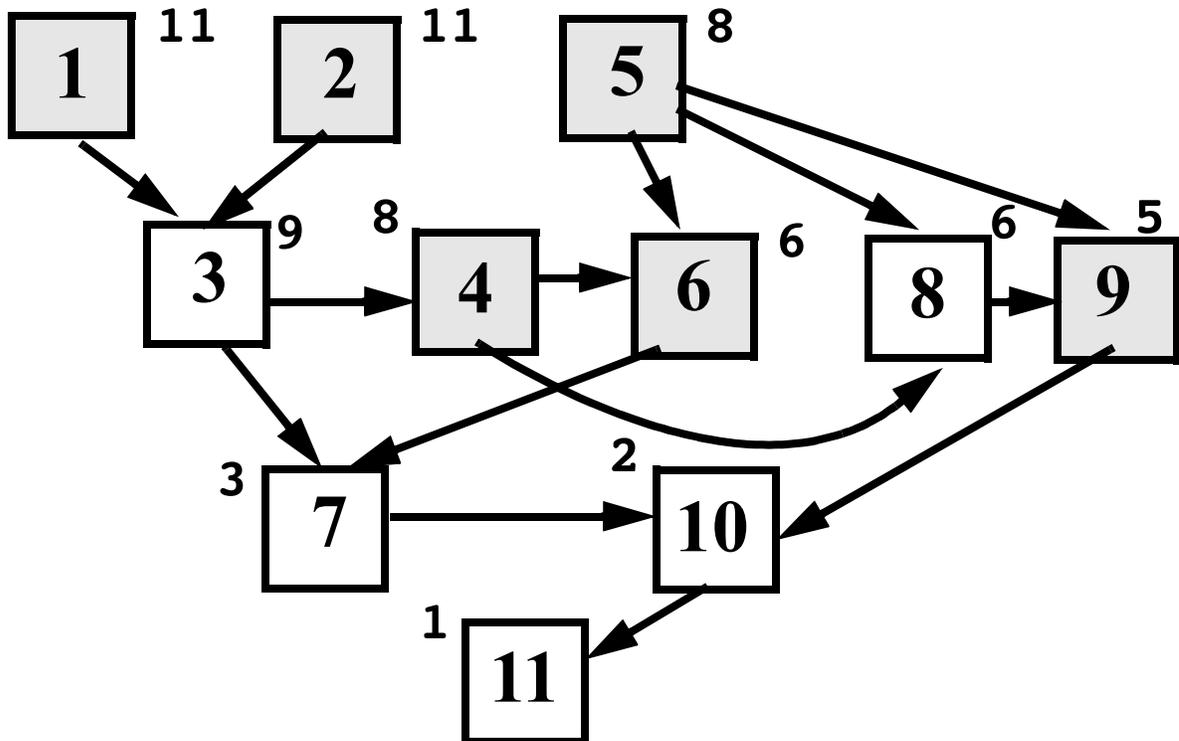
# **Gibbons & Muchnick Postpass Code Scheduler**

- 1. If there is only one root, schedule it.**
- 2. If there is more than one root, choose that root that won't be stalled by instructions already scheduled.**
- 3. If more than one root can be scheduled without stalling, consider the following rules (in order);**
  - (a) Does this root stall any of its successors?  
(If so, schedule it immediately.)**
  - (b) How many new roots are exposed if this node is scheduled?  
(More is better.)**

**(c) Which root has the longest weighted path to a leaf (using instruction delays as the weight). (The “critical path” in the DAG gets priority.)**

# Example

1. ld [a], %r1 //Longest path
2. ld [b], %r2 //Exposes a root
5. ld [d], %r3 //Not delayed
3. add %r1,%r2,%r1 //Only choice
4. ld [c], %r2 //Only choice
6. smul %r2,%r3,%r4 //Stalls succ.
8. add %r2,%r3,%r2 //Not delayed
9. smul %r2,%r3,%r2 //Not delayed
7. add %r1,%r4,%r1 //Only choice
10. add %r1,%r2,%r1 //Only choice
11. st %r1,[a] (2 Stalls Total)



# False Dependencies

**We still have delays in the schedule that was produced because of “false dependencies.”**

**Both `b` and `c` are loaded into `%r2`. This limits the ability to move the load of `c` prior to any use of `%r2` that uses `b`.**

**To improve our schedule we can use a processor that renames registers *or* allocate additional registers to remove false dependencies.**

# Register Renaming

**Many out of order processors automatically rename distinct uses of the same architectural register to distinct internal registers.**

**Thus**

```
ld [a],%r1
ld [b],%r2
add %r1,%r2,%r1
ld [c],%r2
```

**is executed as if it were**

```
ld [a],%r1
ld [b],%r2
add %r1,%r2,%r3
ld [c],%r4
```

**Now the final load can be executed prior to the add, eliminating a stall.**

# Compiler Renaming

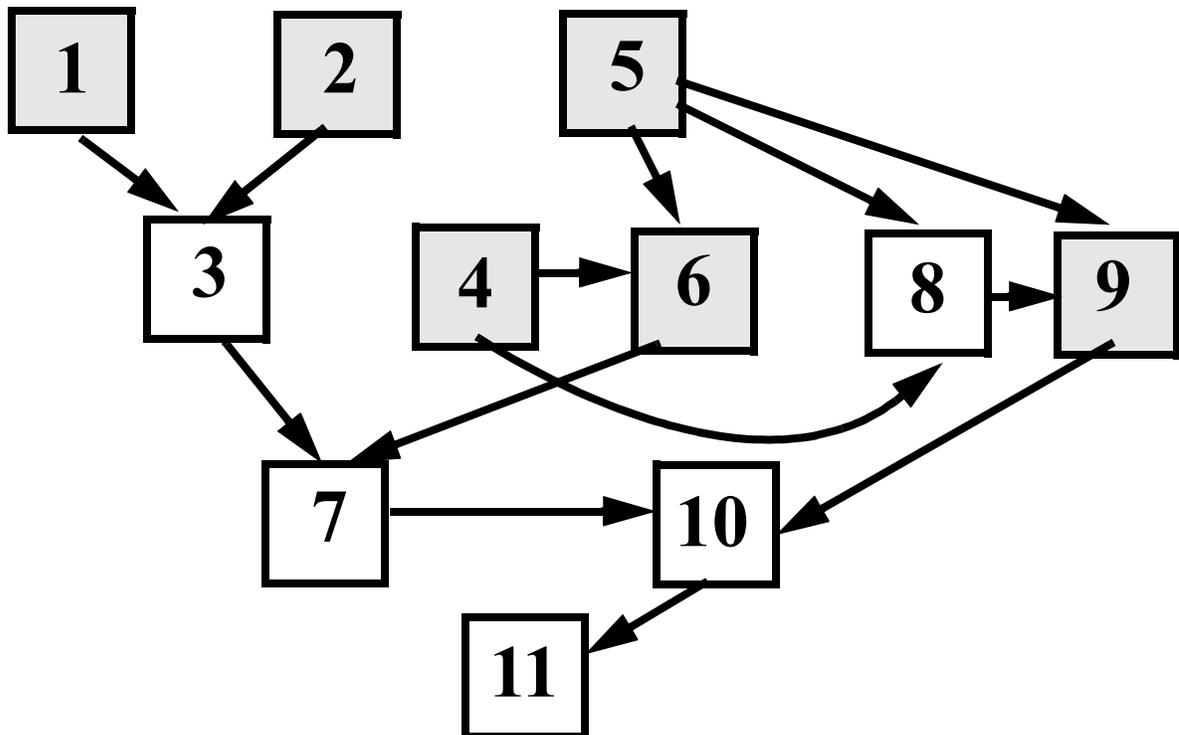
**A compiler can also use the idea of renaming to avoid unnecessary stalls.**

**An extra register may be needed (as was the case for scheduling expression trees).**

**Also, a *round-robin* allocation policy is needed. Registers are reused in a *cyclic* fashion, so that the most recently freed register is reused last, not first.**

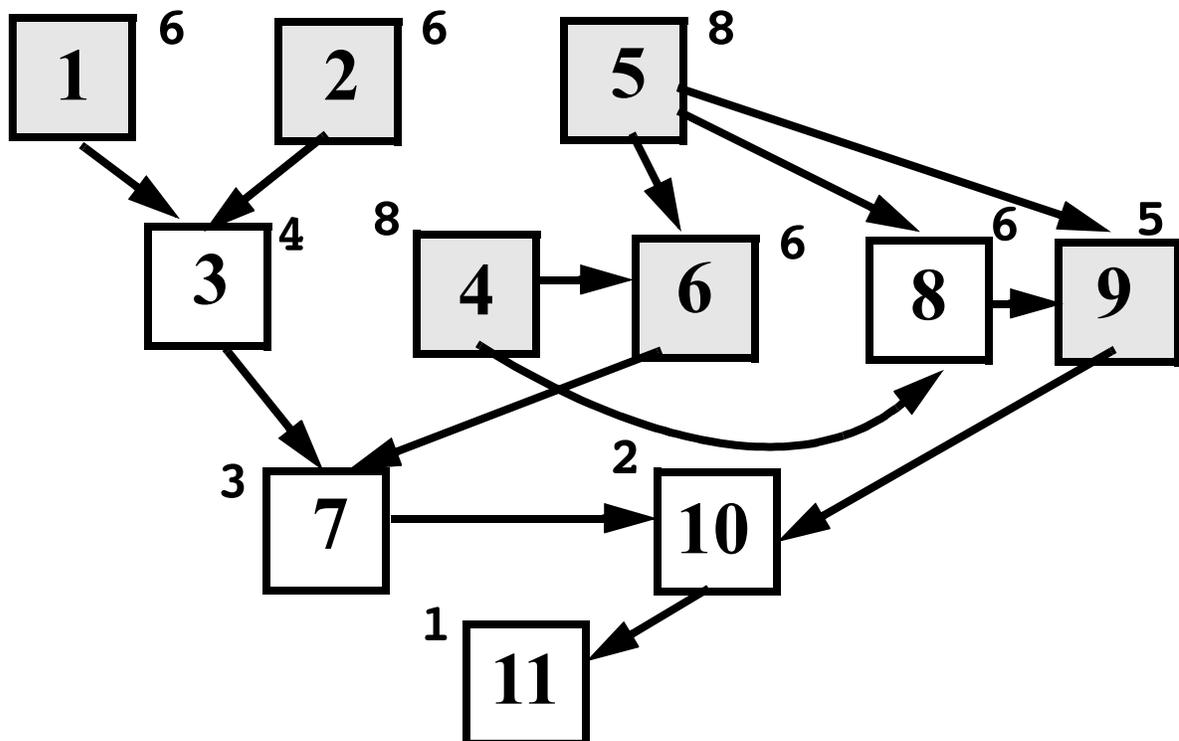
# Example

1. ld [a], %r1
2. ld [b], %r2 ← Stall
3. add %r1, %r2, %r1
4. ld [c], %r3
5. ld [d], %r4
6. smul %r3, %r4, %r5 ← Stall
7. add %r1, %r5, %r2 ← Stall\*2
8. add %r3, %r4, %r3
9. smul %r3, %r4, %r3
10. add %r2, %r3, %r2 ← Stall\*2
11. st %r2, [a] (6 Stalls Total)



# After Scheduling:

4. ld [c], %r3 //Longest path
5. ld [d], %r4 //Exposes a root
1. ld [a], %r1 //Stalls succ.
2. ld [b], %r2 //Exposes a root
6. smul %r3,%r4,%r5 //Stalls succ.
8. add %r3,%r4,%r3 //Longest path
9. smul %r3,%r4,%r3 //Stalls succ.
3. add %r1,%r2,%r1 //Only choice
7. add %r1,%r5,%r2 //Only choice
10. add %r2,%r3,%r2 //Only choice
11. st %r2,[a] (0 Stalls Total)



# Reading Assignment

- **Read Goodman and Hsu’s paper, “Code Scheduling and Register Allocation in Large Basic Blocks.”**
- **Read Bernstein and Rodeh’s paper, “Global Instruction Scheduling for Superscalar Machines.”**  
**(Linked from the class Web page.)**

# Balanced Scheduling

When scheduling a load, we normally anticipate the *best* case, a hit in the primary cache.

On older architectures this makes sense, since we stall execution on a cache miss.

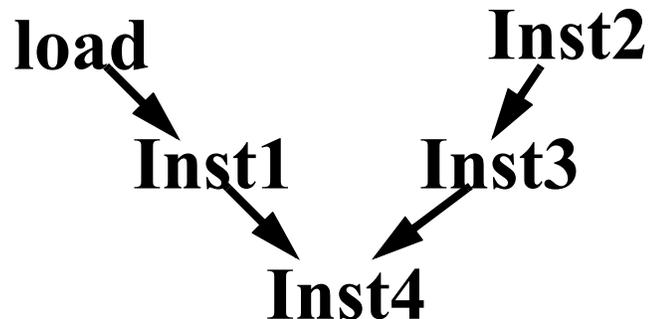
Many newer architectures are *non-blocking*. This means we can continue execution after a miss until the loaded value is used.

Assume a Cache miss takes  $N$  cycles ( $N$  can be from 3 to 100 or more).

Do we schedule a load anticipating a 1 cycle delay (a hit) or an  $N$  cycle delay (a miss)?

**Neither *Optimistic Scheduling* (expect a hit) nor *Pessimistic Scheduling* (expect a miss) is *always better*.**

**Consider**



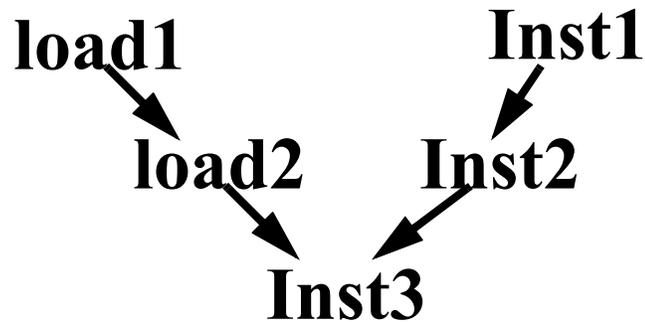
**An Optimistic Schedule is**

<b>load</b>	<b>Fine for a hit; inferior for a miss.</b>
<b>Inst2</b>	
<b>Inst1</b>	
<b>Inst3</b>	
<b>Inst4</b>	

**A Pessimistic Schedule is**

<b>load</b>	<b>Fine for a hit; better for a miss.</b>
<b>Inst2</b>	
<b>Inst3</b>	
<b>Inst1</b>	
<b>Inst4</b>	

**But things become more complex with multiple loads**



**An Optimistic Schedule is**

<b>load1</b>	<b>Better for hits; same for misses.</b>
<b>Inst1</b>	
<b>load2</b>	
<b>Inst2</b>	
<b>Inst3</b>	

**A Pessimistic Schedule is**

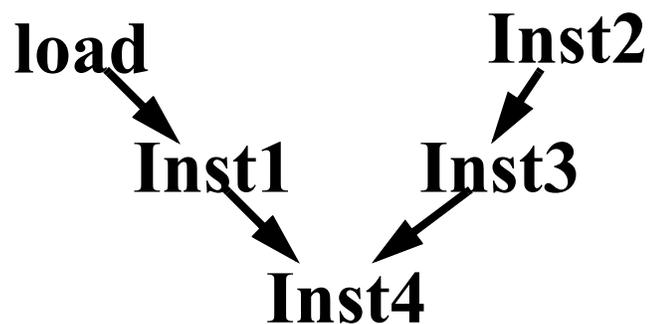
<b>load1</b>	<b>Worse for hits; same for misses.</b>
<b>Inst1</b>	
<b>Inst2</b>	
<b>load2</b>	
<b>Inst3</b>	

# Balance Placement of Loads

Eggers suggests a *balanced scheduler* that spaces out loads, using available independent instructions as “filler.”

The insight is that scheduling should not be driven by worst-case latencies but rather by available *Independent Instructions*.

For



it produces

load  
Inst2  
Inst3  
Inst1  
Inst4

Good; maximum distance between load and Inst1 in case of a miss.

**For**

**load1**

**Inst1**

**load2**

**Inst2**

**Inst3**

**balanced scheduling produces**

**load1**

**Inst1**

**load2**

**Inst2**

**Inst3**

**Good for hits;  
as good as  
possible for misses**

# Idea of the Algorithm

**Look at each Instruction,  $i$ , in the Dependency DAG.**

**Determine which loads can run in parallel with  $i$  and use all (or part) of  $i$ 's execution time to cover the latency of these loads.**

**Compute available latency of each load:**

**Give each load instruction an initial latency of 1.**

**For (each instruction  $i$  in the Dependency DAG) do:**

**Consider Instructions Independent of  $i$ :**

**$G_{\text{ind}} = \text{DepDAG} - (\text{AllPred}(i) \cup \text{AllSucc}(i) \cup \{i\})$**

**For (each connected subgraph  $c$  in  $G_{\text{ind}}$ ) do:**

**Find  $m =$  maximum number of load instructions on any path in  $c$ .**

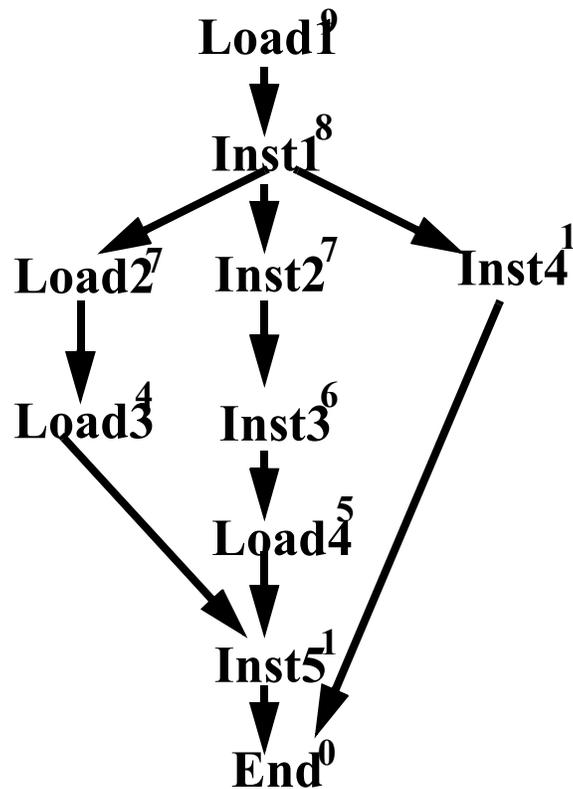
**For (each load  $d$  in  $c$ ) do:  
add  $1/m$  to  $d$ 's latency.**

# Computing the Schedule Using Adjusted Latencies

**Once latencies are assigned to each load (other instructions have a latency of 1), we annotate each instruction in the Dependency DAG with its critical path weight: the maximum latency (along any path) from the instruction to a Leaf of the DAG.**

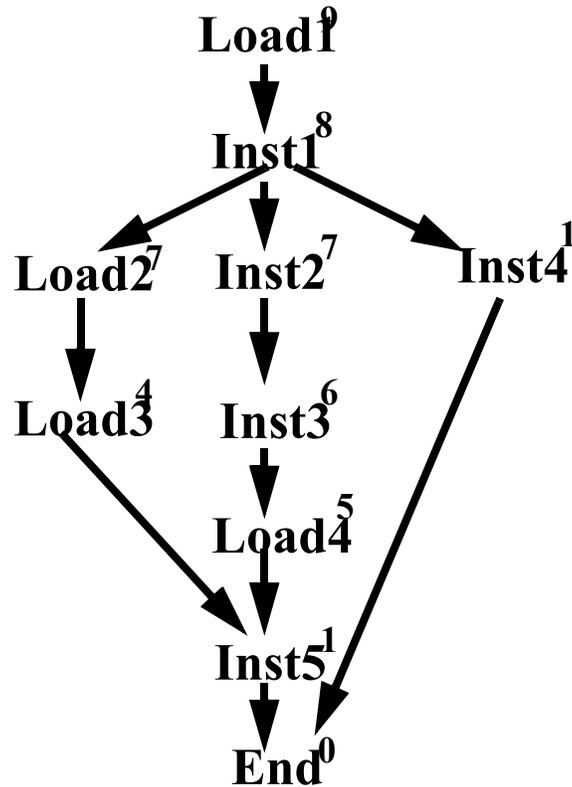
**Instructions are scheduled using critical path values; the root with the highest critical path value is always scheduled next. In cases of ties (same critical path value), operations with the longest latency are scheduled first.**

# Example



	Ld 1	Ld 2	Ld 3	Ld 4	I1	I2	I3	I4	I5	Latency
<b>Load1</b>										$1+0 = 1$
<b>Load2</b>				1/2	1/2	1/2	1/2			$1+2 = 3$
<b>Load3</b>				1/2	1/2	1/2	1/2			$1+2 = 3$
<b>Load4</b>		1	1					1		$1+3 = 4$

**Using the annotated Dependency Dag,  
instructions can be scheduled:**



<b>Load1</b>	<b>(0 latency; unavoidable)</b>
<b>Inst1</b>	
<b>Load2</b>	<b>(3 instruction latency)</b>
<b>Inst2</b>	
<b>Inst3</b>	
<b>Load4</b>	<b>(2 instruction latency)</b>
<b>Load3</b>	<b>(1 instruction latency)</b>
<b>Inst4</b>	
<b>Inst5</b>	

# **Goodman/Hsu Integrated Code Scheduler**

## **Prepass Schedulers:**

**Schedule code prior to register allocation.**

**Can overuse registers—Always using a “fresh” register maximizes freedom to rearrange Instructions.**

## **Postpass Schedulers:**

**Schedule code after register allocation.**

**Can be limited by “false dependencies” induced by register reuse.**

**Example is Gibbons/Muchnick heuristic.**

# Integrated Schedulers

**Capture best of both approaches.**

**When registers are plentiful, use additional registers to avoid stalls.**

**Goodman & Hsu call this *CSP*:  
Code Scheduling for Pipelines.**

**When registers are scarce, switch to a policy that frees registers.**

**Goodman & Hsu call this *CSR*:  
Code Scheduling to free Registers.**

**Assume code is generated in single assignment form, with a unique pseudo-register for each computed value.**

**We schedule from a DAG where nodes are operations (to be mapped to instructions), and arcs represent data dependencies.**

**Each node will have an associated Cost, that measures the execution and stall time of the instruction that the node represents.**

**Nodes are labeled with a critical path cost, used to select the “most critical” instructions to schedule.**

# Definitions

## **Leader Set:**

**Set of DAG nodes ready to be scheduled, possibly with an interlock.**

## **Ready Set:**

**Subset of Leader Set; Nodes ready to be scheduled without an interlock.**

## **AvailReg:**

**A count of currently unused registers.**

## **MinThreshold:**

**Threshold at which heuristic will switch from avoiding interlocks to reducing registers in use.**

## Goodman/Hsu Heuristic:

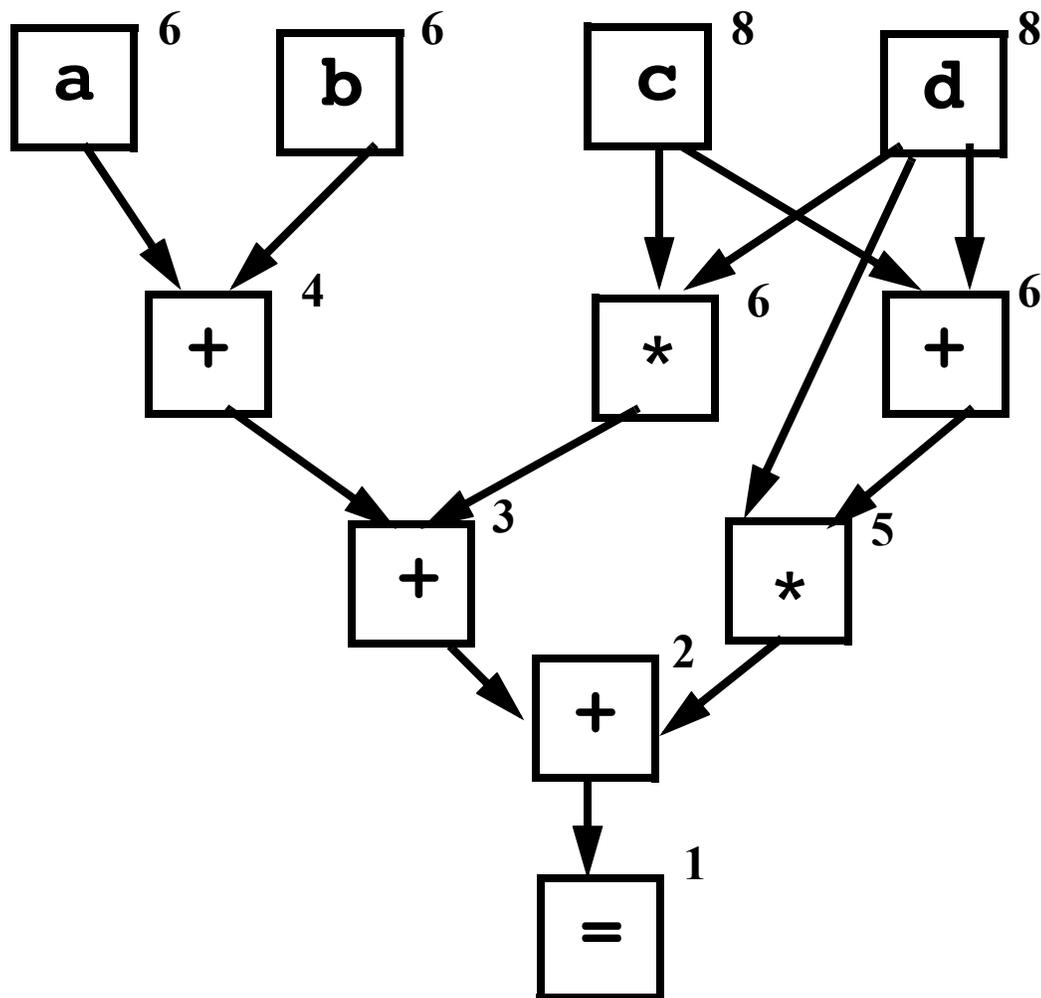
```
while (DAG  $\neq$   $\phi$ ) {  
  if ( AvailReg > MinThreshold)  
    if (ReadySet  $\neq$   $\phi$ )  
      Select Ready node with Max cost  
    else Select Leader node with Max cost  
  else // Reduce Registers in Use  
    if ( $\exists$  node  $\in$  ReadySet that frees registers){  
      Select node that frees most registers  
      If (selected node isn't unique)  
        Select node with Max cost  }  
    elseif ( $\exists$  node  $\in$  LeaderSet that frees regs){  
      Select node that frees most registers  
      If (selected node isn't unique)  
        Select node with fewest interlocks}  
    else find a partially evaluated path and  
      select a leader from this path  
    else Select any node in ReadySet  
    else Select any node in LeaderSet  
  Schedule Selected node  
  Update AvailReg count }//end while
```

# Example

We'll again consider

$$a = ((a+b) + (c*d)) + ((c+d) * d);$$

Again, assume a 1 cycle stall between a load and use of its value and a 2 cycle stall between a multiplication and first use of the product.



**We'll try 4 registers (the minimum), then 5 registers.**

**Should `MinThreshold` be 0 or 1?**

**At `MinThreshold = 1`, we always have a register to hold a result, but we may force a register to be spilled too soon!**

**At `MinThreshold = 0`, we may be forced to spill a register to free a result register.**

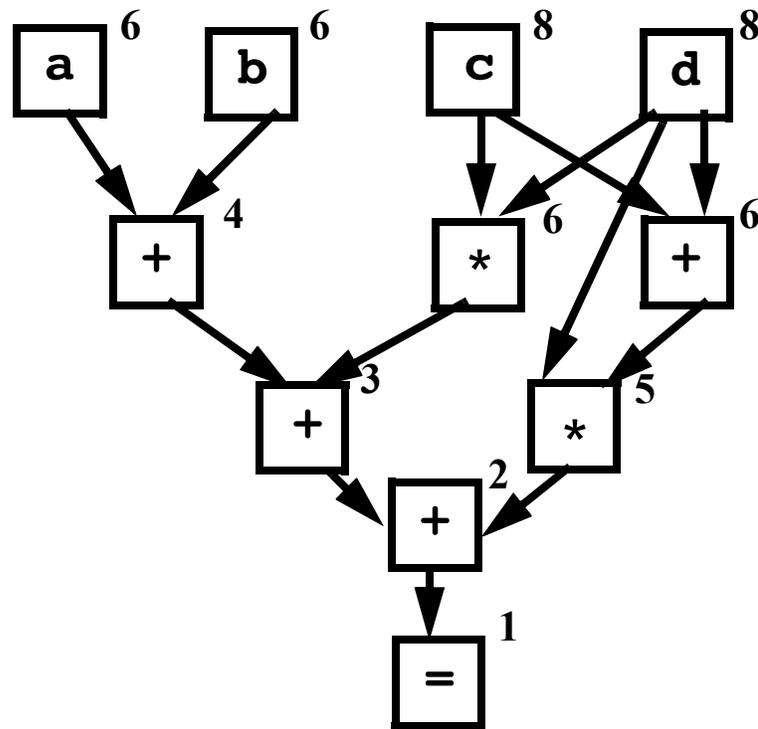
**But, we'll also be able to schedule more aggressively.**

**Is a spill or stall worse?**

**Note that we may be able to “hide” a spill in a delay slot!**

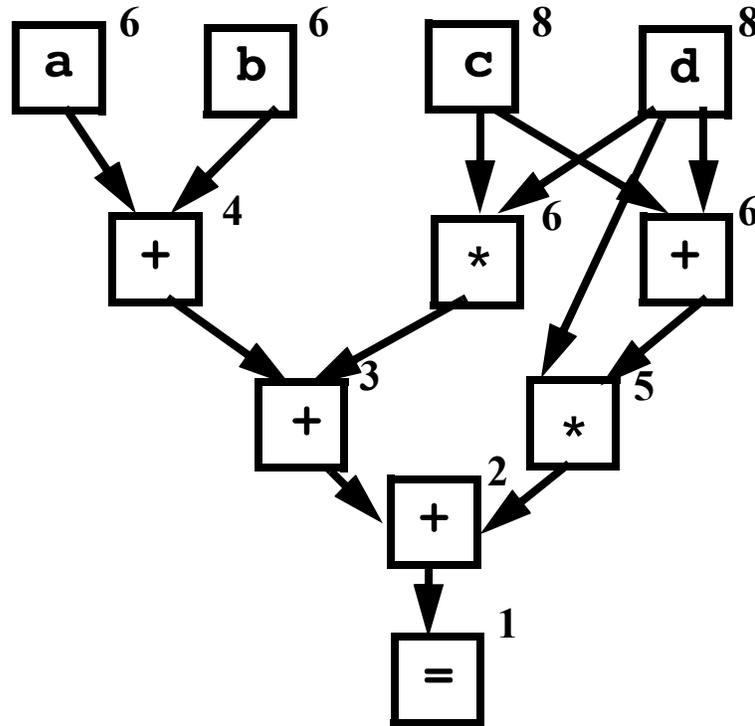
**We'll be aggressive and use `MinThreshold = 0`.**

# 4 Registers Used (1 Stall)



Instruction	Comment	Regs Used
<b>ld</b> [c], %r1	Choose ready, cost=8	1
<b>ld</b> [d], %r2	Choose ready, cost=8	2
<b>ld</b> [a], %r3	Choose ready, cost=6	3
<b>smul</b> %r1,%r2,%r4	Choose ready, cost=6	4
<b>add</b> %r1,%r2,%r1	Free a register	4
<b>smul</b> %r1,%r2,%r1	Free a register	3
<b>ld</b> [b], %r2	Choose ready, cost=6	4
<b>add</b> %r3,%r2,%r3 ←	Choose a leader	3
<b>add</b> %r3,%r4,%r3	No choice	2
<b>add</b> %r3,%r1,%r3	No choice	1
<b>st</b> %r3,[a]	No choice	0

# 5 Registers Used (No Stalls)



Instruction	Comment	Regs Used
<b>ld</b> [c], %r1	Choose ready, cost=8	1
<b>ld</b> [d], %r2	Choose ready, cost=8	2
<b>ld</b> [a], %r3	Choose ready, cost=6	3
<b>smul</b> %r1,%r2,%r4	Choose ready, cost=6	4
<b>add</b> %r1,%r2,%r1	Choose ready, cost=6	4
<b>ld</b> [b], %r5	Choose ready, cost=6	5
<b>smul</b> %r1,%r2,%r1	Free a register	4
<b>add</b> %r3,%r5,%r3	Choose ready, cost=4	3
<b>add</b> %r3,%r4,%r3	No choice	2
<b>add</b> %r3,%r1,%r3	No choice	1
<b>st</b> %r3,[a]	No choice	0

# Scheduling for Superscalar & Multiple Issue Machines

**A number of computers have the ability to issue more than one instruction per cycle *if* the instructions are independent and satisfy constraints on available functional units.**

**Thus the instructions**

```
add %r1, 1, %r2
```

```
sub %r1, 2, %r3
```

**can issue and execute in parallel, but**

```
add %r1, 1, %r2
```

```
sub %r2, 2, %r3
```

**must execute sequentially.**

**Instructions that are linked by true or output dependencies must execute sequentially, but instructions that are linked by an anti dependence may execute concurrently.**

**For example,**

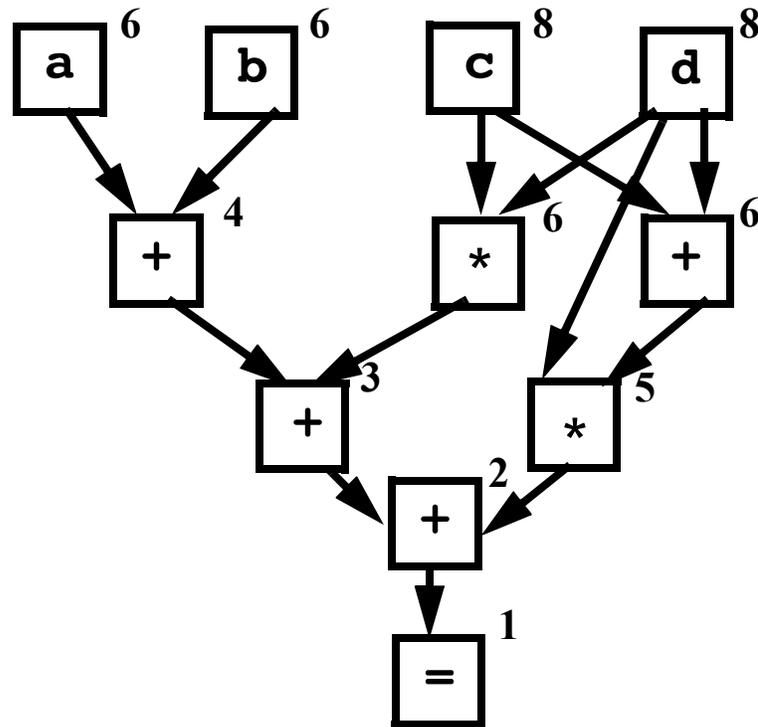
```
add %r1, 1, %r2  
sub %r3, 2, %r1
```

**can issue and execute in parallel.**

**The code scheduling techniques we've studied can be used to schedule machines that can issue 2 or more independent instructions simultaneously.**

**We select pairs (or triples or n-tuples), verifying (with the Dependence Dag) that they are independent or anti dependent.**

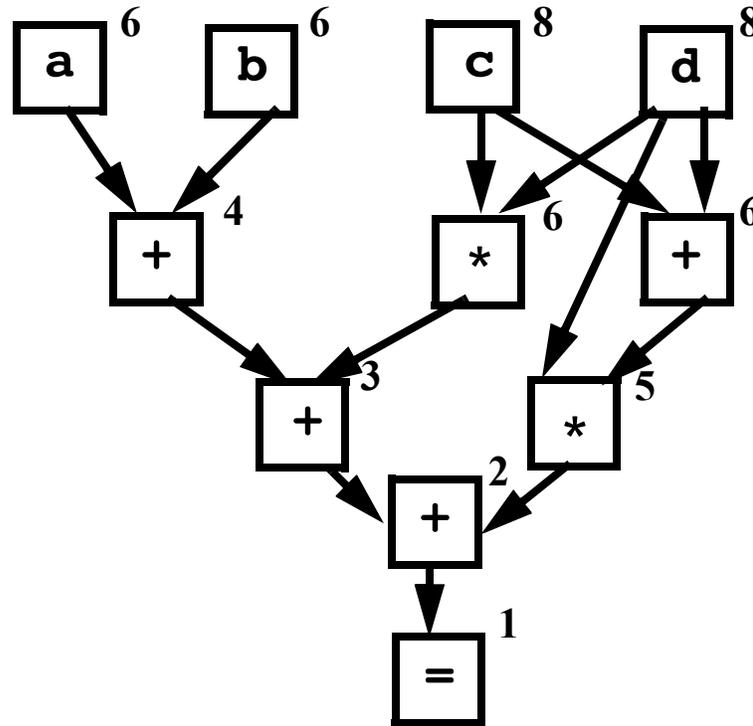
# Example: 5 Registers (2 Wide Issue)



1	ld [c], %r1	ld [d], %r2
2	ld [a], %r3	ld [b], %r4
3	smul %r1,%r2,%r5	add %r1,%r2,%r1
4	add %r3,%r4,%r3	smul %r1,%r2,%r1
5	nop	nop
6	add %r3,%r5,%r3	nop
7	add %r3,%r1,%r3	nop
8	st %r3,[a]	nop

**We need only 8 cycles rather than 11.**

# 5 Registers (3 Wide Issue)



1	ld [c], %r1	ld [d], %r2	ld [a], %r3
2	ld [b], %r4	nop	nop
3	smul %r1, %r2, %r5	add %r1, %r2, %r1	nop
4	add %r3, %r4, %r3	smul %r1, %r2, %r1	nop
5	nop	nop	nop
6	add %r3, %r5, %r3	nop	nop
7	add %r3, %r1, %r3	nop	nop
8	st %r3, [a]	nop	nop

**We still need 8 cycles!**

# **Finding Additional Independent Instructions for Parallel Issue**

**We can extend the capabilities of processors:**

- **Out of order execution allows a processor to “search ahead” for independent instructions to launch.**
- *But*, **since basic blocks are often quite small, the processor may need to accurately predict branches, issuing instructions before the execution path is fully resolved.**
- *But*, **since branch predictions may be wrong, it will be necessary to “undo” instructions executed speculatively.**

# Reading Assignment

- **Read pp 367-386 of Allan et. al.'s paper, "Software Pipelining."  
(Linked from the class Web page.)**

# Compiler Support for Extended Scheduling

- **Trace Scheduling**

**Gather sequences of basic blocks together and schedule them as a unit.**

- **Global Scheduling**

**Analyze the control flow graph and move instructions across basic block boundaries to improve scheduling.**

- **Software Pipelining**

**Select instructions from several loop iterations and schedule them together.**

# Trace Scheduling

## Reference:

**J. Fisher, “Trace Scheduling: A Technique for Global Microcode Compaction,” IEEE Transactions on Computers, July 1981.**

## Idea:

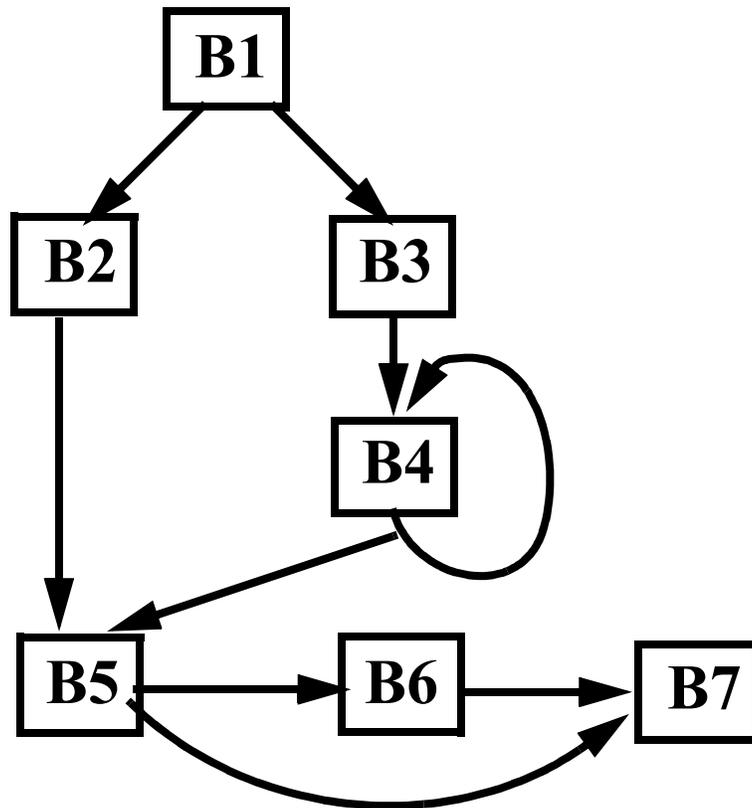
**Since basic blocks are often too small to allow effective code scheduling, we will *profile* a program’s execution and identify the most frequently executed paths in a program.**

**Sequences of contiguous basic blocks on frequently executed paths will be gathered together into *traces*.**

# Trace

- **A sequence of basic blocks (excluding loops) executed together can form a trace.**
- **A trace will be scheduled as a unit, allowing a larger span of instructions for scheduling.**
- **A loop can be unrolled or scheduled individually.**
- ***Compensation code* may need to be added when a branch into, or out of, a trace occurs.**

# Example



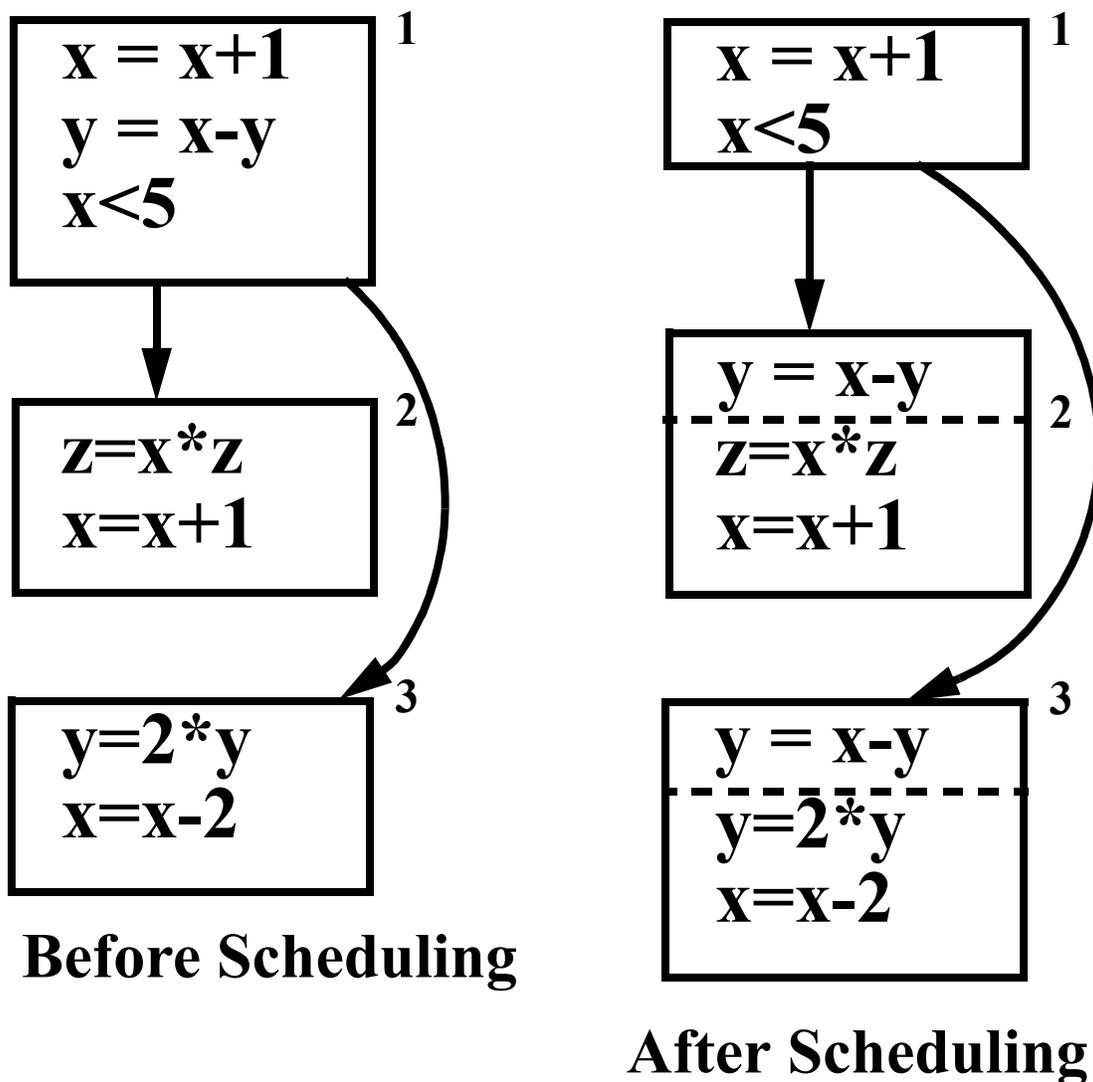
Assume profiling shows that  $B1 \rightarrow B3 \rightarrow B4^+ \rightarrow B5 \rightarrow B7$  is the most common execution path. The traces extracted from this path are  $B1 \rightarrow B3$ ,  $B4$ , and  $B5 \rightarrow B7$ .

# Compensation Code

**When we move instructions across basic block boundaries within a trace, we may need to add extra instructions that preserve program semantics on paths that enter or leave the trace.**

# Example

In the previous example, basic block B1 had B2 and B3 as successors, and B1→B3 formed a trace.



# Advantages & Disadvantages

- **Trace scheduling allows scheduling to span multiple basic blocks. This can significantly increase the effectiveness of scheduling, especially in the context of superscalar processors (which need ILP to be effective).**
- **Trace Scheduling can also increase code size (because of compensation code).**  
**It is also sensitive to the accuracy of trace estimates.**

# Global Code Scheduling

- **Bernstein and Rodeh approach.**
- **A *prepass scheduler***  
**(does scheduling before register allocation).**
- **Can move instructions across basic block boundaries.**
- **Prefers to move instructions that *must* eventually be executed.**
- **Can move Instructions *speculatively*, possibly executing instructions unnecessarily.**

# Data & Control Dependencies

**When moving instructions across basic block boundaries, we must respect both data dependencies and control dependencies.**

*Data dependencies specify necessary orderings among instructions that produce a value and instructions that use that value.*

*Control dependencies determine when (and if) various instructions are executed. Thus an instruction is control dependent on expressions that affect flow of control to that instruction.*

# Definitions used in Global Scheduling

- **Basic Block A dominates Basic Block B** if and only if A appears on *all* paths to B.
- **Basic Block B postdominates Basic Block A** if and only if B appears on *all* paths from A to an exit point.
- **Basic Blocks A and B are equivalent** if and only if A dominates B and B postdominates A.
- **Moving an Instruction from Basic Block B to Basic Block A is useful** if and only if A and B are equivalent.
- **Moving an Instruction from Basic Block B to Basic Block A is speculative** if B does not postdominate A.

- **Moving an Instruction from Basic Block B to Basic Block A requires *duplication* if A does not dominate B.**

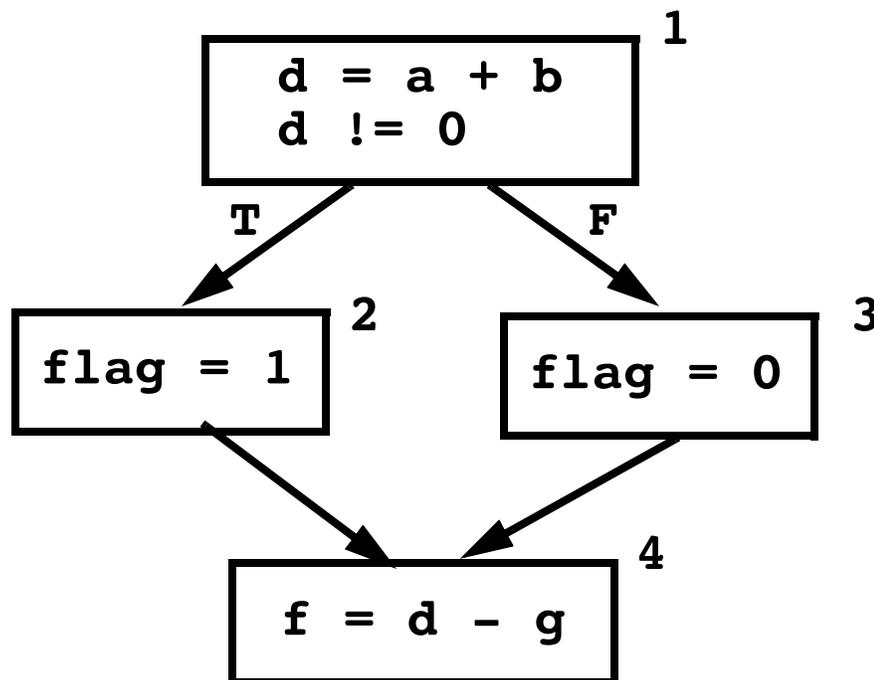
**We prefer a move that does not require duplication. (Why?)**

**The degree of speculation in moving an instruction from one basic block to another can be quantified:**

- **Moving an Instruction from Basic Block B to Basic Block A is *n-branch* speculative if n conditional branches occur on a path from A to B.**

# Example

```
d = a + b;  
if ( d != 0 )  
    flag = 1;  
else flag = 0;  
f = d - g;
```



**Blocks 1 and 4 are equivalent.**

**Moving an Instruction from B2 to B1  
(or B3 to B1) is 1-branch speculative.**

**Moving an Instruction from B4 to B2  
(or B4 to B3) requires duplication.**

# Limits on Code Motion

**Assume that pseudo registers are used in generated code (prior to register allocation).**

**To respect data dependencies:**

- A use of a Pseudo Register can't be moved above its definition.**
- Memory loads can't be moved ahead of Stores to the same location.**
- Stores can't be moved ahead of either loads or stores to the same location.**
- A load of a memory location *can* be moved ahead of another load of the same location (such a load may often be optimized away by equivalencing the two pseudo registers).**

# Example (Revisited)

```
block1:
    ld    [a],Pr1
    ld    [b],Pr2
    add   Pr1,Pr2,Pr3    ← Stall
    st    Pr3,[d]
    cmp   Pr3,0
    be    block3

block2:
    mov   1,Pr4
    st    Pr4,[flag]
    b     block4

block3:
    st    0,[flag]

block4:
    ld    [d],Pr5
    ld    [g],Pr6
    sub   Pr5,Pr6,Pr7    ← Stall
    st    Pr7,[f]
```

**In B1 and B4, the number of available registers is *irrelevant* in avoiding stalls. There are too few independent instructions in each block.**

# **Global Scheduling Restrictions (in Bernstein/ Rodeh Heuristic)**

- 1. Subprograms are divided into *Regions*. A region is a loop body or the subprogram body without enclosed loops.**
- 2. Regions are scheduled inside-out.**
- 3. Instructions never cross region boundaries.**
- 4. All instructions move “upward” (to earlier positions in the instruction order).**
- 5. The original order of branches is preserved.**

## **Lesser (temporary) restrictions**

**Include:**

**6. No code duplication.**

**7. Only 1-branch speculation.**

**8. No new basic blocks are created or added.**

## **Scheduling Basic Blocks in a CFG**

**Basic blocks are visited and scheduled in *Topological Order*. Thus all of a block's predecessors are scheduled before it is.**

**Two levels of scheduling are possible (depending on whether speculative execution is allowed or not):**

**1. When Basic Block A is scheduled, only Instructions in A and blocks equivalent to A that A dominates are considered.**

**(Only “useful” instructions are considered.)**

**2. Blocks that are immediate successors of those considered in (1) are also considered.**

**(This allows 1-branch speculation.)**

# Candidate Instructions

**We first compute the set of basic blocks that may contribute instructions when block A is scheduled. (Either blocks equivalent to A or blocks at most 1-branch speculative.)**

**An individual Instruction, Inst, in this set of basic blocks may be scheduled in A if:**

- 1. It is located in A.**
- 2. It is in a block equivalent to A and may be moved across block boundaries.  
(Some instructions, like calls, can't be moved.)**
- 3. It is not in a block equivalent to A, but may be scheduled speculatively.  
(Some instructions, like stores, can't be executed speculatively.)**

# Selecting Instructions to Issue

- **A list of “ready to issue” instructions in block A and in blocks equivalent to A (or 1-branch distant from A) is maintained.**
- **All data dependencies must be satisfied and stalls avoided (if possible).**
- **N independent instructions are selected, where N is the processor’s issue-width.**
- **But what if more than N instructions are ready to issue?**
- **Selection is by *Priority*, using two *Scheduling Heuristics*.**

# Delay Heuristic

**This value is computed on a per-basic block basis.**

**It estimates the worst-case delay (stalls) from an Instruction to the end of the basic block.**

**$D(I) = 0$  if  $I$  is a leaf.**

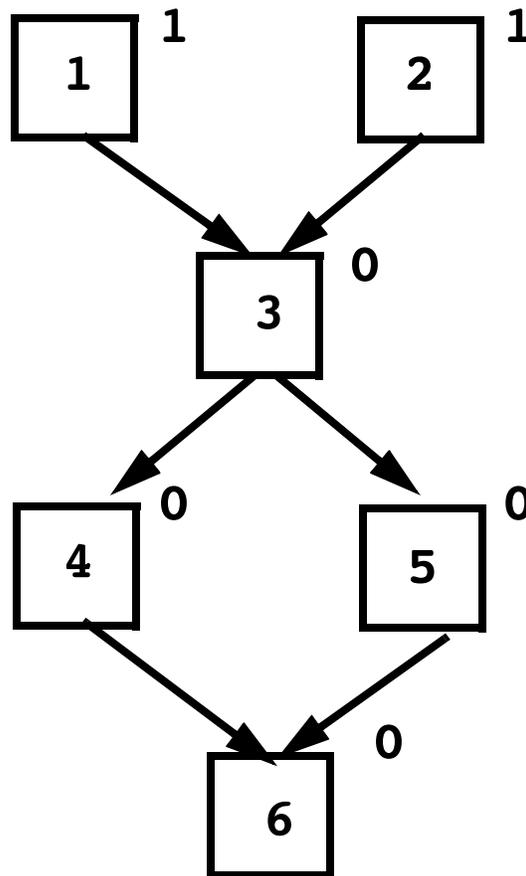
**Let  $d(I,J)$  be the delay if instruction  $J$  follows instruction  $I$  in the code schedule.**

$$D(I) = \text{Max} (D(J_i) + d(I, J_i))$$
$$J_i \in \text{Succ}(I)$$

# Example of Delay Values

**block1:**

1. ld [a], Pr1
2. ld [b], Pr2
3. add Pr1, Pr2, Pr3
4. st Pr3, [d]
5. cmp Pr3, 0
6. be block3



**(Assume only loads can stall.)**

# Critical Path Heuristic

**This value is also computed on a per-basic block basis.**

**It estimates how long it will take to execute Instruction I, and all I's successors, assuming unlimited parallelism.**

**$E(I)$  = Execution time for instruction I**

**(normally 1 for pipelined machines)**

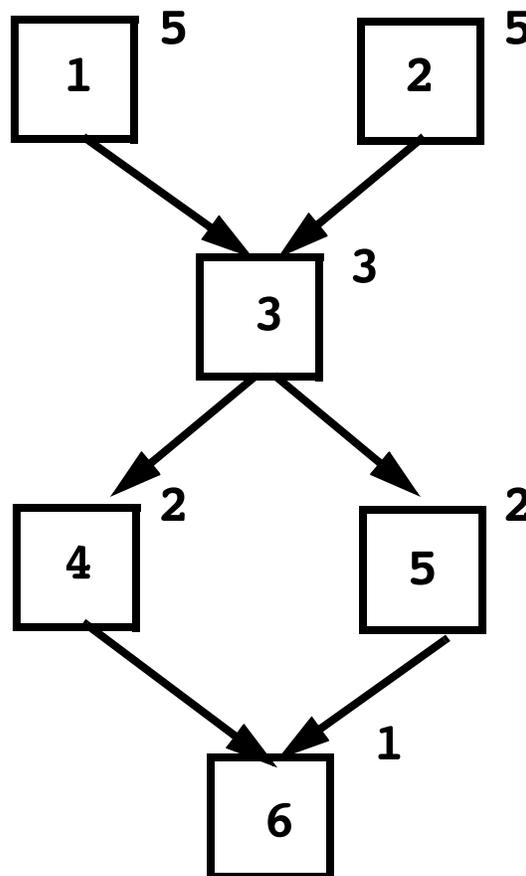
**$CP(I) = E(I)$  if I is a leaf.**

$$CP(I) = E(I) + \text{Max}_{J_i \in \text{Succ}(I)} (CP(J_i) + d(I, J_i))$$

# Example of Critical Path Values

**block1:**

1. **ld** [a],Pr1
2. **ld** [b],Pr2
3. **add** Pr1,Pr2,Pr3
4. **st** Pr3,[d]
5. **cmp** Pr3,0
6. **be** block3



# Selecting Instructions to Issue

**From the Ready Set (instructions with all dependencies satisfied, and which will not stall) use the following priority rules:**

- 1. Instructions in block A and blocks equivalent to A have priority over other (speculative) blocks.**
- 2. Instructions with the highest D values have priority.**
- 3. Instructions with the highest CP values have priority.**

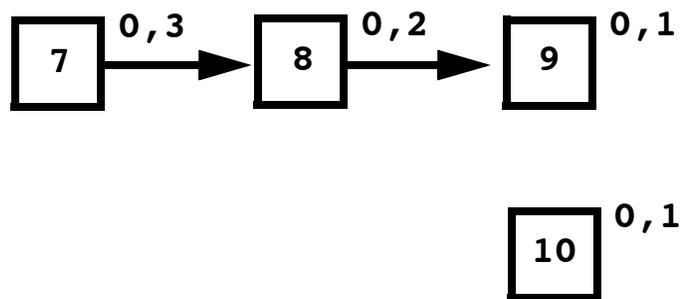
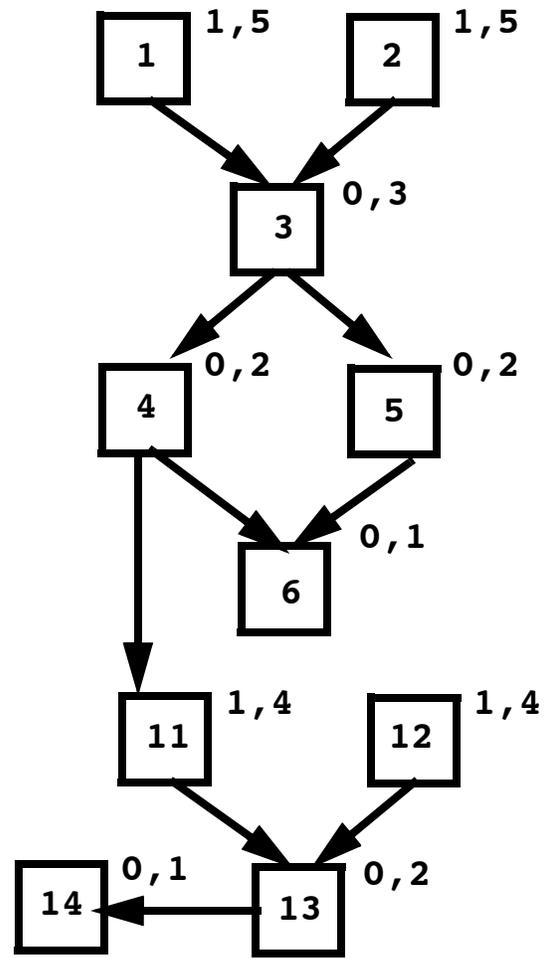
**These rules imply that we schedule useful instructions before speculative ones, instructions on paths with potentially many stalls over those with fewer stalls, and instructions on critical paths over those on non-critical paths.**

# Example

```

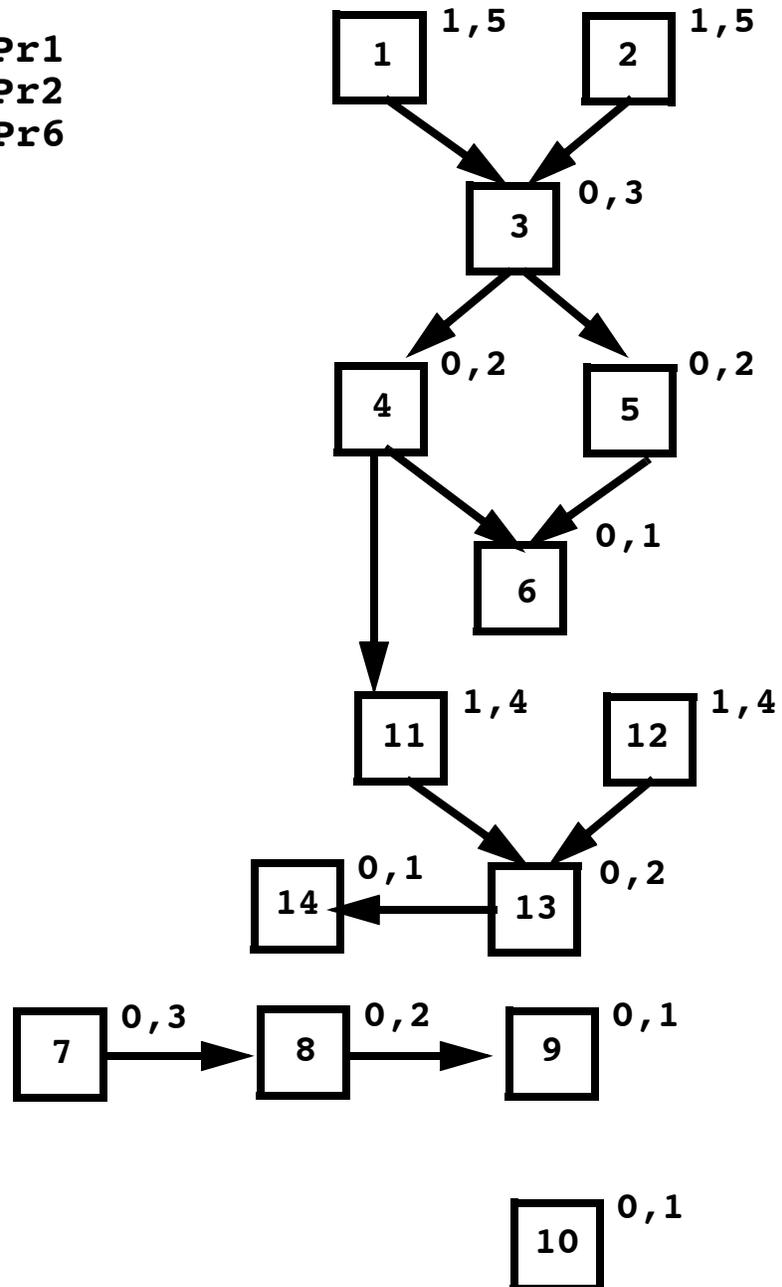
block1:
1.  ld  [a],Pr1
2.  ld  [b],Pr2
3.  add Pr1,Pr2,Pr3
4.  st  Pr3,[d]
5.  cmp Pr3,0
6.  be  block3
block2:
7.  mov 1,Pr4
8.  st  Pr4,[flag]
9.  b   block4
block3:
10. st  0,[flag]
block4:
11. ld  [d],Pr5
12. ld  [g],Pr6
13. sub Pr5,Pr6,Pr7
14. st  Pr7,[f]

```



**We'll schedule without speculation;  
highest D values first, then highest CP  
values.**

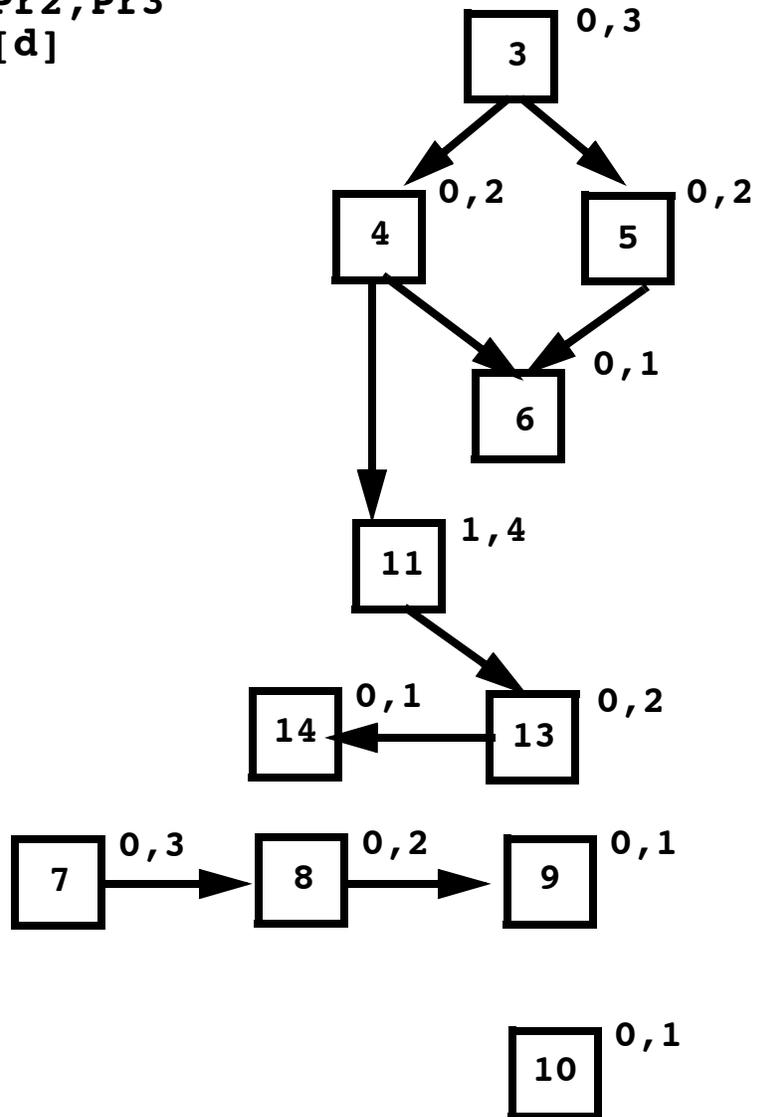
**block1:**  
 1. ld [a],Pr1  
 2. ld [b],Pr2  
 12. ld [g],Pr6



# Next, come Instructions 3 and 4.

**block1:**

- 1. ld [a],Pr1
- 2. ld [b],Pr2
- 12. ld [g],Pr6
- 3. add Pr1,Pr2,Pr3
- 4. st Pr3,[d]

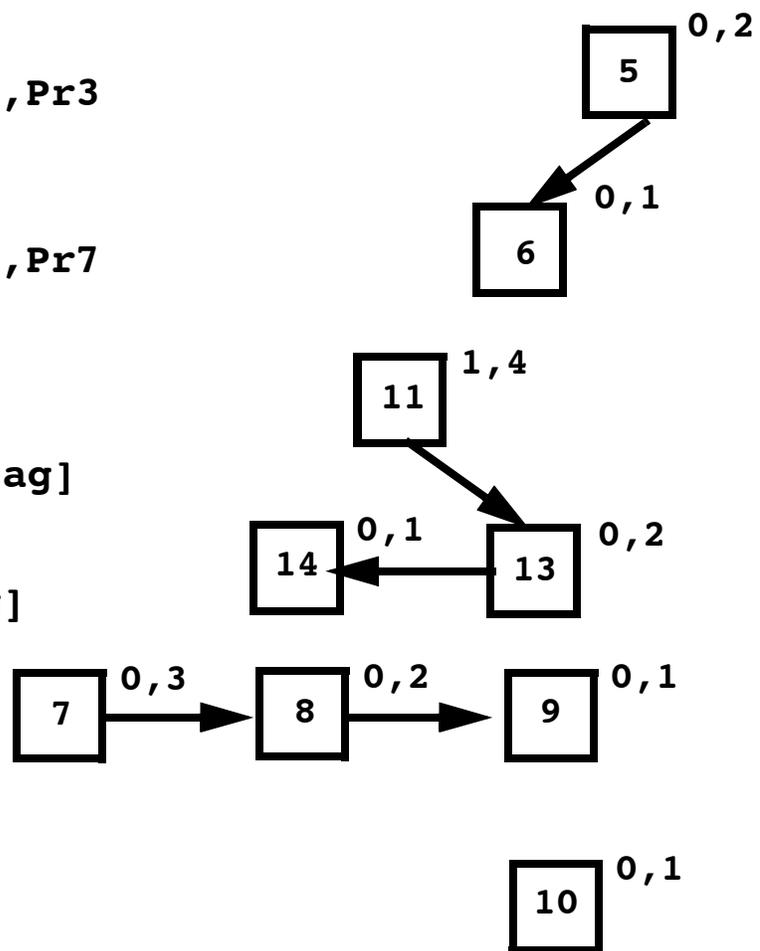


Now 11 can issue (D=1), followed by 5, 13, 6 and 14. Block B4 is now empty, so B2 and B3 are scheduled.

```

block1:
1.  ld  [a],Pr1
2.  ld  [b],Pr2
12. ld  [g],Pr6
3.  add Pr1,Pr2,Pr3
4.  st  Pr3,[d]
11. ld  [d],Pr5
5.  cmp Pr3,0
13. sub Pr5,Pr6,Pr7
6.  be  block3
14. st  Pr7,[f]
block2:
7.  mov 1,Pr4
8.  st  Pr4,[flag]
9.  b   block4
block3:
10. st 0,[flag]
block4:

```



There are no stalls. In fact, if we equivalence Pr3 and Pr5, Instruction 11 can be removed.

# Hardware Support for Global Code Motion

We want to be aggressive in scheduling loads, which incur high latencies when a cache miss occurs.

In many cases, control and data dependencies may force us to restrict how far we may move a critical load.

Consider

```
p = Lookup(Id);  
...  
if (p != null)  
    print(p.a);
```

It may well be that the object returned by `Lookup` is not in the L1 cache. Thus we'd like to schedule the load generated by `p.a` as soon as possible; ideally right after the lookup.

But moving the load above the `p != null` check is clearly unsafe.

**A number of modern machine architectures, including Intel's Itanium, have proposed a *speculative load* to allow freer code motion when scheduling.**

**A speculative load,**

**`ld.s [adr], %reg`**

**acts like an ordinary load as long as the load does not force an interrupt. If it does, the interrupt is suppressed and a special NaT (not a thing) bit is set in the register (a hidden 65th bit). A NaT bit can be propagated through instructions before being tested.**

**In some cases (like our table lookup example), a register containing a NaT bit may simply not be used because control doesn't reach its intended uses.**

**However a NaT bit need not indicate an outright error. A load may force a TLB (translation lookaside buffer) fault or a**

**page fault. These interrupts are probably too costly to do speculatively, but if we decide the loaded value is really needed, we will want to allow them.**

**A special check instruction, of the form,**

**`chk.s %reg, adr`**

**checks whether `%reg` has its `NaT` bit set. If it does, control passes to `adr`, where user-supplied “fixup” code is placed. This code can redo the load non-speculatively, allowing necessary interrupts to occur.**

# Hardware Support for Data Speculation

**In addition to supporting control speculation (moving instructions above conditional branches), it is useful to have hardware support for data speculation.**

**In data speculation, we may move a load above a store if we believe the chance of the load and store conflicting is slim.**

**Consider a variant of our earlier lookup example,**

```
p = Lookup(Id);  
...  
q.a = init();  
print(p.a);
```

**We'd like to move the load implied by `p.a` above the assignment to `q.a`. This allows `p` to miss in the L1 cache, using the execution of `init()` to cover the miss latency.**

*But*, we need to be sure that `q` and `p` don't reference the same object and that `init()` doesn't indirectly change `p.a`. Both possibilities may be remote, but proving non-interference may be difficult.

**The Intel Itanium provides a special “advanced load” that supports this sort of load motion.**

**The instruction**

**`ld.a [adr], %reg`**

**loads the contents of memory location `adr` into `%reg`. It also stores `adr` into**

**special *ALAT* (Advanced Load Address Table) hardware.**

**When a store to address *x* occurs, an *ALAT* entry corresponding to address *x* is removed (if one exists).**

**When we wish to use the contents of *%reg*, we execute a**

**`ld.c [adr], %reg`**

**instruction (a *checked* load).**

**If an *ALAT* entry for *adr* is present, this instruction does nothing; *%reg* contains the correct value. If there is no corresponding *ALAT* entry, the `ld.c` simply acts like an ordinary load.**

**(Two versions of `ld.c` exist; one preserves an *ALAT* entry while the other purges it).**

**And yes, a speculative load (ld.s) and an advanced load (ld.a) may be combined to form a speculative advanced load (ld.sa).**

# **Speculative Multi-threaded Processors**

**The problem of moving a load above a store that may conflict with it also appears in multi-threaded processors.**

**How do we know that two threads don't interfere with one another by writing into locations both use?**

**Proofs of non-interference can be difficult or impossible. Rather than severely restrict what independent threads can do, researchers have proposed *speculative* multi-threaded processors.**

**In such processors, one thread is primary, while all other threads are secondary and speculative. Using hardware tables to remember locations read and written, a secondary thread**

**can commit (make its updates permanent) only if it hasn't read locations the primary thread later wrote and hasn't written locations the primary thread read or wrote. Access conflicts are automatically detected, and secondary threads are automatically restarted as necessary to preserve the illusion of serial memory accesses.**

# Software Pipelining

Often loop bodies are too small to allow effective code scheduling. But loop bodies, being “hot spots,” are exactly where scheduling is most important.

Consider

```
void f (int a[],int last) {
    for (p=&a[0];p!=&a[last];p++)
        (*p)++;
}
```

The body of the loop might be:

```
L: ld    [%g3],%g2
    nop
    add  %g2,1,%g2
    st  %g2,[%g3]
    add  %g3,4,%g3
    cmp  %g3,%g4
    bne  L
    nop
```

**Scheduling this loop body in isolation is ineffective—each instruction depends upon its immediate predecessor.**

**So we have a loop body that takes 8 cycles to execute 6 “core” instructions.**

**We could unroll the loop body, but for how many iterations? What if the loop ends in the “middle” of an expanded loop body? Will extra registers be a problem?**

**In this case *software pipelining* offers a nice solution. We expand the loop body *symbolically*, intermixing instructions from several iterations. Instructions can overlap, increasing parallelism and forming a “tighter” loop body:**

```
    ld    [%g3], %g2
    nop
    add   %g2, 1, %g2
L:   st   %g2, [%g3]
    add   %g3, 4, %g3
    ld    [%g3], %g2
    cmp   %g3, %g4
    bne   L
    add   %g2, 1, %g2
```

**Now the loop body is ideal—exactly 6 instructions. Also, no extra registers are needed!**

***But*, we do “overshoot” the end of the loop a bit, loading one element past the exit point. (How serious is this?)**

# Key Insight of Software Pipelining

Software pipelining exploits the fact that a loop of the form  $\{A\ B\ C\}^n$ , where **A**, **B** and **C** are individual instructions, and  $n$  is the iteration count, is equivalent to  $A\ \{B\ C\ A\}^{n-1}\ B\ C$  and is also equivalent to  $A\ B\ \{C\ A\ B\}^{n-1}\ C$ .

Mixing instructions from several iterations may increase the effectiveness of code scheduling, and may perhaps allow for more parallel execution.

# **Software Pipelining is Hard**

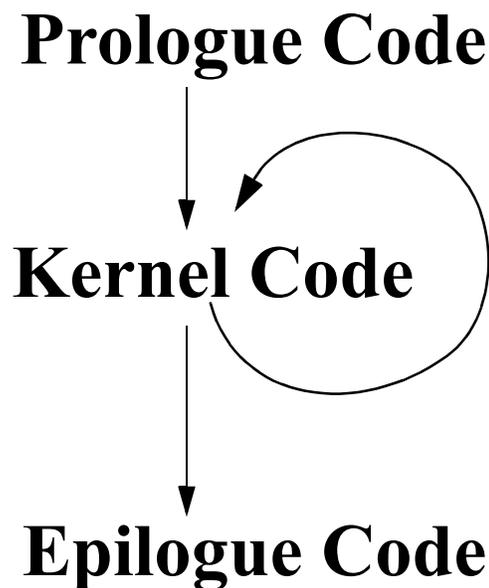
**In fact, it is NP-complete:**

**Hsu and Davidson, “Highly concurrent scalar processing,” 13th ISCA (1986).**

# The Iteration Interval

**We seek to initiate the next iteration of a loop as soon as possible, squeezing each iteration of the loop body into as few machine cycles as possible.**

**The general form of a software pipelined loop is:**



**The prologue code “sets up” the main loop, and the epilogue code “cleans up” after loop termination. Neither the prolog nor the epilogue need be optimized, since they execute only once.**

**Optimizing the kernel is key in software pipelining. The kernel’s execution time (in cycles) is called the *initiation interval (II)*; it measures how quickly the next iteration of a loop can start.**

**We want the smallest possible initiation interval. Determining the smallest viable  $II$  is itself NP-complete. Because of parallel issue and execution in superscalar and multiple issue processors, very small  $II$  values are possible (even less than 1!)**

# Factors that Limit the Size of the Initiation Interval

**We want the initiation interval to be as small as possible. Two factors limit how small the II can become:**

- Resource Constraints**
- Dependency Constraints**

# Resource Constraints

**A small II normally means that we are doing steps of several iterations simultaneously. The number of registers and functional units (that execute instructions) can become limiting factors of the size of II.**

**For example, if a loop body contains 4 floating point operations, and our processor can issue and execute no more than 2 floating point operations per cycle, then the loop's II can't be less than 2.**

# Dependency Constraints

**A loop body can often contain a *loop-carried dependence*. This means one iteration of a loop depends on values computed in an earlier iteration. For example, in**

```
void f (int a[]) {  
    for (i=1; i<1000; i++)  
        a[i]=(a[i-1]+a[i])/2;  
}
```

**there is a loop carried dependence from the use of `a[i-1]` to the computation of `a[i]` in the previous iteration. This means the computation of `a[i]` can't begin until the computation of `a[i-1]` is completed.**

**Let's look at the code that might be generated for this loop:**

```

f:
    mov    %o0, %o2      !a in %o2
    mov    1, %o1        !i=1 in %o1

L:
    sll    %o1, 2, %o0   !i*4 in %o0
    add    %o0, %o2, %g2 !&a[i] in %g2
    * ld    [%g2-4], %g2  !a[i-1] in %g2
    ld     [%o2+%o0], %g3 !a[i] in %g3
    * add    %g2, %g3, %g2 !a[i-1]+a[i]
    * srl    %g2, 31, %g3 !s=0 or 1=sign
    * add    %g2, %g3, %g2 !a[i-1]+a[i]+s
    * sra    %g2, 1, %g2  !a[i-1]+a[i]/2
    add    %o1, 1, %o1   !i++
    cmp    %o1, 999
    ble    L
    * st    %g2, [%o2+%o0] !store a[i]
    retl
    nop

```

**The 6 marked instructions form a cyclic dependency chain from a use of  $a[i-1]$  to its computation (as  $a[i]$ ) in the previous cycle. This cycle means that the loop's  $\Pi$  can never be less than 6.**

# Modulo Scheduling

**There are many approaches to software pipelining. One of the simplest, and best known, is *modulo scheduling*. Modulo scheduling builds upon the postpass basic block schedulers we've already studied.**

**First, we estimate the II of the loop we will create. How?**

**We can compute the minimum II based on resource considerations ( $II_{res}$ ) and the minimum II based on cyclic loop-carried dependencies ( $II_{dep}$ ). Then  $\max(II_{res}, II_{dep})$  is a reasonable estimate of the best possible II. We'll try to build a loop with a kernel size of II. If this fails, we'll try  $II+1$ ,  $II+2$ , etc.**

**In modulo scheduling we'll schedule instructions one by one, using the dependency dag and whatever heuristic we prefer to choose among multiple roots.**

**Now though, if we place an instruction at cycle  $c$  (many independent instructions may execute in the same cycle), then we'll place additional copies of the instruction at cycle  $c+II$ ,  $c+2*II$ , etc.**

**Placement must respect dependency constraints and resource limits at all positions. We consider placements only until a kernel (of size  $II$ ) forms. The kernel must begin before cycle  $s-1$ , where  $s$  is the size of the loop body (in instructions). The loop's conditional branch is placed *after* the kernel is formed.**

**If we can't form a kernel of size  $\Pi$  (because of dependency or resource conflicts), we increase  $\Pi$  by 1 and try again. At worst, we get a kernel equal in size to the original loop body, which guarantees that the modulo scheduler eventually terminates.**

**Depending on how many iterations are intermixed in the kernel, the loop termination condition may need to be adjusted (since the initial and final iterations may appear as part of the loop prologue and epilogue).**

# Example

Consider the following simple function which adds an array index to each element of an array and copies the results into a second array:

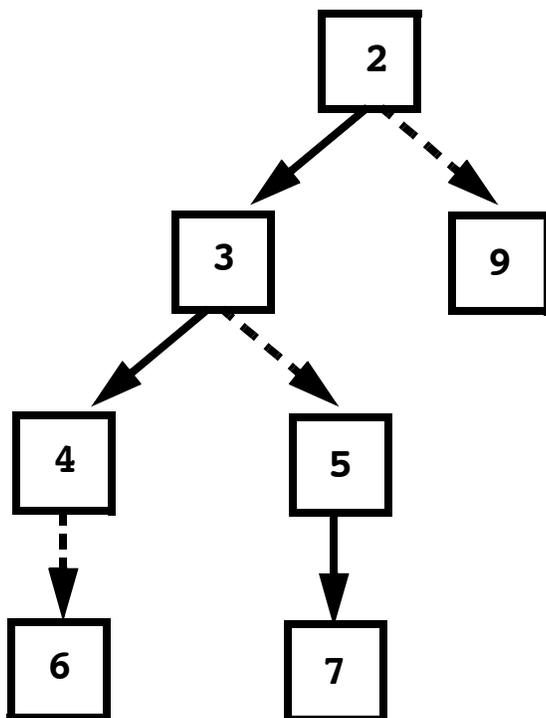
```
void f (int a[],int b[]) {  
    t1 = &a[0];  
    t2 = &b[0];  
    for (i=0;i<1000;i++,t1++,t2++)  
        *t1 = *t2 + i;  
}
```

The code for  $f$  (compiled as a leaf procedure) is:

```

1.  f:  mov    0, %g3
2.  L:  ld     [%o1], %g2
3.      add   %g3, %g2, %g4
4.      st   %g4, [%o0]
5.      add   %g3, 1, %g3
6.      add   %o0, 4, %o0
7.      cmp   %g3, 999
8.      ble  L
9.      add   %o1, 4, %o1
10.     retl
11.     nop

```



Dashed arcs are anti dependencies.

**We'll software pipeline the loop body, excluding the conditional branch (which is placed after the loop kernel is formed).**

**This loop body contains 2 loads/stores, 5 arithmetic and logical operations (including the compare) and one conditional branch.**

**Let's assume the processor we are compiling for has 1 load/store unit, 3 arithmetic/logic units, and 1 branch unit. That means the processor can (ideally) issue and execute simultaneously 1 load or store, 3 arithmetic and logic instructions, and 1 branch. Thus its maximum issue width is 5. (Current superscalars have roughly this capability.)**

**Considering resource requirements, we will need at least two cycles to process the contents of the loop body. There are no loop-carried dependencies.**

**Thus we will estimate this loop's best possible Initiation Interval to be 2.**

**Since the only instruction that can stall is the root of the dependency dag, we'll schedule using estimated critical path length, which is just the node's height in the tree. Hence we'll schedule the nodes in the order: 2,3,4,5,6,7,9.**

**We'll schedule all instructions in a legal execution order (respecting dependencies), and we'll try to choose as many instructions as possible to execute in the same cycle.**

**Starting with the root, instruction 2,  
we schedule it at cycles 1, 3 ( $=1+II$ ),  
5 ( $=1+2*II$ ):**

<b>cycle</b>	<b>instruction</b>
1.	ld [%o1], %g2
2.	
3.	ld [%o1], %g2
4.	
5.	ld [%o1], %g2

**No conflicts so far, since each of the  
loads starts an independent  
iteration.**

**We'll schedule instruction 3 next. It must be placed at cycles 3, 5 and 7 since it uses the result of the load.**

<b>cycle</b>	<b>instruction</b>
1.	ld [%o1], %g2
2.	
3.	add %g3, %g2, %g4
3.	ld [%o1], %g2
4.	
5.	add %g3, %g2, %g4
5.	ld [%o1], %g2
6.	
7.	add %g3, %g2, %g4

**Note that in cycles 3 and 5 we use the current value of %g2 *and* initiate a load into %g2.**

**Instruction 4 is next. It uses the result of the add we just scheduled, so it is placed at cycles 4 and 6.**

<b>cycle</b>	<b>instruction</b>
1.	ld [%o1], %g2
2.	
3.	add %g3, %g2, %g4
3.	ld [%o1], %g2
4.	st %g4, [%o0]
5.	add %g3, %g2, %g4
5.	ld [%o1], %g2
6.	st %g4, [%o0]
7.	add %g3, %g2, %g4

**Instruction 5 is next. It is anti dependent on instruction 3, so we can place it in the same cycles that 3 uses (3, 5 and 7).**

<b>cycle</b>	<b>instruction</b>	
1.	ld	[%o1], %g2
2.		
3.	add	%g3, %g2, %g4
3.	ld	[%o1], %g2
3.	add	%g3, 1, %g3
4.	st	%g4, [%o0]
5.	add	%g3, %g2, %g4
5.	ld	[%o1], %g2
5.	add	%g3, 1, %g3
6.	st	%g4, [%o0]
7.	add	%g3, %g2, %g4
7.	add	%g3, 1, %g3

**Instruction 6 is next. It is anti dependent on instruction 4, so we can place it in the same cycles that 4 uses (4 and 6).**

<b>cycle</b>	<b>instruction</b>	
1.	ld	[%o1], %g2
2.		
3.	add	%g3, %g2, %g4
3.	ld	[%o1], %g2
3.	add	%g3, 1, %g3
4.	st	%g4, [%o0]
4.	add	%o0, 4, %o0
5.	add	%g3, %g2, %g4
5.	ld	[%o1], %g2
5.	add	%g3, 1, %g3
6.	st	%g4, [%o0]
6.	add	%o0, 4, %o0
7.	add	%g3, %g2, %g4
7.	add	%g3, 1, %g3

**Next we place instruction 7. It uses the result of instruction 5 (%g3), so it is placed in the cycles following instruction 5 (4 and 6).**

<b>cycle</b>	<b>instruction</b>	
1.	ld	[%o1], %g2
2.		
3.	add	%g3, %g2, %g4
3.	ld	[%o1], %g2
3.	add	%g3, 1, %g3
4.	st	%g4, [%o0]
4.	add	%o0, 4, %o0
4.	cmp	%g3, 999
5.	add	%g3, %g2, %g4
5.	ld	[%o1], %g2
5.	add	%g3, 1, %g3
6.	st	%g4, [%o0]
6.	add	%o0, 4, %o0
6.	cmp	%g3, 999
7.	add	%g3, %g2, %g4
7.	add	%g3, 1, %g3

**Finally we place instruction 9. It is anti dependent on instruction 2 so it is placed in the same cycles as instruction 2 (1, 3 and 5).**

<b>cycle</b>	<b>instruction</b>
1.	ld [%o1], %g2
1.	add %o1, 4, %o1
3.	add %g3, %g2, %g4
3.	ld [%o1], %g2
3.	add %o1, 4, %o1
3.	add %g3, 1, %g3
4.	st %g4, [%o0]
4.	add %o0, 4, %o0
4.	cmp %g3, 999
5.	add %g3, %g2, %g4
5.	ld [%o1], %g2
5.	add %o1, 4, %o1
5.	add %g3, 1, %g3
6.	st %g4, [%o0]
6.	add %o0, 4, %o0
6.	cmp %g3, 999
7.	add %g3, %g2, %g4
7.	add %g3, 1, %g3

**We look for a 2 cycles kernel that contains all 7 instructions of the loop body that we have scheduled. We also want a kernel that sets the condition code (via the cmp) during its first cycle so that it can be tested during its second (and final) cycle. Cycles 4 and 5 meet these criteria, and will form our kernel.**

**We place the conditional branch just before the last instruction in cycle 5 (to give the conditional branch a useful instruction for its delay slot).**

## We now have:

<b>cycle</b>		<b>instruction</b>
1.		ld [%o1], %g2
1.		add %o1, 4, %o1
3.		add %g3, %g2, %g4
3.		ld [%o1], %g2
3.		add %o1, 4, %o1
3.		add %g3, 1, %g3
4.	L:	st %g4, [%o0]
4.		add %o0, 4, %o0
4.		cmp %g3, 999
5.		add %g3, %g2, %g4
5.		ld [%o1], %g2
5.		add %o1, 4, %o1
5.		ble L
5.		add %g3, 1, %g3
6.		st %g4, [%o0]
6.		add %o0, 4, %o0
6.		cmp %g3, 999
7.		add %g3, %g2, %g4
7.		add %g3, 1, %g3

**A couple of final issues must be dealt with:**

- **Does the iteration count need to be changed?**

**In this case no, since the final valid value of  $i$ , 999, is used to compute  $\%g4$  in cycle 5, before the loop exits.**

- **What instructions do we keep as the loop's epilogue?**

**None! Instructions past the kernel aren't needed since they are part of future iterations (past  $i==999$ ) which aren't needed or wanted.**

- **Note that  $b[1000]$  and  $b[1001]$  are "touched" even though they are never used. This is probably OK as long as arrays aren't placed at the very end of a page or segment.**

## Our final loop is:

<b>cycle</b>		<b>instruction</b>	
1.		ld [%o1], %g2	!N <sub>0</sub>
1.		add %o1, 4, %o1	!N <sub>0</sub>
3.		add %g3, %g2, %g4	!N <sub>0</sub>
3.		ld [%o1], %g2	!N <sub>1</sub>
3.		add %o1, 4, %o1	!N <sub>1</sub>
3.		add %g3, 1, %g3	!N <sub>0</sub>
4.	L:	st %g4, [%o0]	!N <sub>0</sub>
4.		add %o0, 4, %o0	!N <sub>0</sub>
4.		cmp %g3, 999	!N <sub>0</sub>
5.		add %g3, %g2, %g4	!N <sub>1</sub>
5.		ld [%o1], %g2	!N <sub>2</sub>
5.		add %o1, 4, %o1	!N <sub>2</sub>
5.		ble L	!N <sub>0</sub>
5.		add %g3, 1, %g3	!N <sub>1</sub>

This is very efficient code—we use the full parallelism of the processor, executing 5 instructions in cycle 5 and 8 instructions in just 2 cycles. All resource limitations are respected.

# False Dependencies & Loop Unrolling

**A limiting factor in how “tightly” we can software pipeline a loop is reuse of registers and the false dependencies reuse induces.**

**Consider the following simple function that copies array elements:**

```
void f (int a[],int b[], int lim) {  
    for (i=0;i<lim;i++)  
        a[i]=b[i];  
}
```

**The loop that is generated takes 3 cycles:**

<b>cycle</b>		<b>instruction</b>
<b>1.</b>	<b>L:</b>	<b>ld [%g3+%o1], %g2</b>
<b>1.</b>		<b>addcc %o2, -1, %o2</b>
<b>3.</b>		<b>st %g2, [%g3+%o0]</b>
<b>3.</b>		<b>bne L</b>
<b>3.</b>		<b>add %g3, 4, %g3</b>

**We'd like to tighten the iteration interval to 2 or less. One cycle is unlikely, since doing a load and a store in the same cycle is problematic (due to a possible dependence through memory).**

**If we try to use modulo scheduling, we can't put a second copy of the load in cycle 2 because it would overwrite the contents of the first load. A load in cycle 3 will clash with the store.**

**The solution is to unroll the loop into two copies, using different registers to hold the contents of the load and the current offset into the arrays.**

**The use of a "count down" register to test for loop termination is**

**helpful, since it allows an easy exit from the middle of the loop.**

**With the renaming of the registers used in the two expanded iterations, scheduling to “tighten” the loop is effective.**

**After expansion we have:**

<b>cycle</b>		<b>instruction</b>
1.	<b>L:</b>	<b>ld [%g3+%o1], %g2</b>
1.		<b>addcc %o2, -1, %o2</b>
3.		<b>st %g2, [%g3+%o0]</b>
3.		<b>beq L2</b>
3.		<b>add %g3, 4, %g4</b>
4.		<b>ld [%g4+%o1], %g5</b>
4.		<b>addcc %o2, -1, %o2</b>
6.		<b>st %g5, [%g4+%o0]</b>
6.		<b>bne L</b>
6.		<b>add %g4, 4, %g3</b>

**L2:**

**We still have 3 cycles per iteration, because we haven't scheduled yet.**

**Now we can move the increment of %g3 (into %g4) above other uses of %g3. Moreover, we can move the load into %g5 above the store from %g2 (if the load and store are independent):**

<b>cycle</b>		<b>instruction</b>
1.	<b>L:</b>	<b>ld [%g3+%o1], %g2</b>
1.		<b>addcc %o2, -1, %o2</b>
1.		<b>add %g3, 4, %g4</b>
2.		<b>ld [%g4+%o1], %g5</b>
3.		<b>st %g2, [%g3+%o0]</b>
3.		<b>beq L2</b>
3.		<b>addcc %o2, -1, %o2</b>
4.		<b>st %g5, [%g4+%o0]</b>
4.		<b>bne L</b>
4.		<b>add %g4, 4, %g3</b>
	<b>L2:</b>	

**We can normally test whether %g4+%o1 and %g3+%o0 can be equal at compile-time, by looking at the actual array parameters.  
(Can &a[0] == &b[1]?)**

# Predication

**We have seen that conditional execution complicates code scheduling by creating small basic blocks and limiting code movement across conditional branches.**

**However, the problems conditionals introduce are even more fundamental.**

**Consider the following code fragment:**

```
if (a<b)
    a++;
else b++;
if (c<d)
    c++;
else d++;
```

**The two conditionals are completely independent, but they can't be evaluated concurrently in a single thread.**

**Why?**

**Look at the Sparc code generated:**

```
cmp      %o0, %g1  
bge,a   L1  
add     %g1, 1, %g1  
add     %o0, 1, %o0
```

**L1:**

```
cmp      %o5, %o4  
bge,a   L2  
add     %o4, 1, %o4  
add     %o5, 1, %o5
```

**L2:**

**The two compares can't be executed concurrently (because there is only one condition code register).**

**We can't do two conditional branches to two different places simultaneously.**

**And we must select the correct combination of two of the four adds to execute.**

**We could restructure this code into a four-way switch, but this far beyond what a code scheduler is expected to do.**

**The problem is that while **values** can easily be computed in parallel, flow of control **can't**.**

**The solution?**

**Convert flow of control computations into value computations.**

**Our first step is to generalize a single condition code register into a set of predicate registers. The Itanium, for example, includes 64 predicate registers that hold a single boolean value. For our purposes, let's denote a predicate register as %p0 to %p63.**

**Predicate registers are set by doing compare or test instructions.**

**Thus**

```
cmpeq    %o0, %g1, %p1
```

**sets %p1 true if the two operands are equal and false otherwise.**

**The real power of predication is that most instructions can be controlled (predicated) by a predicate register.**

**Thus**

**`add(%p1) %r1, %r2, %r3`**

**does an ordinary add but only commits the result (into %r3) if %p1 is true.**

**A negated form is often included too:**

**`add(~%p1) %r1, %r2, %r3`**

**In this form, the add is completed only if %p1 is false.**

**Using predication, we can eliminate many conditional branches. Now **both** legs of a conditional can be evaluated, with only one leg allowed to commit.**

**Returning to our earlier example,**

```
if (a<b)
    a++;
else b++;
if (c<d)
    c++;
else d++;
```

**we now generate**

<b>1. <code>cmplt</code></b>	<b><code>%o0, %g1, %p1</code></b>
<b>1. <code>cmplt</code></b>	<b><code>%o5, %o4, %p2</code></b>
<b>2. <code>add(%p1)</code></b>	<b><code>%g1, 1, %g1</code></b>
<b>2. <code>add(~p1)</code></b>	<b><code>%o0, 1, %o0</code></b>
<b>2. <code>add(%p2)</code></b>	<b><code>%o4, 1, %o4</code></b>
<b>2. <code>add(%~p2)</code></b>	<b><code>%o5, 1, %o5</code></b>

**This entire code fragment can now execute in two cycles, since the two compares and four adds are independent of each other.**

# Predication Enhances Software Pipelining

Conditionals in a loop body greatly complicate software pipelining since we usually won't know exactly what instructions future iterations will execute.

Consider this minor variant of our earlier example:

```
void f (int a[],int b[]) {
    t1 = &a[0];
    t2 = &b[0];
    for (i=0;i<1000;i++,t1++,t2++)
        if (i%2)
            *t1 = *t2 + i;
        else *t1 = *t2 - i;
}
```

```

1.   f:   mov     0, %g3
2.   L:   andcc  %g3, 1, %g0
3.           bne   L1
4.           ld    [%o1], %g2
5.           b     L2
6.           sub   %g3, %g2, %g4
7.   L1:  add    %g3, %g2, %g4
8.   L2:  st     %g4, [%o0]
9.           add   %g3, 1, %g3
10.          add   %o0, 4, %o0
11.          cmp   %g3, 999
12.          ble   L
13.          add   %o1, 4, %o1
14.          retl
15.          nop

```

**We've added an `andcc` (to do the `i%2` computation) as well as a conditional and unconditional branch. Each iteration will do an add or a subtract.**

**A two cycle per iteration schedule seems most unlikely.**

**But predication helps immensely!**

**The generated code becomes much cleaner:**

```
1.   f:   mov     0, %g3
2.   L:   and     %g3, 1, %p1
3.     ld     [%o1], %g2
4.     sub(~%p1) %g3, %g2, %g4
5.     add(%p1)  %g3, %g2, %g4
6.     st     %g4, [%o0]
7.     add   %g3, 1, %g3
8.     add   %o0, 4, %o0
9.     cmp   %g3, 999
10.    ble   L
11.    add   %o1, 4, %o1
12.    retl
13.    nop
```

**And guess what? We can still software pipeline this into 2 cycles per iteration:**

cycle	instruction
1.	ld [%o1], %g2
1.	add %o1, 4, %o1
2.	and %g3, 1, %p1
3.	add(%p1) %g3, %g2, %g4
3.	sub(~%p1) %g3, %g2, %g4
3.	ld [%o1], %g2
3.	add %o1, 4, %o1
3.	add %g3, 1, %g3
4.	L: st %g4, [%o0]
4.	add %o0, 4, %o0
4.	and %g3, 1, %p1
4.	cmp %g3, 999
5.	add(%p1) %g3, %g2, %g4
5.	sub(~%p1) %g3, %g2, %g4
5.	ld [%o1], %g2
5.	add %o1, 4, %o1
5.	ble L
5.	add %g3, 1, %g3

**We now do need to be able to issue four ALU operations per cycle (since we issue both the add and subtract in the same cycle).**

# Reading Assignment

- **Read Section 13.5 (Automatic Instruction Selection) of *Crafting a Compiler*.**

# Automatic Instruction Selection

**Besides register allocation and code scheduling, a code generator must also do *Instruction Selection*.**

**For CISC (Complex Instruction Set Computer) Architectures, like the Intel x86, DEC Vax, and many special purpose processors (like Digital Signal Processors), instruction selection is often *challenging* because so many choices exist.**

**In the Vax, for example, one, two and three address instructions exist. Each address may be a register, memory location (with or without indexing), or an immediate operand.**

**For RISC (Reduced Instruction Set Computer) Processors, instruction formats and addressing modes are far more limited.**

**Still, it is necessary to handle immediate operands, commutative operands and special case null operands (add of 0 or multiply of 1).**

**Moreover, automatic instruction selection supports *automatic retargeting* of a compiler to a new or extended instruction set.**

# Tree-Structured Intermediate Representations

**For purposes of automatic code generation, it is convenient to translate a source program into a *Low-level, Tree-Structured IR*.**

**This representation exposes translation details (how locals are accessed, how conditionals are translated, etc.) without assuming a particular instruction set.**

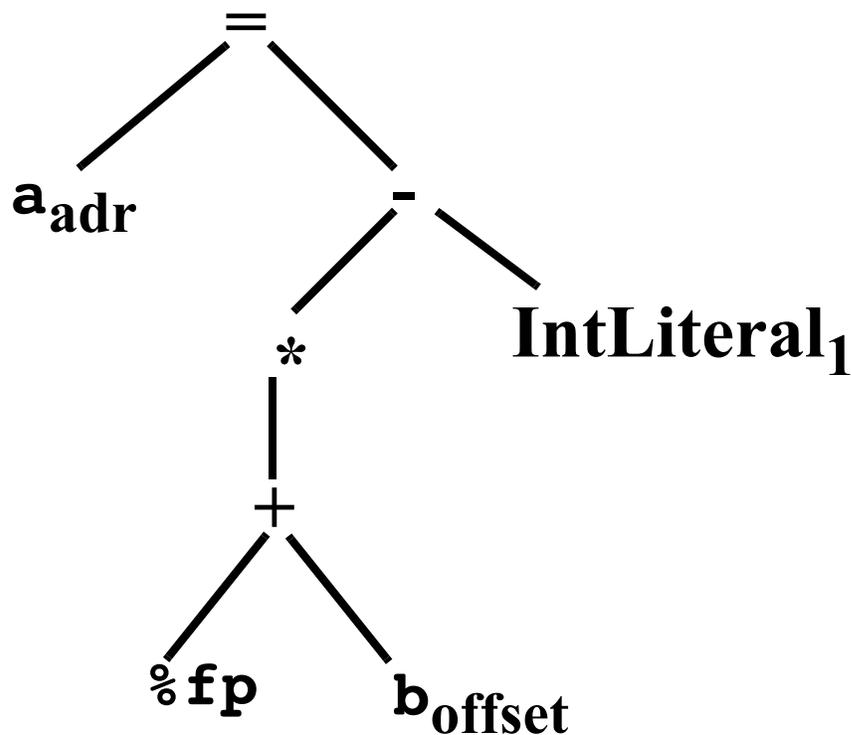
**In a low-level, tree-structured IR, leaves are registers or bit-patterns and internal nodes are machine-level primitives, like load, store, add, etc.**

# Example

Let's look at how

`a = b - 1;`

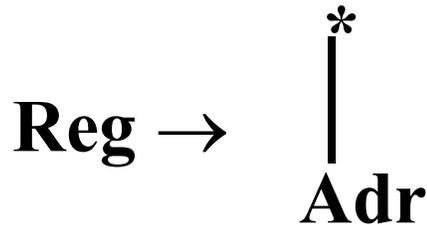
is represented, where `a` is a global integer variable and `b` is a local (frame allocated) integer variable.



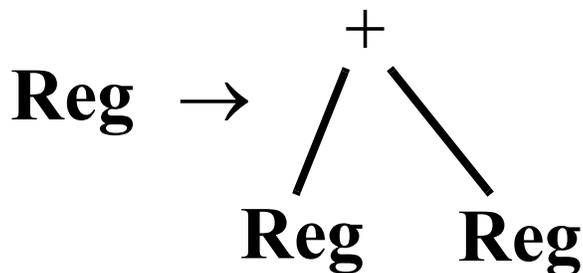
# Representation of Instructions

**Individual instructions can be represented as trees, rooted by the operation they implement.**

**For example:**

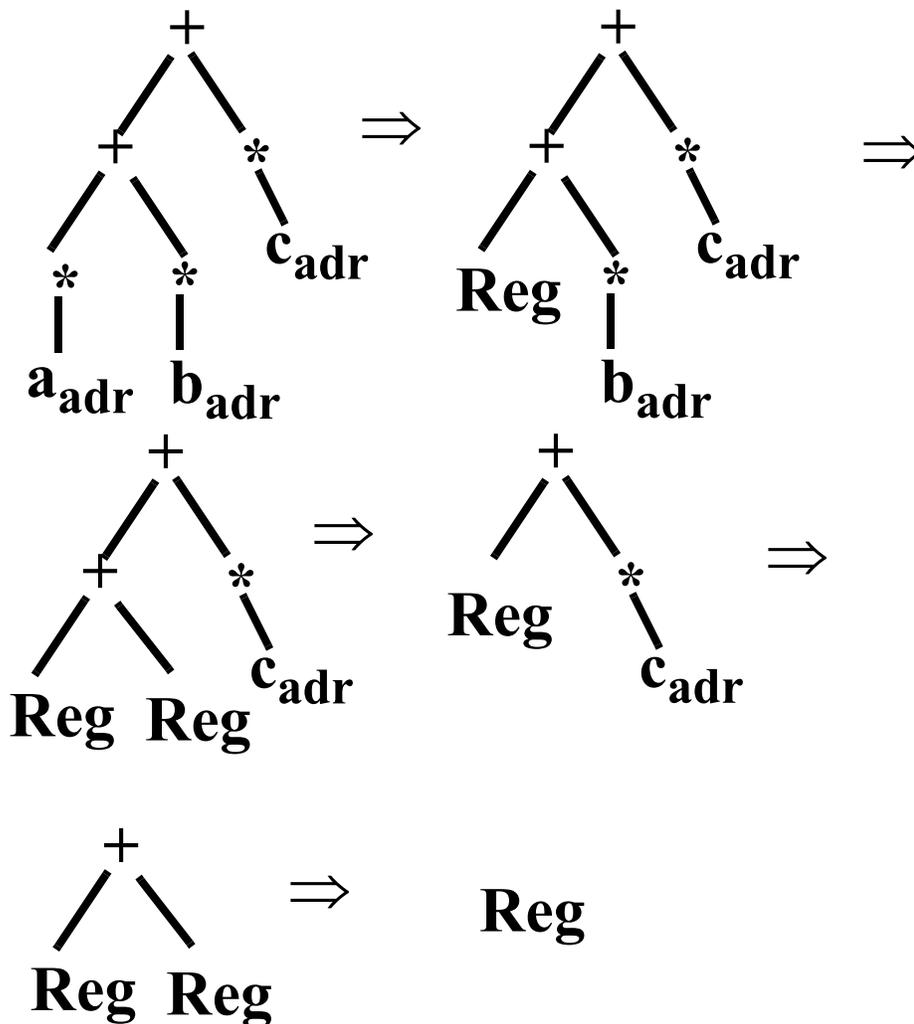


**This is an instruction that loads a register with the value at an absolute address.**



**This is an instruction that adds the contents of two registers and stores the sum into a third register.**

**Using the above pair of instruction definitions, we can repeatedly match instructions in the following program IR:**



**Each match of an instruction pattern can have the side-effect of generating an instruction:**

```
ld    [a], %R1
ld    [b], %R2
add   %R1, %R2, %R3
ld    [c], %R4
add   %R3, %R4, %R5
```

**Registers can be allocated on-the-fly as Instructions are generated or instructions can be generated using pseudo-registers, with a subsequent register allocation phase.**

**Using this view of instruction selection, choosing instructions involves finding a *cover* for an IR tree using Instruction Patterns.**

***Any cover is a valid translation.***

# **Tree Parsing vs. String Parsing**

**This process of selecting instructions by matching instruction patterns is very similar to how strings are parsed using Context-free Grammars.**

**We repeatedly identify a sub-tree that corresponds to an instruction, and simplify the IR-tree by replacing the instruction sub-tree with a nonterminal symbol. The process is repeated until the IR-tree is reduced to a single nonterminal.**

**The theory of reducing an IR-tree using rewrite rules has been studied as part of BURS (Bottom-Up Rewrite Systems) Theory by Pelegri-Llopart and Graham.**

# **Automatic Instruction Selection Tools**

**Just as tools like Yacc and Bison automatically generate a string parser from a specification of a Context-free Grammar, there exist tools that will automatically generate a tree-parser from a specification of tree productions.**

**Two such tools are BURG (Bottom Up Rewrite Generator) and IBURG (Interpreted BURG). Both automatically generate parsers for tree grammars using BURS theory.**

# Least-Cost Tree Parsing

**BURG (and IBURG) guarantee to find a cover for an input tree (if one exists).**

**But tree grammars are usually *very* ambiguous.**

**Why?—Because there is usually more than one code sequence that can correctly implement a given IR-tree.**

**To deal with ambiguity, BURG and IBURG allow each instruction pattern (tree production) to have a *cost*.**

**This cost is typically the size or execution time for the corresponding target-machine instructions.**

**Using costs, BURG (and IBURG) not only guarantee to find a cover, but also a *least-cost cover*.**

**This means that when a generated tree-parser is used to cover (and thereby translate) an IR-Tree, the *best possible code sequence* is guaranteed.**

**If more than one least-cost cover exists, an arbitrary choice is made.**

# Using BURG to Specify Instruction Selection

We'll need a tree grammar to specify possible partial covers of a tree.

For simplicity, BURG requires that all tree productions be of the form

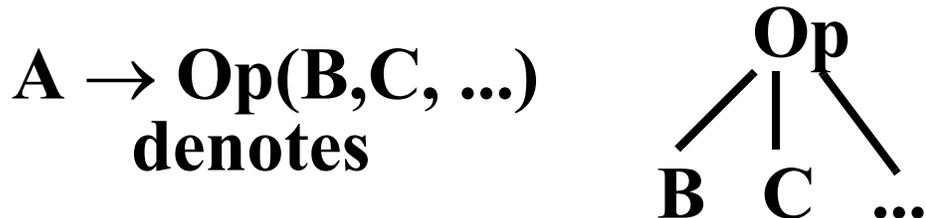
$$A \rightarrow b$$

(where  $b$  is a single terminal symbol)

or

$$A \rightarrow \text{Op}(B, C, \dots)$$

**(where Op is a terminal that is a subtree root and B,C, ... are non-terminals)**



**All tree grammars can be put into this form by adding new nonterminals and productions as needed.**

**We must specify terminal symbols (leaves and operators in the IR-Tree) and nonterminals that are used in tree productions.**

# Example

**A subset of a SPARC instruction selector.**

## Terminals

### Leaf Nodes

<b>int32</b>	<b>(32 bit integer)</b>
<b>s13</b>	<b>(13 bit signed integer)</b>
<b>r</b>	<b>(0-31, a register name)</b>

### Operator Nodes

<b>*</b>	<b>(unary indirection)</b>
<b>-</b>	<b>(binary minus)</b>
<b>+</b>	<b>(binary addition)</b>
<b>=</b>	<b>(binary assignment)</b>

# Nonterminals

<b>UInt</b>	<b>(32 bit unsigned</b>
<b>integer)</b>	
<b>Reg</b>	<b>(Loaded register value)</b>
<b>Imm</b>	<b>(Immediate operand)</b>
<b>Adr</b>	<b>(Address expression)</b>
<b>Void</b>	<b>(Null value)</b>

# Productions

Rule #	Production	Cost	SPARC Code
R0	UInt $\rightarrow$ Int32	0	
R1	Reg $\rightarrow$ r	0	
R2	Adr $\rightarrow$ r	0	
R3	Adr $\rightarrow$ $\begin{array}{c} + \\ / \quad \backslash \\ \text{Reg} \quad \text{Imm} \end{array}$	0	
R4	Imm $\rightarrow$ s13	0	
R5	Reg $\rightarrow$ s13	1	<code>mov s13,Reg</code>
R6	Reg $\rightarrow$ int32	2	<code>sethi</code> <code>  %hi(int32),%g1</code> <code>or %g1,</code> <code>  %lo(int32),Reg</code>
R7	Reg $\rightarrow$ $\begin{array}{c} - \\ / \quad \backslash \\ \text{Reg} \quad \text{Reg} \end{array}$	1	<code>sub Reg,Reg,Reg</code>

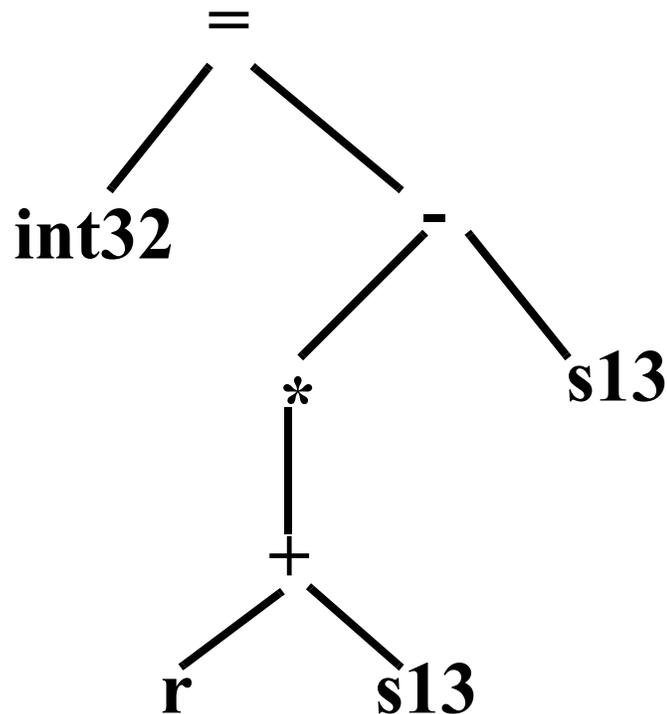
Rule #	Production	Cost	SPARC Code
R8	$\text{Reg} \rightarrow \begin{array}{c} \text{---} \\ / \quad \backslash \\ \text{Reg} \quad \text{Imm} \end{array}$	1	<code>sub Reg, Imm, Reg</code>
R9	$\text{Reg} \rightarrow \begin{array}{c} * \\   \\ \text{Adr} \end{array}$	1	<code>ld [Adr], Reg</code>
R10	$\text{Void} \rightarrow \begin{array}{c} = \\ / \quad \backslash \\ \text{UInt} \quad \text{Reg} \end{array}$	2	<code>sethi</code> <code>  %hi(UInt), %g1</code> <code>st Reg,</code> <code>  [%g1+%lo(UInt)]</code>

# Example

Let's look at instruction selection for

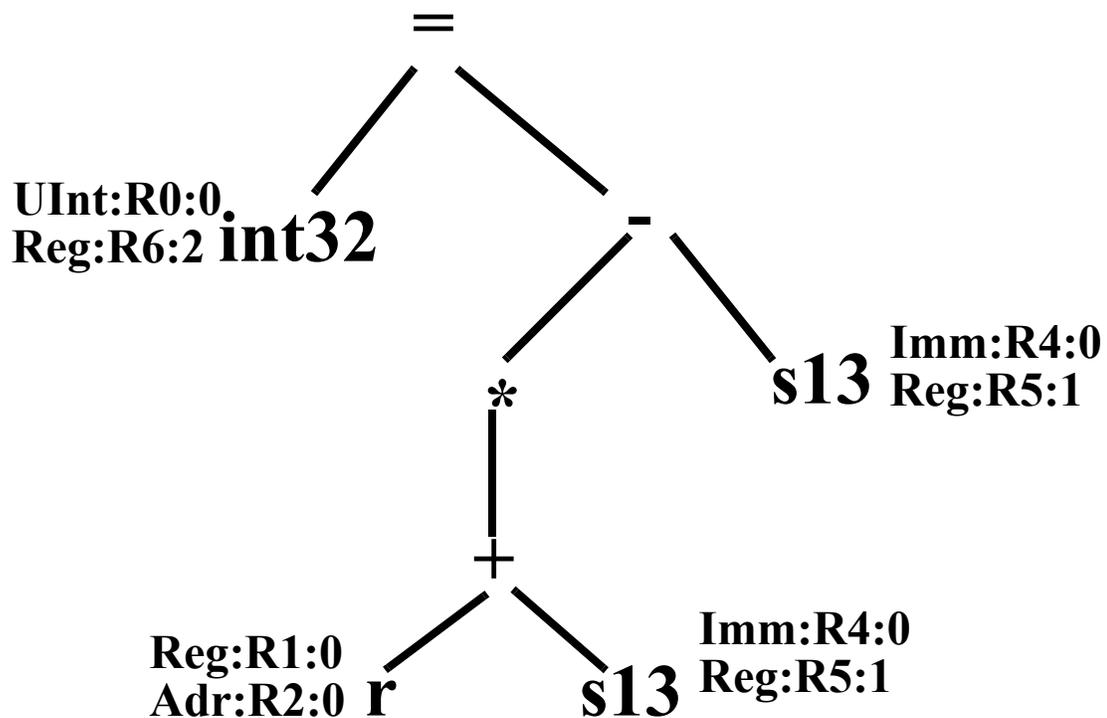
`a = b - 1;`

where `a` is a global int, accessed with a 32 bit address and `b` is a local int, accessed as an offset from the frame pointer.



We match tree nodes *bottom-up*.  
 Each node is labeled with the nonterminals it can be reduced to, the production used to produce the nonterminal, and the cost to generate the node (and its children) from the nonterminal.

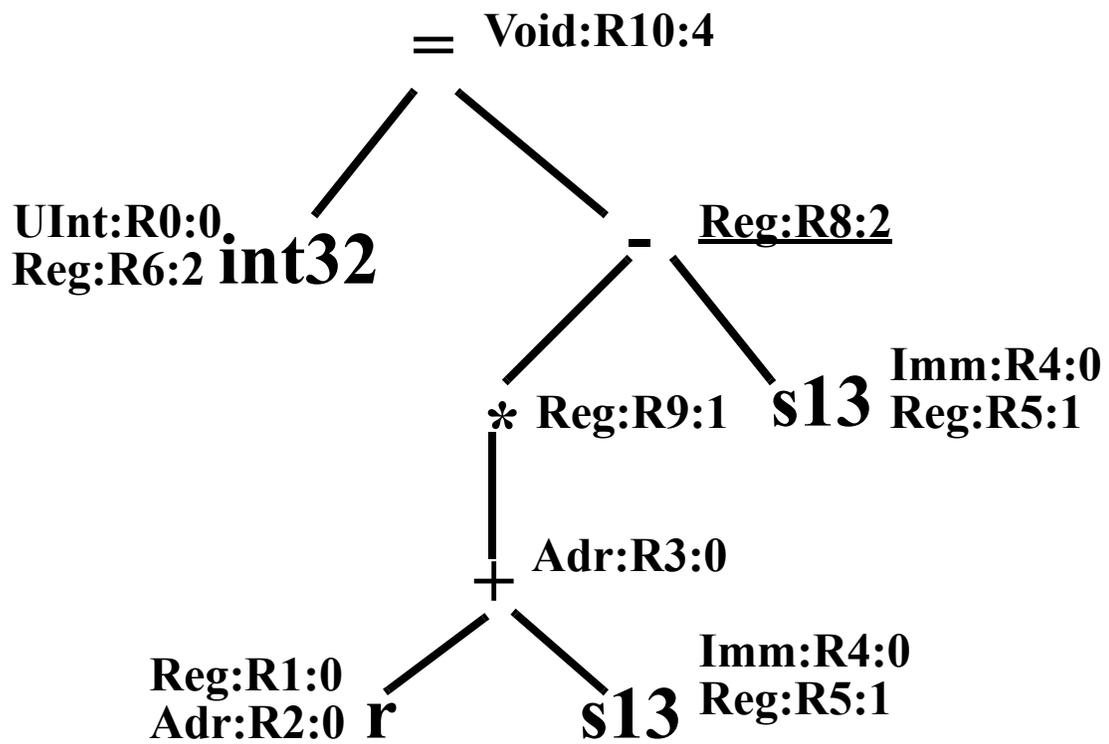
We match leaves first:



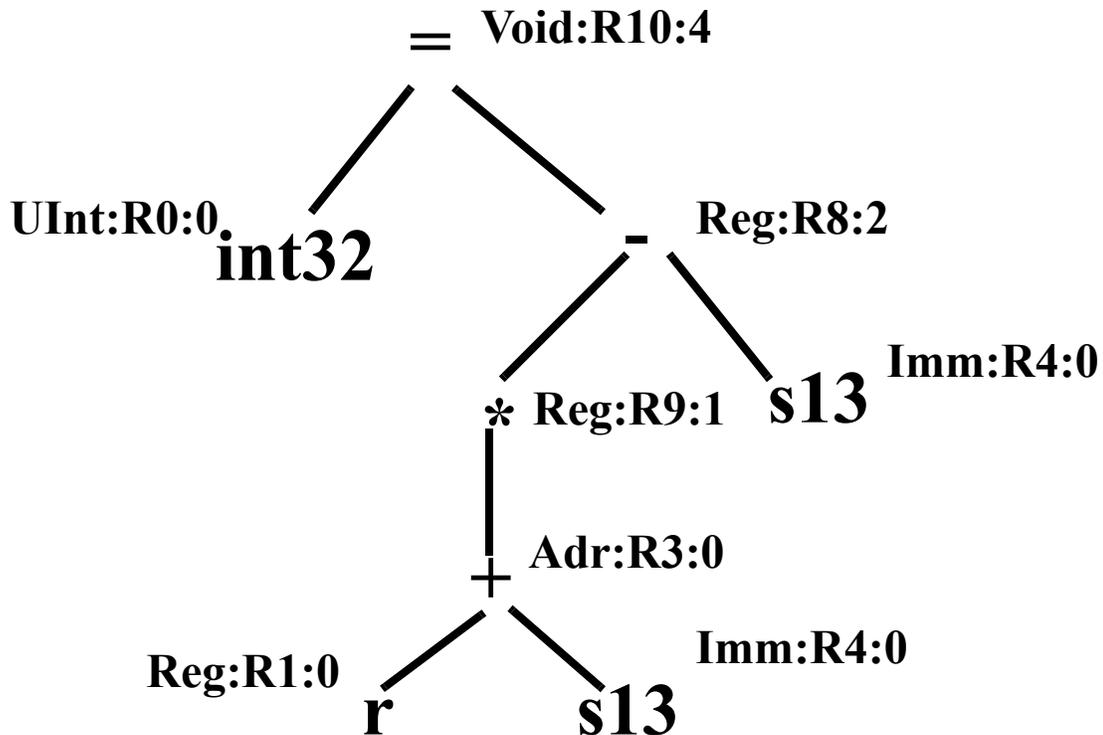
**We now work *upward*, considering operators whose children have been labeled. Again, if an operator can be generated by a nonterminal, we mark the operator with the nonterminal, the production used to generate the operator, and the total cost (including the cost to generate all children).**

**If a nonterminal can generate the operator using more than one production, the *least-cost* derivation is chosen.**

**When we reach the root, the nonterminal with the lowest overall cost is used to generate the tree.**

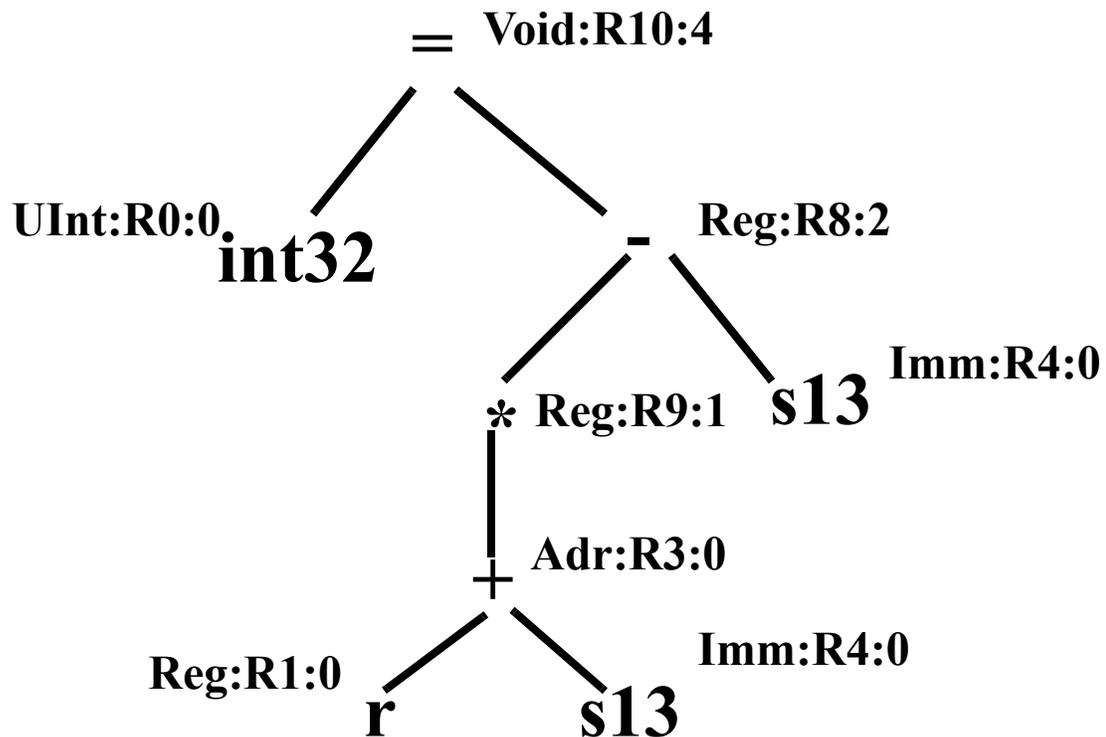


**Note that once we know the production used to generate the root of the tree, we know the productions used to generate each subtree too:**



We generate code by doing a depth-first traversal, generating code for a production after all the production's children have been processed.

We need to do register allocation too; for our example, a simple on-the-fly generator will suffice.



```

ld    [%fp+b], %10
sub   %10, 1, %10
sethi %hi(a), %g1
st    %10, [%g1+%lo(a)]
  
```

**Had we translated a slightly  
difference expression,**

**a = b - 1000000;**

**we would *automatically* get a  
different code sequence (because  
1000000 is an int32 rather than an  
s13):**

```
ld      [%fp+b], %10  
sethi  %hi(1000000), %g1  
or     %g1, %lo(1000000), %11  
sub    %10, %11, %10  
sethi  %hi(a), %g1  
st     %10, [%g1+%lo(a)]
```

# Adding New Rules

Since instruction selectors can be automatically generated, it's easy to add "extra" rules that handle optimizations or special cases.

For example, we might add the following to handle addition of a left immediate operand or subtraction of 0 from a register:

Rule #	Production	Cost	SPARC Code
R11	$\text{Reg} \rightarrow \begin{array}{c} + \\ / \quad \backslash \\ \text{Imm} \quad \text{Reg} \end{array}$	1	<code>add Reg, Imm, Reg</code>
R12	$\text{Reg} \rightarrow \begin{array}{c} - \\ / \quad \backslash \\ \text{Reg} \quad \text{Zerc} \end{array}$	0	

# Improving the Speed of Instruction Selection

**As we have presented it, instruction selection looks rather slow—for each node in the IR tree, we must match productions, compare costs, and select least-cost productions.**

**Since compilers routinely generate program with tens or hundreds of thousands of instructions, doing a lot of computation to select one instruction (even if it's the *best* instruction) could be too slow.**

**Fortunately, this need not be the case.**

**Instruction selection using BURS can be made *very* fast.**

# Adding States to BURG

**We can *precompute* a set of *states* that represent possible labelings on IR tree nodes. A table of node names and subtree states then is used to select a node's state. Thus labeling becomes nothing more than repeated table lookup.**

**For example, we might create a state  $s_0$  that corresponds to the labeling  $\{\text{Reg:R1:0}, \text{Adr:R2:0}\}$ .**

**A state selection function, *label*, defines  $\text{label}(r) = s_0$ . That is, whenever  $r$  is matched as a leaf, it is to be labeled with  $s_0$ .**

**If a node is an operator, *label* uses the name of the operator and the**

**labeling assigned to its children to choose the operator's label. For example,**

**label(+,s0,s1)=s2**

**says that a + with children labeled as s0 and s1 is to be labeled as s2.**

**In theory, that's all there is to building a fast instruction selector.**

**We generate possible labelings, encode them as states, and table all combinations of labelings.**

*But,*

**how do we know the set of possible labelings is even finite?**

**In fact, it isn't!**

# Normalizing Costs

**It is possible to generate states that are identical except for their costs.**

**For example, we might have**

**$s_1 = \{\text{Reg:R1:0}, \text{Adr:R2:0}\},$**

**$s_2 = \{\text{Reg:R1:1}, \text{Adr:R2:1}\},$**

**$s_3 = \{\text{Reg:R1:2}, \text{Adr:R2:2}\}, \text{ etc.}$**

**Here an important insight is needed—the *absolute* costs included in states aren't really essential.**

**Rather *relative* costs are what is important. In  $s_1$ ,  $s_2$ , and  $s_3$ , Reg and Adr have the same cost. Hence the same decision in choosing between Reg and Adr will be made in all three states.**

**We can limit the number of states needed by *normalizing* costs within states so that the lowest cost choice is always 0, and other costs are differences (deltas) from the lowest cost choice.**

**This observation keeps costs bounded within states (except for pathologic cases).**

**Using additional techniques to further reduce the number of states needed, and the time needed to generate them, fast and compact BURS instruction selectors are achievable. See**

**“Simple and Efficient BURS Table Generation,” T. Proebsting, 1992 PLDI Conference.**

# Example

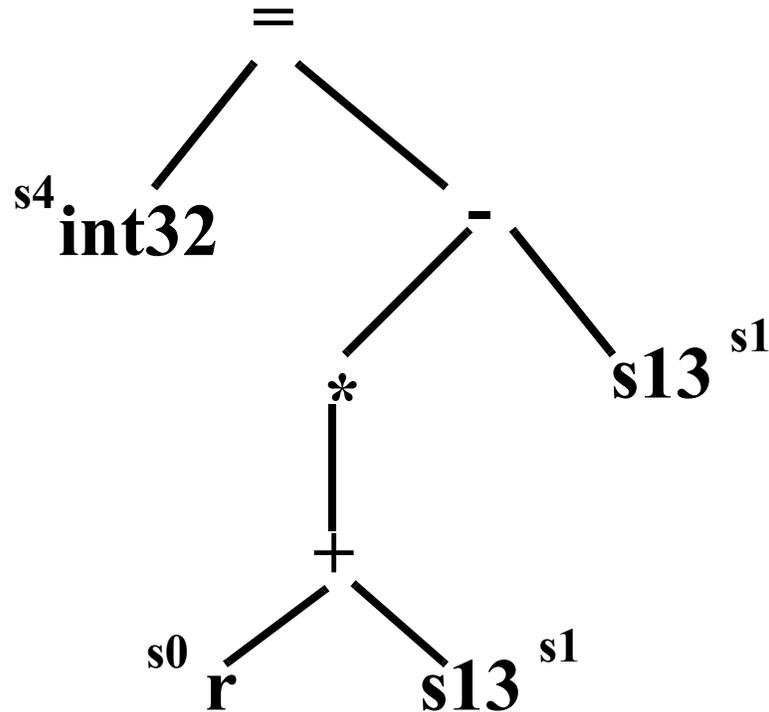
State	Meaning
<b>s0</b>	<b>{Reg:R1:0, Adr:R2:0}</b>
<b>s1</b>	<b>{Imm:R4:0, Reg:R5:1}</b>
<b>s2</b>	<b>{adr:R3:0}</b>
<b>s3</b>	<b>{Reg:R9:0}</b>
<b>s4</b>	<b>{UInt:R0:0}</b>
<b>s5</b>	<b>{Reg:R8:0}</b>
<b>s6</b>	<b>{Void:R10:0}</b>
<b>s7</b>	<b>{Reg:R7:0}</b>

Node	Left Child	Right Child	Result
<b>r</b>			<b>s0</b>
<b>s13</b>			<b>s1</b>
<b>int32</b>			<b>s4</b>
<b>+</b>	<b>s0</b>	<b>s1</b>	<b>s2</b>
<b>*</b>	<b>s2</b>		<b>s3</b>
<b>-</b>	<b>s3</b>	<b>s1</b>	<b>s5</b>
<b>-</b>	<b>s1</b>	<b>s3</b>	<b>s7</b>
<b>=</b>	<b>s4</b>	<b>s5</b>	<b>s6</b>

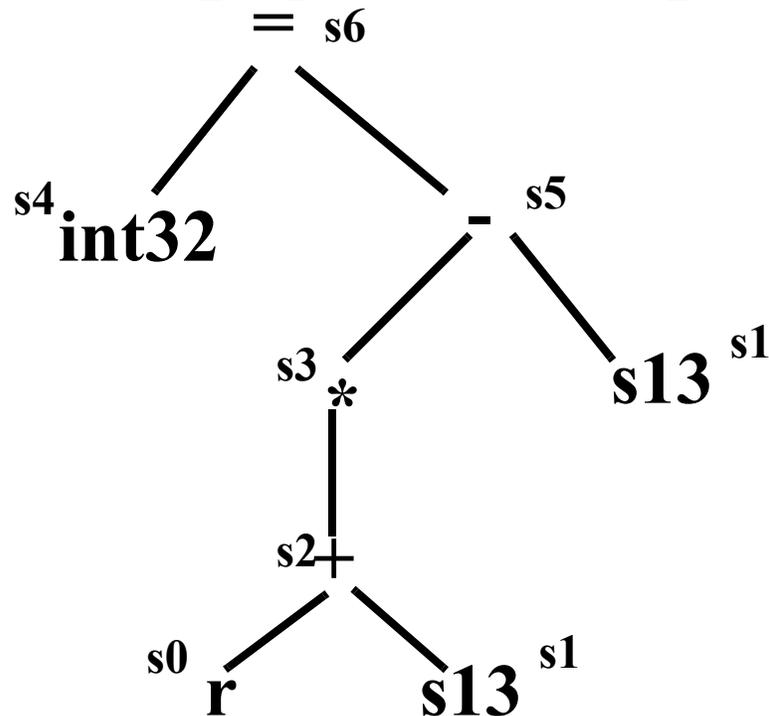
**We start by looking up the state assigned to each leaf. We then work upward, choosing the state of a parent based on the parent's kind and the states assigned to the children. These are all table lookups, and hence very fast.**

**At the root, we select the nonterminal and production based on the state assigned to the root (any entry with 0 cost). Knowing the production used at the root tells us the nonterminal used at each child. Each state has only one entry per nonterminal, so knowing a node's state and the nonterminal used to generate it immediately tells us the production used. Hence identifying the production used for each node is again very fast.**

## Step 1 (Label leaves with states):

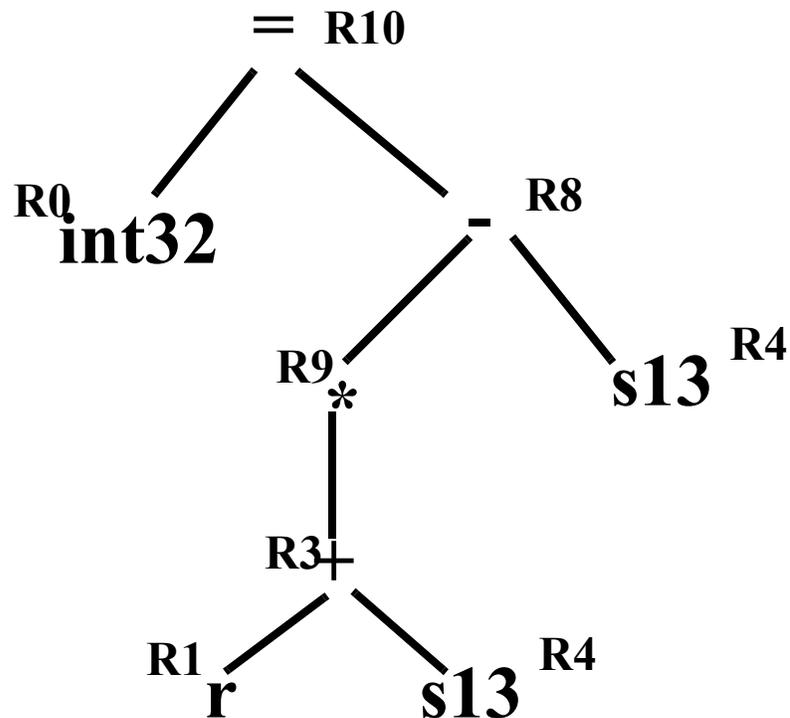


## Step 2 (Propagate states upward):



**Step 3 (Choose production used at root): R10.**

**Step 4 (Propagate productions used downward to children):**



# **Code Generation for x86 Machines**

**The x86 presents several special difficulties when generating code.**

- There are only 8 architecturally visible registers, and only 6 of these are allocatable. Deciding what values to keep in registers, and for how long, is a difficult, but crucial, decision.**
- Operands may be addressed directly from memory in some instructions. Such instructions avoid using a register, but are longer and add to I-cache pressure.**

**In “Optimal Spilling for CISC  
Machines with Few Registers,” Appel**

**and George address both of these difficulties.**

**They use Integer Programming techniques to directly and optimally solve the crucial problem of deciding which live ranges are to be register-resident at each program point. Stores and loads are automatically added to split long live ranges.**

**Then a variant of Chaitin-style register allocation is used to assign registers to live ranges chosen to be register-resident.**

**The presentation of this paper, at the 2001 PLDI Conference, is at**

**`www.cs.wisc.edu/~fischer/  
cs701/cisc.spilling.pdf`**

# Optimistic Coalescing

Given  $R$  allocatable registers, Appel and George guarantee that no more than  $R$  live ranges are marked as register resident.

This doesn't always guarantee that an  $R$  coloring is possible.

Consider the following program fragment:

```
x=0;
while (...) {
    y = x+1;
    print(x);
    z = y+1;
    print(y);
    x = z+1;
    print(z);
}
```

**At any given point in the loop body only 2 variables are live, but 3 registers are needed ( $x$  interferes with  $y$ ,  $y$  interferes with  $z$  and  $z$  interferes with  $x$ ).**

**We know that we have enough registers to handle all live ranges marked as register-resident, but we may need to “shuffle” register allocations at certain points.**

**Thus at one point  $x$  might be allocated R1 and at some other point it might be placed in R2. Such shuffling implies register to register copies, so we'd like to minimize their added cost.**

**Appel and George suggest allowing changes in register assignments between program points. This is done by creating multiple variable names for a live range ( $x_1, x_2, x_3, \dots$ ), one for each program point. Variables are connected by assignments between points. Using coalescing, it is expected that most of the assignments will be optimized away.**

**Using our earlier example, we have the following code with each variable expanded into 3 segments (one for each assignment). Copies of dead variables are removed to simplify the example:**

```
x3=0;  
while (...) {  
    x1 = x3;  
    y1 = x1+1;  
    print(x1);  
    y2 = y1;  
    z2 = y2+1;  
    print(y2);  
    z3 = z2;  
    x3 = z3+1;  
    print(z3);  
}
```

**Now a 2 coloring is possible:**

**x<sub>1</sub>: R1, y<sub>1</sub>: R2**

**z<sub>2</sub>: R1, y<sub>2</sub>: R2**

**z<sub>3</sub>: R1, x<sub>3</sub>: R2**

**(and only x<sub>1</sub> = x<sub>3</sub> is retained).**

**Appel and George found that iterated coalescing wasn't effective (too many copies, most of which are useless).**

**Instead they recommend *Optimistic Coalescing*. The idea is to first do Chaitin-style reckless coalescing of all copies, even if colorability is impaired.**

**Then we do graph coloring register allocation, using the cost of copies as the “spill cost.” As we select colors, a coalesced node that can't be colored is simply split back to the original source and target variables. Since we always limit the number of live ranges to the number of colors, we know the live ranges must be colorable (with register to register copies sometimes needed).**

Using our earlier example, we initially merge  $x_1$  and  $x_3$ ,  $y_1$  and  $y_2$ ,  $z_2$  and  $z_3$ . We already know this can't be colored with two registers. All three pairs have the same costs, so we arbitrarily stack  $x_1$ - $x_3$ , then  $y_1$ - $y_2$  and finally  $z_2$ - $z_3$ .

When we unstack,  $z_2$ - $z_3$  gets R1, and  $y_1$ - $y_2$  gets R2.  $x_1$ - $x_3$  must be split back into  $x_1$  and  $x_3$ .  $x_1$  interferes with  $y_1$ - $y_2$  so it gets R1.  $x_3$  interferes with  $z_2$ - $z_3$  so it gets R2, and coloring is done.

$x_1$ : R1,  $y_1$ : R2

$z_2$ : R1,  $y_2$ : R2

$z_3$ : R1,  $x_3$ : R2

# Procedure & Code Placement

**We have seen many optimizations that aim to reduce the number of instructions executed by a program.**

**Another important class of optimizations derives from the fact that programs often must be paged in virtual memory and almost always are far bigger than the I-cache.**

**Hence how procedures and basic blocks are placed in memory is important. Page faults and I-cache misses can be *very* costly.**

**In “Profile Guided Code Positioning,” Pettis and Hansen explore three kinds of code placement optimizations:**

**1. Procedure Positioning.**

**Try to keep procedures that often call each other close together.**

**2. Basic Block Positioning.**

**Try to place the most frequently executed series of basic blocks “in sequence.”**

**3. Procedure Splitting.**

**Place infrequently executed “fluff” in a different memory area than heavily executed code.**

# Procedure Placement

**Procedures (and classes in Java) are normally separately compiled. They are then placed in memory by a linker or loader in an arbitrary order.**

**This arbitrary ordering can be problematic:**

**If A calls B frequently, and A and B happen to be placed far apart in memory, the calls will cross page boundaries and perhaps cause I-cache conflicts (if code in A and B happen to map to common cache locations).**

*However,*

**if A and B are placed close together in memory, they may both fit on the same page *and* fit into the I-cache without conflicts.**

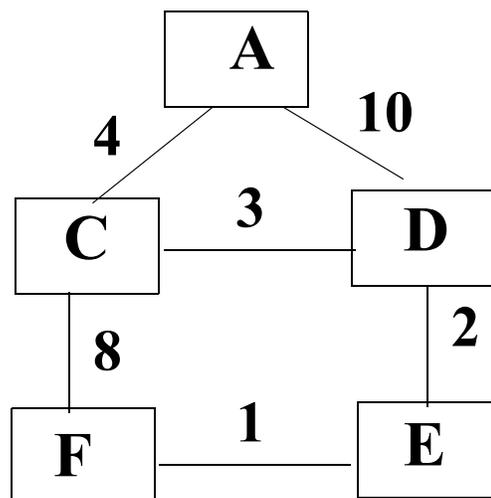
**Pettis & Hansen suggest a “closest is best” procedure placement policy.**

**That is, they recommend that we place procedures that often call each other as close together as possible.**

**How?**

**First, we must obtain dynamic call frequencies using a profiling tool like gprof or qpt.**

**Given call frequencies, we create a call graph, with edges annotated with call frequencies:**

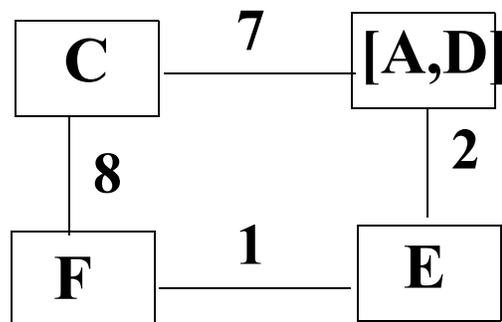


# Group Procedures by Call Frequency

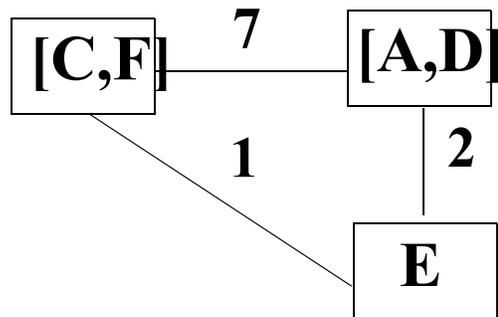
We find the pair of procedures that call each other most often, and group them for contiguous positioning.

The notation  $[A,D]$  means  $A$  and  $D$  will be adjacent (either in order  $A-D$  or  $D-A$ ).

The two procedures chosen are combined in the call graph, which is simplified (much like move-related nodes in an interference graph):



**Now C and F are grouped, without their relative order set (as yet):**



**Next [A,D] and [C,F] are to be joined, but in what exact order?**

**Four orderings are possible:**

**A-D-C-F  $\equiv$  F-C-D-A**

**A-D-F-C  $\equiv$  C-F-D-A**

**D-A-C-F  $\equiv$  F-C-A-D**

**D-A-F-C  $\equiv$  C-F-A-D**

**Are these four orderings equivalent?**

**No—Look at the original call graph.  
At the boundary between [A,D] and [C,F], which of the following is best:**

**D-C (3 calls),**

**D-F (0 calls)**

**A-C (4 calls)**

**A-F (0 calls)**

**A-C has the highest call frequency,  
so we choose D-A-C-F.**

**Finally, we have:**



**We place E near D (call frequency  
2) rather than near F (call  
frequency 1).**

**Our final ordering is  
E-D-A-C-F.**

# Basic Block Placement

We often see conditionals of the form

```
if (error-test)
    {Handle error case}
{Rest of Program}
```

Since error tests rarely succeed (we hope!), the error handling code “pollutes” the I-cache.

In general, we’d like to order basic blocks not in their order of appearance in the source program, but rather in order of their execution along frequently executed paths.

Placing frequently executed basic blocks together in memory fills the I-cache nicely, leads to a smaller

**working set *and* makes branch prediction easier.**

**Pettis & Hansen suggest that we profile execution to determine the frequency of inter-block transitions. We then will group blocks together that execute in sequence most often.**

**At the start, all basic blocks are grouped into singleton chains of one block each.**

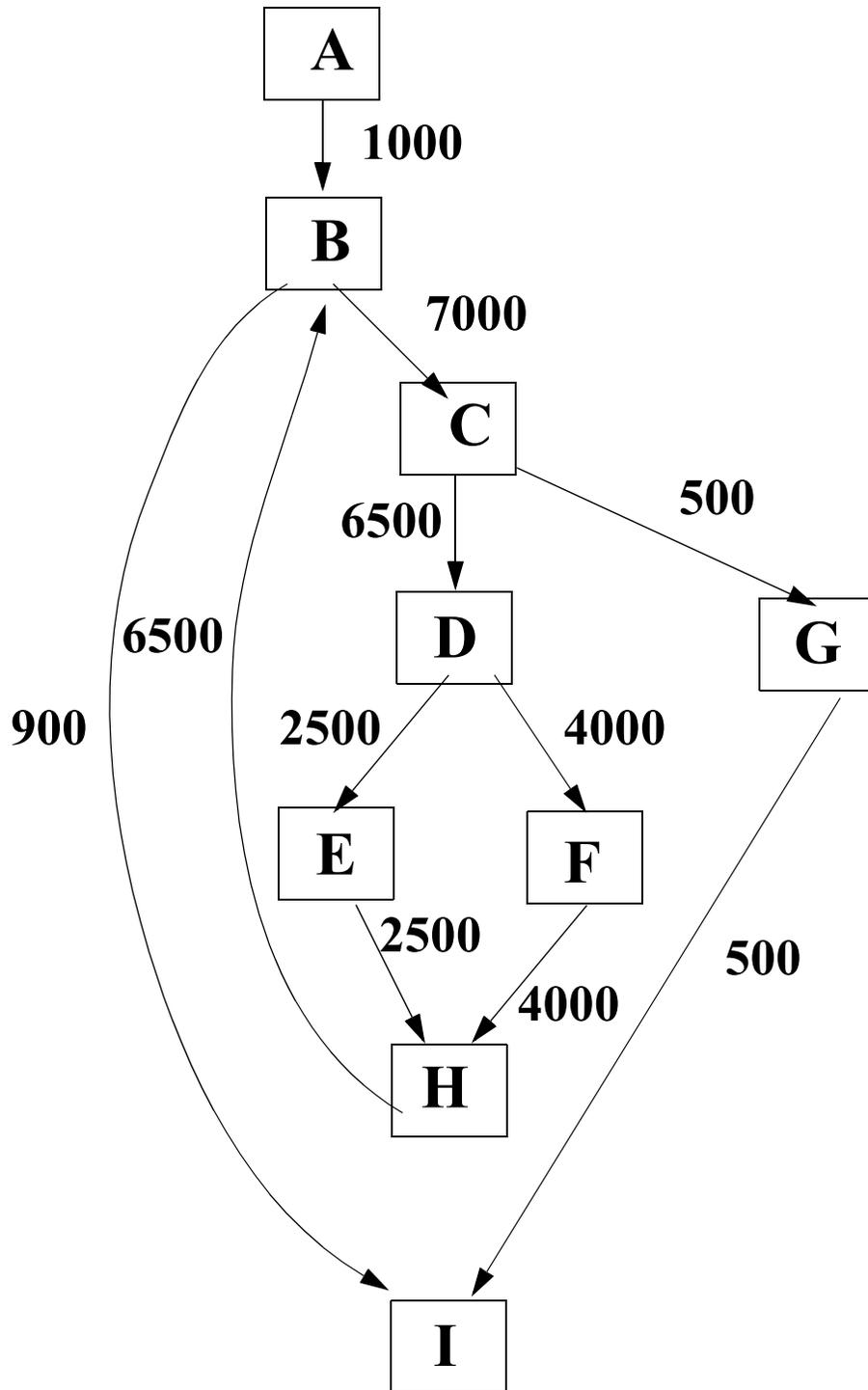
**Then, in decreasing order of transition frequency, we visit arcs in the CFG.**

**If the blocks in the source and target can be linked into a longer chain then do so, else skip to the next transition.**

**When we are done, we have linked together blocks in paths in the CFG that are most frequently executed.**

**Linked basic blocks are allocated together in memory, in the sequence listed in the chain.**

# Example



**Initially, each block is in its own chain.**

<b>Frequency</b>	<b>Action</b>
<b>7000</b>	<b>Form B-C</b>
<b>6500</b>	<b>Form B-C-D</b>
<b>6500</b>	<b>Form H-B-C-D</b>
<b>4000</b>	<b>Form H-B-C-D-F</b>
<b>4000</b>	<b>H is already placed</b>
<b>2500</b>	<b>E can't be placed after D, leave it alone</b>
<b>2500</b>	<b>H is already placed</b>
<b>1000</b>	<b>A can't be placed before B, leave it alone</b>
<b>900</b>	<b>I can't be placed after B, leave it alone</b>
<b>500</b>	<b>G can't be placed after C, leave it alone</b>

**500**

**Form G-I**

**We will place in memory the following chains of basic blocks:**

**H-B-C-D-F, E, A, G-I**

**On some computers, the direction of a conditional branch predicts whether the branch is expected to be taken or not (e.g., the HP PA-RISC). On such machines, a backwards branch (forming a loop) is assumed taken; a forward branch is assumed not taken.**

**If the target architecture makes such assumptions regarding conditional branches, we place chains to (where possible) correctly predict the branch outcome.**

**Thus E and G-I are placed after H-B-C-D-F since  $D \rightarrow E$  and  $C \rightarrow G$  normally aren't taken.**

**On the SPARC (V 9) you can set a bit in each conditional branch indicating expected taken/not taken status.**

**On many machines internal branch prediction hardware can over-rule poorly made (or absent) static predictions.**

# Procedure Splitting

**When we profile the basic blocks within a procedure, we'll see some that are frequently executed, and others that are executed rarely or never.**

**If we allocate all the blocks of a procedure contiguously, we'll intermix frequently executed blocks with infrequently executed ones.**

**An alternative is “fluff removal.” We can split a procedure's body into two sets of basic blocks: these executed frequently and those executed infrequently (the dividing line is, of course, somewhat arbitrary).**

**Now when procedure bodies are placed in memory, frequently executed basic blocks will be placed near each other, and infrequently executed blocks will be placed elsewhere (though infrequently executed blocks are still placed near each other). In this way we expect to make better use of page frames and I-cache space, filling them with mostly active basic blocks.**