Partitioning SSA Variables

Initially, all SSA variables will be partitioned by the *form* of the expression assigned to them.

Expressions involving different constants or operators won't (in general) be equivalent, even if their operands happen to be equivalent.

Thus

$v_1 = 2$ and $w_1 = a_2 + 1$

are always considered inequivalent.

But,

$v_3 = a_1 + b_2$ and $w_1 = d_1 + e_2$

may *possibly* be equivalent since both involve the same operator.

Phi functions are potentially equivalent only if they are in the same basic block.

All variables are initially considered equivalent (since they all initially are considered uninitialized until explicit initialization).

After SSA variables are grouped by assignment form, groups are split.

If a_i op b_y and c_k op d_l are in the same group (because they both have the same operator, op) and $a_i \not\equiv c_k$ or $b_j \not\equiv d_l$ then we split the two expressions apart into different groups.

We continue splitting based on operand inequivalence, until no more splits are possible. Values still grouped are equivalent.

Example

```
if (...) {
  $a_1$=0
  if (...)
    $b_1$=0
  else {
    $a_2$=$x_0$
    $b_2$=$x_0$ }
  $a_3$=$\phi$($a_1$,$a_2$)
  $b_3$=$\phi$($b_1$,$b_2$)
  $c_2$=*$a_3$
  $d_2$=*$b_3$ }
else {
  $b_4$=10 }
$a_5$=$\phi$($a_0$,$a_3$)
$b_5$=$\phi$($b_3$,$b_4$)
$c_3$=*$a_5$
$d_3$=*$b_5$
$e_3$=*$a_5$
```

Initial Groupings:

G_1=[a_0,b_0,c_0,d_0,e_0,x_0]
G_2=[a_1=0, b_1=0]
G_3=[a_2=x_0, b_2=x_0]
G_4=[b_4=10]
G_5=[a_3=ϕ(a_1,a_2),
b_3=ϕ(b_1,b_2)]
G_6=[a_5=ϕ(a_0,a_3),
b_5=ϕ(b_3,b_4)]
G_7=[c_2=*a_3,
d_2=*b_3,
d_3=*b_5,
c_3=*a_5,
e_3=*a_5]

Now b_4 isn't equivalent to anything, so split a_5 and b_5. In G_7 split operands b_3, a_5 and b_5. We have

```
if (...) {
  $a_1$=0
  if (...)
    $b_1$=0
  else {
    $a_2$=$x_0$
    $b_2$=$x_0$ }
  $a_3$=$\phi$($a_1$,$a_2$)
  $b_3$=$\phi$($b_1$,$b_2$)
  $c_2$=*$a_3$
  $d_2$=*$b_3$ }
else {
  $b_4$=10 }
$a_5$=$\phi$($a_0$,$a_3$)
$b_5$=$\phi$($b_3$,$b_4$)
$c_3$=*$a_5$
$d_3$=*$b_5$
$e_3$=*$a_5$
```

Final Groupings:

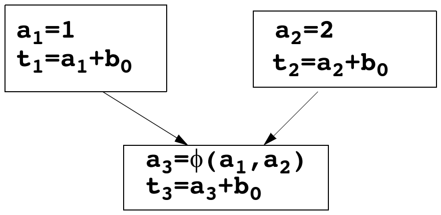
G_1=[a_0,b_0,c_0,d_0,e_0,x_0]
G_2=[a_1=0, b_1=0]
G_3=[a_2=x_0, b_2=x_0]
G_4=[b_4=10]
G_5=[a_3=ϕ(a_1,a_2),
b_3=ϕ(b_1,b_2)]
G_{6a}=[a_5=ϕ(a_0,a_3)]
G_{6b}=[b_5=ϕ(b_3,b_4)]
G_{7a}=[c_2=*a_3,
d_2=*b_3]
G_{7b}=[d_3=*b_5]
G_{7c}=[c_3=*a_5,
e_3=*a_5]

Variable e_3 can use c_3's value and d_2 can use c_2's value.

Limitations of Global Value Numbering

As presented, our global value numbering technique doesn't recognize (or handle) computations of the same expression that produce different values along different paths.

Thus in

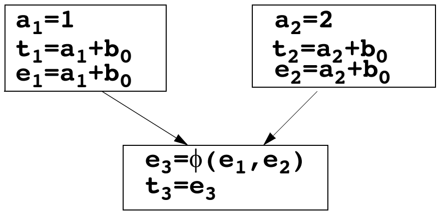

a_1=1
t_1=a_1+b_0

a_2=2
t_2=a_2+b_0

a_3=ϕ(a_1,a_2)
t_3=a_3+b_0

variable `a`$_3$ isn't equivalent to either `a`$_1$ or `a`$_2$.

But,
we can still remove a redundant computation of `a+b` by moving the computation of `t`$_3$ to each of its predecessors:

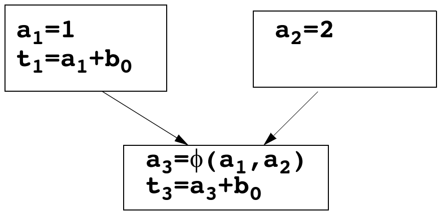

a_1=1
t_1=a_1+b_0
e_1=a_1+b_0

a_2=2
t_2=a_2+b_0
e_2=a_2+b_0

e_3=ϕ(e_1,e_2)
t_3=e_3

Now a redundant computation of `a+b` is evident in each predecessor block. Note too that this has a nice register targeting effect—`e`$_1$, `e`$_2$ and `e`$_3$ can be readily mapped to the same live range.

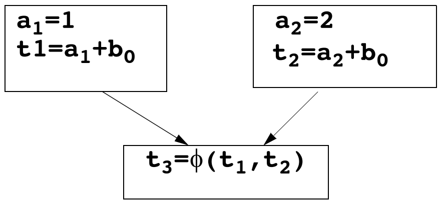
The notion of moving expression computations above phi functions also meshes nicely with notion of partial redundancy elimination. Given


a_1=1
t_1=a_1+b_0

a_2=2

a_3=ϕ(a_1,a_2)
t_3=a_3+b_0

moving `a+b` above the phi produces


a_1=1
t1=a_1+b_0

a_2=2
t_2=a_2+b_0

t_3=ϕ(t_1,t_2)

Now `a+b` is computed only once on each path, an improvement.

Reading Assignment

- **Read "Pointer Analysis," by Susan Horwitz.
(Linked from the class Web page.)**

Points-To Analysis

All compiler analyses and optimizations are limited by the potential effects of assignments through pointers and references.

Thus in C:

```
b = 1;
*p = 0;
print(b);
```

is 1 or 0 printed?

Similarly, in Java:

```
a[1] = 1;
b[1] = 0;
print(a[1]);
```

is 1 or 0 printed?

Points-to analysis aims to determine what variables or heap objects a pointer or reference may access.

To simplify points-to analysis, a number of reasonable assumptions are commonly made:

- Points to analysis is usually flow-insensitive. We don't analyze flow of control within a subprogram, but rather gather points-to information for the subprogram as a whole. Thus in

```
if (b)
     p = &a;
else p = &c;
```

we conclude `p` may point to either `a` or `c`.

- Points to analysis is usually context-insensitive (with respect to calls). This means individual call sites for the same subprogram are not differentiated.
  Therefore in

```
 *int echo (*int r) {
   return r; }
p = echo (&a);
q = echo (&b);
```

we determine that `r` may point to either `a` or `b` and therefore `p` can point to either `a` or `b`.

- Heap objects are named by the call site at which they are created. In:

```
p = new int; //Site 1
q = new int; //Site2
```

we know **`p`** and **`q`** can't interfere since each refers to distinct call site.

- Aggregates (arrays, structs, classes) are collapsed. Pointers or references to individual components are not distinguished. Given

```
p = &a[1];
 q = &a[2];
```

pointers **`p`** and **`q`** are assumed to interfere.

Similarly in

```
p = Obj.a;
 q = Obj.b;
```

pointers **`p`** and **`q`** are assumed to interfere.

- **Complex pointer expressions are assumed to be simplified prior to points-to analysis.**
  For example,

```
**p = 1;
```

is transformed into

```
temp = *p;
*temp = 1;
```

Points-To Representation

There are several ways to represent points-to information. We will use a points-to graph, which is concise and easy to understand.

Nodes are pointer variables and "pointed to" locations.

An arc connects a pointer to a location it may potentially reference.

Given

```
p = &a
```

we create:

$$\begin{tikzcd} p \arrow[r] & a \end{tikzcd}$$

Therefore in

$$\begin{tikzcd} p \arrow[r] \arrow[dr] & a \\ q \arrow[r] \arrow[dr] & b \\ r \arrow[r] & c \end{tikzcd}$$

we see `p` and `q` may both point to `b`, but `p` and `r` can't interfere (since their points-to sets are disjoint).

Simple Point-To Information

A primitive points-to analysis can be done using type or "address taken" information.

In a type-safe language like Java, a reference to type `T` can only point to objects of type `T` (or a subtype of `T`).

Given

```
ref1 = new Integer();
ref2 = new Float();
```

we trivially know `ref1` and `ref2` can't interfere.

Similarly, in C no pointer can access a variable `v` unless its address is taken (using the `&` operator). With very little effort we can limit the points-to sets of pointer `p` to only those variables of the correct type (excluding casting) whose address has been explicitly taken.

In practice both of these observations are too broad to be of much use.

Andersen's Algorithm

An algorithm to build a points-to graph for a C program is presented in:

"Program Analysis and Specialization for the C programming Language," L.O. Andersen, 1994.

The algorithm examines statements that create pointers, one by one, in textual order (the algorithm is flow-insensitive). Each statement updates the points-to graph if it can create new points-to relationships.

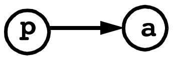
Six kinds of statements are considered:

- `p = &a;`
- `p = q;`
- `p = *r;`
- `*p = &a;`
- `*p = q;`
- `*p = *r;`

We will detail the points-to graph updates each of the statements induces.

1. `p = &a;`

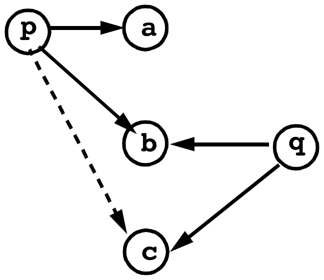
   We add an arc from `p` to `a`, showing `p` can possibly point to `a`:



2. `p = q;`

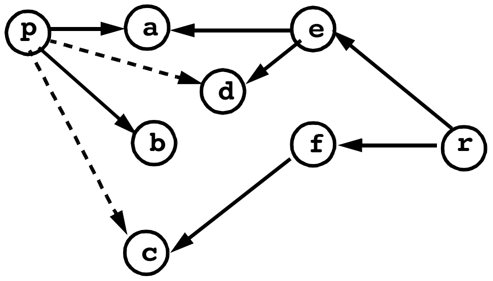
   We add arcs from `p` to everything `q` points to. If new arcs from `q` are later added, corresponding arcs from `p` must also be added (this implies an iterative or worklist algorithm).

   For example (the dashed arc is newly added):



3. `p = *r;`

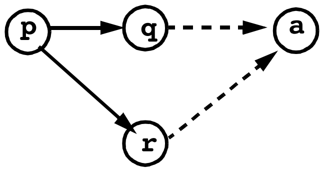
Let S be all the nodes `r` points to. Let T be all the nodes members of S point to. We add arcs from `p` to all nodes in T. If later pointer assignments increase S or T, new arcs from `p` must also be added (this again implies an iterative or worklist algorithm).
For example (dashed arcs are newly added):



4. `*p = &a;`

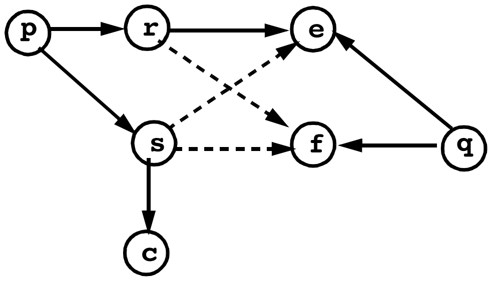
Add an arc to `a` from all nodes `p` points to. If new arcs from `p` are later added, new arcs to `a` must be added (this implies an iterative or worklist algorithm).

For example (dashed arcs are newly added):



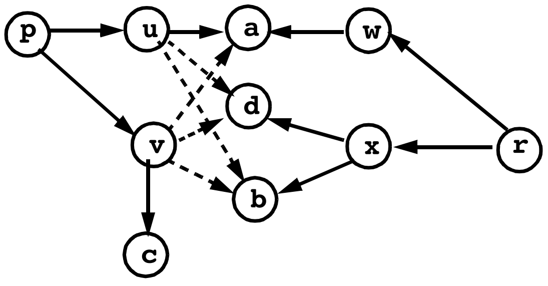
5. `*p = q;`

Nodes pointed to by `p` must be linked to all nodes pointed to by `q`. If later pointer assignments add arcs from `p` or `q`, this assignment must be revisited (this again implies an iterative or worklist algorithm).
For example (dashed arcs are newly added):



6. `*p = *r;`

Let S be all the nodes `r` points to. Let T be all the nodes members of S point to. We add arcs from all nodes `p` points to to all nodes in T. If later pointer assignments increase S or T or link new nodes to `p`, this assignment must be revisited (this again implies an iterative or worklist algorithm).
For example (dashed arcs are newly added):



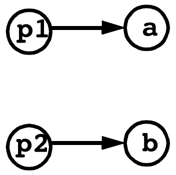
Example

Consider the following pointer manipulations:

```
p1 = &a;
p2 = &b;
p1 = p2;
r = &p1;
*r = &c;
p3 = *r;
p2 = &d;
```

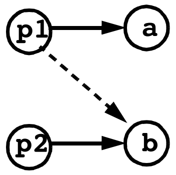
We start with:

```
p1 = &a;
p2 = &b;
```



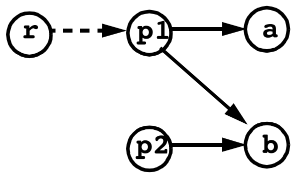
Next:

```
p1 = p2;
```



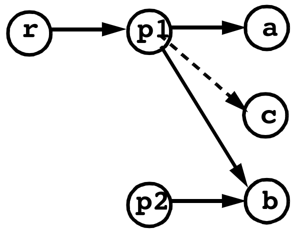
Then:

```
r = &p1;
```



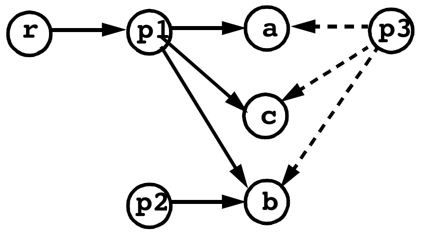
Next:

```
*r = &c;
```



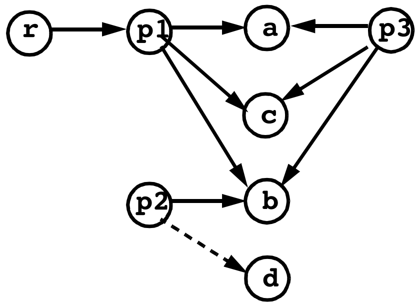
Then:

```
p3 = *r;
```

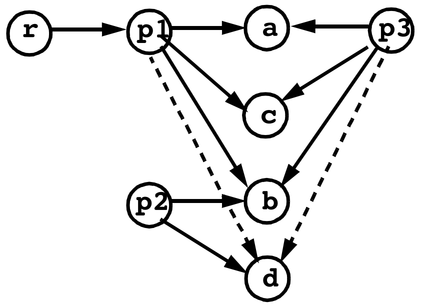


Finally:

```
p2 = &d;
```



But we aren’t quite done yet. This algorithm is flow-insensitive, so we must consider other execution orders (and iterative re-execution). If we make another pass through the assignments, we see that the final assignment to `p2` can flow to `p1`, and then to `p3` through `r`:



This points-to graph is rather dense, but it does capture all the ways pointer values might propagate through the various pointer assignments.

Calls are handled by treating pointer parameters and pointer returns as assignments, done at the points of call and return. Subprogram bodies are effectively inlined to capture the points-to relations they induce.

Given

```
*int echo (*int r) {
  return r; }
p = echo (&a);
```

we see the implicit assignments

```
r = &a;
p = r;
```

and add the following points-to information:

$$\begin{tikzcd} p \arrow[r] & a & r \arrow[l] \end{tikzcd}$$

As an optimization, libraries can be pre-analyzed to determine the points-to relations they induce. Most may use (read) pointers but don't create any new points-to relations visible outside their bodies. Call to such library routines can be ignored as far as the caller's points-to graph is concerned.

Performance of Andersen's Algorithm

Experience has shown that Andersen's Algorithm gives useful points-to data and is far superior to the naive address-taken approach.

Interestingly, experiments show that making the technique flow-sensitive or calling context-sensitive doesn't improve results very much on typical benchmarks.

But execution time for moderate to large programs can be a problem.

Careful analysis shows that Andersen's Algorithm can require $O(n^3)$ time (where n is the number of nodes in the points-to graph).

The reason for this larger-than-expected analysis time is that a statement like

```
  p = *q;
```

can force the algorithm to visit n^2 nodes (`q` may point to n nodes and each of these nodes may point to n nodes). The number of pointer statements analyzed can be O(n), leading to an $O(n^3)$ execution time.

Steensgaard’s Algorithm

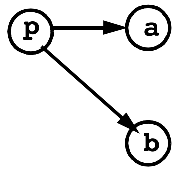
It would be useful to have a reasonably accurate points-to analysis that runs in essentially linear time so that really large programs could be handled.

This is what Steensgaard’s Algorithm offers.

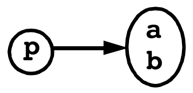
(Points-to Analysis in Almost Linear Time, B. Steensgaard, 1996 Principles of Programming Languages Conference.)

Steensgaard’s Algorithm is essentially Andersen’s Algorithm, simplified by merging nodes `a` and `b` if any pointer can reference both.

That is, in Andersen’s Algorithm we might have

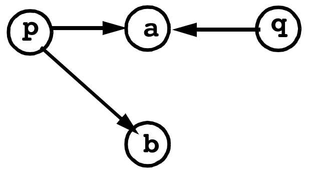


In Steensgaard’s Algorithm we would instead have

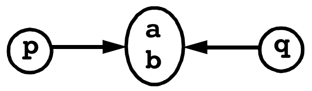


In effect any two locations that might be pointed to by the same pointer are placed in a single equivalence class.

Steensgaard’s Algorithm is sometimes less accurate than Andersen’s Algorithm. For example, the following points-to graph, created by Andersen’s Algorithm, shows that `p` may point to `a` or `b` whereas `q` may only point to `a`:



In Steensgaard’s Algorithm we get



incorrectly showing that if `p` may point to `a` or `b` then so may `q`.

But now statements like

```
p = *q;
```

can’t force the algorithm to visit n^2 nodes, because multiple nodes referenced by the same pointer are always merged. Using the fast union-find algorithm, we can get an execution time of O(n α(n)) which is essentially linear in n. Now very large programs can be analyzed, and without too much of a loss in precision.

Andersen vs. Steensgaard in Practice

- Horwitz and Shapiro examined 61 C programs, ranging in size from 300 to 24,300 lines.
- As expected, Steensgaard is less precise: On average points-to sets are 4 times bigger; at worst 15 times bigger.
- As expected, Andersen is slower. On average 1.5 times slower: at worst 31 times slower.
- Both are much better than the naive “address taken” approach.
- Bottom line: Use Andersen for small programs, use Steensgaard (or something else) for large programs.

Reading Assignment

- Read "Fast and Accurate Flow-Insensitive Points-To Analysis,” by Shapiro and Horwitz.
  (Linked from the class Web page.)

The Horwitz-Shapiro Approach

It would be nice to have a points-to analysis that is parameterizable, ranging between the accuracy of Andersen and the speed of Steensgaard.

Horwitz and Shapiro (Fast and Accurate Flow-Insensitive Points-To Analysis, 1997 Principles of Programming Languages Conference) present a technique intermediate to those proposed by Andersen and Steensgaard.

Horwitz and Shapiro suggest each node in the points-to graph be limited to out degree k, where $1 \leq k \leq n$.

If k =1 then they have Steensgaard's approach.

If k =n (n is number of nodes in points to graph), then they have Andersen's approach.

Their worst case run-time is

$O(k^2 n)$, which is not much worse than Steensgaard if k is kept reasonably small.

To use their approach assign each variable that may be pointed to to one of k categories.

Now if `p` may point to `x` and `p` may also point to `y`, we merge `x` and `y` only if they both are in the same category.

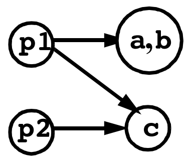
If `x` and `y` are in different categories, they aren't merged, leading to more accurate points-to estimates.

Example

```
p1 = &a;
p1 = &b;
p1 = &c;
p2 = &c;
```

Say we have k = 2 and place `a` and `b` in category 1 and `c` in category 2.

We then build:

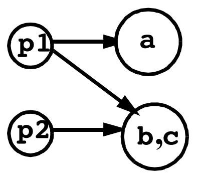


This points-to graph is just as accurate as that built by Andersen's approach.

But...

What if we chose to place `a` in category 1 and `b` and `c` in category 2.

We now have:



This graph is inexact, since it tells us `p2` may point to `b`, which is false.

(Steensgaard would have been worse still, incorrectly telling us `p2` may point to `a` as well as `b` and `c`).

Another Good Idea

What if we ran Shapiro and Horwitz's points-to analysis twice, each with different category assignments?

Each run may produce a different points-to graph. One may say `p2` points to `b` whereas the other says it does not.

Which do we believe?

Neither analysis misses a genuine points-to relation. Rather, merging of nodes sometimes creates false points-to information.

So we will believe `p2` may point to `b` only if all runs say so. This means multiple runs may "filter out" false points-to relations due to merging.

How Many Runs are Needed?

How are Categories to be Set?

We want to assign categories so that during at least one run, any pair of pointed-to variables are in different categories.

This guarantees that if all the runs tell us `p` may point to `a` and `b`, it is not just because `a` and `b` always happened to be assigned the same category.

To force different category assignments for each pair of variables, we assign each pointed-to variable an index and write that index in base k (the number of categories chosen).

For example, if we had variables `a`, `b`, `c` and `d`, and chose k = 2, we'd use the following binary indices:

`a`	00
`b`	01
`c`	10
`d`	11

Note that the number of base k digits needed to represent indices from 0 to n-1 is just ceiling($\log_k$ n).

This number is just the number of runs we need!

Why?

In the first run, we'll use the right most digit in a variable's index as its category.

In the next run, we'll use the second digit from the right, then the third digit from the right, ...

Any two distinct variables have different index values, so they must differ in at least digit position.

Returning to our example,

`a`	**00**
`b`	**01**
`c`	**10**
`d`	**11**

On run #1 we give `a` and `c` category 0 and `b` and `d` category 1.

On run #2, `a` and `b` get category 0 and `c` and `d` get category 1.

So using just 2 runs in this simple case, we eliminate much of the inaccuracy Steensgaard's merging introduces.

Run time is now $O(\log_k(n)\ k^2\ n)$.

How Well does this Approach Work?

On 25 tests, using 3 categories, Horwitz & Shapiro points-to sets on average are 2.67 larger than those of Andersen (Steensgaard's are 4.75 larger).

This approach is slower than Steensgaard but on larger programs it is 7 to 25 times faster than Andersen.

How Well do Points-to Analyses Work in Real Data Flow Problems?

In "Which Pointer Analysis Should I Use," Hind and Pioli survey the effectiveness of a number of points-to analyses in actual data flow analyses (mod/ref, liveness, reaching defs, interprocedural constant propagation).

Their conclusions are essentially the same across all these analyses:

- **Steensgaard's analysis is significantly more precise than address-taken analysis and not significantly slower.**
- **Andersen's analysis produces modest, but consistent,**

improvements over Steensgaard's analysis.

- **Both context-sensitive points-to analysis and flow-sensitive points-to analysis give little improvement over Andersen's analysis.**