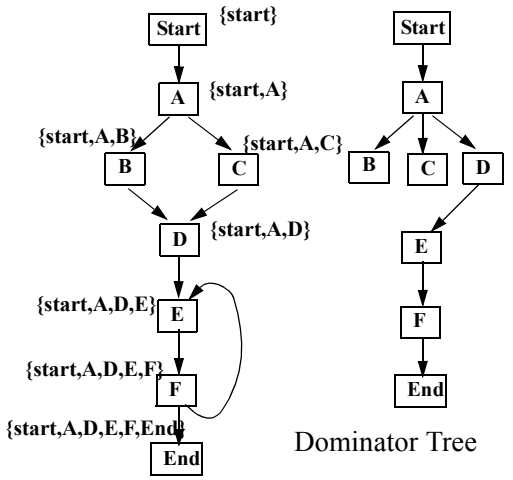


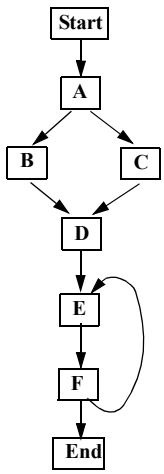
Control Flow Graph

Dominator Tree

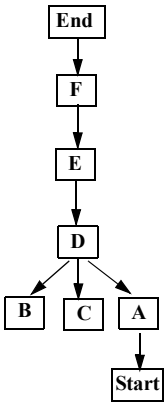
Postdominance

A block Z *postdominates* a block Y (Z pdom Y) if and only if all paths from Y to an exit block must pass through Z. Notions of immediate postdominance and a postdominator tree carry over.

Note that if a CFG has a single exit node, then postdominance is equivalent to dominance if flow is reversed (going from the exit node to the start node).



Control Flow Graph



Postdominator Tree

Reading Assignment

- **Section 14.5 - 14.7 of CaC**
- **Pages 31 - 63 of "Automatic Program Optimization"**
- **Assignment 2**

Dominance Frontiers

Dominators and postdominators tell us which basic block must be executed prior to, of after, a block N.

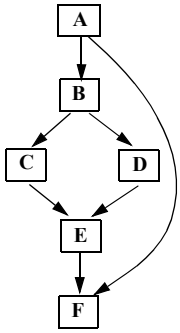
It is interesting to consider blocks "just before" or "just after" blocks we're dominated by, or blocks we dominate.

The Dominance Frontier of a basic block N, DF(N), is the set of all blocks that are immediate successors to blocks dominated by N, but which aren't themselves strictly dominated by N.

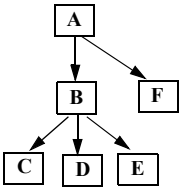
DF(N) =
{Z | M→Z & (N dom M) &
¬(N sdom Z)}

The dominance frontier of N is the set of blocks that are not dominated N and which are "first reached" on paths from N.

Example



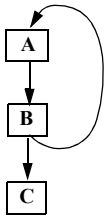
Control Flow Graph



Dominator Tree

Block	A	B	C	D	E	F
Domi-nance Frontier	ϕ	{F}	{E}	{E}	{F}	ϕ

A block can be in its own Dominance Frontier:



Here, DF(A) = {A}

Why? Reconsider the definition:

DF(N) =
{Z | M→Z & (N dom M) &
¬(N sdom Z)}

Now B is dominated by A and B→A.

Moreover, A does not *strictly* **dominate itself. So, it meets the definition.**

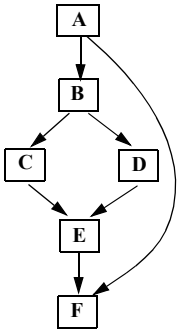
Postdominance Frontiers

The Postdominance Frontier of a basic block N, PDF(N), is the set of all blocks that are immediate predecessors to blocks postdominated by N, but which aren't themselves postdominated by N.

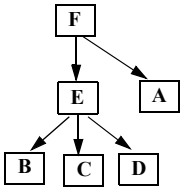
PDF(N) =
{Z | Z→M & (N pdom M) &
¬(N pdom Z)}

The postdominance frontier of N is the set of blocks closest to N where a choice was made of whether to reach N or not.

Example



Control Flow Graph



Postominator Tree

Block	A	B	C	D	E	F
Postdominance Frontier	ϕ	{A}	{B}	{B}	{A}	ϕ

Control Dependence

Since CFGs model flow of control, it is useful to identify those basic blocks whose execution is controlled by a branch decision made by a predecessor.

We say Y is *control dependent* **on X if, reaching X, choosing one out arc will force Y to be reached, while choosing another arc out of X allows Y to be avoided.**

Formally, Y is control dependent on X if and only if,

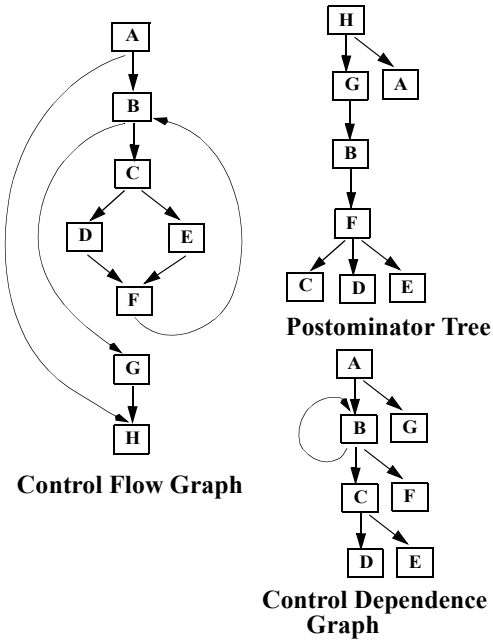
(a) Y postdominates a successor of X.

(b) Y does not postdominate all successors of X.

X is the most recent block where a choice was made to reach Y or not.

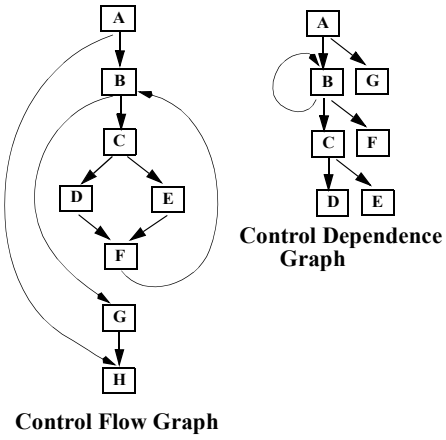
Control Dependence Graph

We can build a *Control Dependence Graph* that shows (in graphical form) all Control Dependence relations.
(A Block *can be* Control Dependent on itself.)



What happened to H in the CD Graph?

Let's reconsider the CD Graph:



Blocks C and F, as well as D and E, seem to have the same control dependence relations with their parent. But this isn't so!

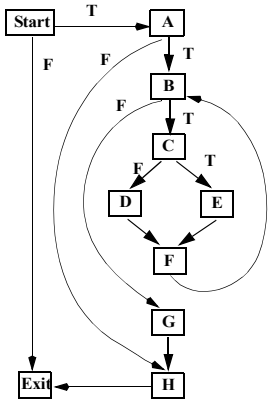
C and F *are* control equivalent, but D and E are *mutually exclusive*!

Improving the Representation of Control Dependence

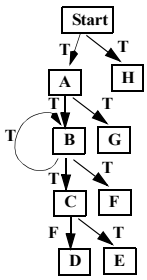
We can label arcs in the CFG and the CD Graph with the condition (T or F or some switch value) that caused the arc to be selected for execution.

This labeling then shows the conditions that lead to the execution of a given block.

To allow the exit block to appear in the CD Graph, we can also add "artificial" start and exit blocks, linked together.



Control Flow Graph



Control Dependence Graph

C and F have the same Control Dependence relations. They are part of the same extended basic block.

But D and E aren't identically control dependent. A and H are control equivalent, as are B and G.

Data Flow Frameworks Revisited

Recall that a Data Flow problem is characterized as:

(a) A Control Flow Graph

(b) A Lattice of Data Flow values

(c) A Meet operator to join solutions from Predecessors or Successors

(d) A Transfer Function
$Out = f_b(In)$ or $In = f_b(Out)$

Value Lattice

The lattice of values is usually a *meet semilattice* defined by:

A: a set of values

T and $\perp$ ("top" and "bottom"): distinguished values in the lattice

$\leq$: A reflexive partial order relating values in the lattice

$\wedge$: An associative and commutative meet operator on lattice values

Lattice Axioms

The following axioms apply to the lattice defined by A, T, $\perp$, $\leq$ and $\wedge$:

$a \leq b \Leftrightarrow a \wedge b = a$

$a \wedge a = a$

$(a \wedge b) \leq a$

$(a \wedge b) \leq b$

$(a \wedge T) = a$

$(a \wedge \perp) = \perp$

Monotone Transfer Function

Transfer Functions, f_b:L $\rightarrow$ L (where L is the Data Flow Lattice) are normally required to be monotone.

That is $x \leq y \Rightarrow f_b(x) \leq f_b(y)$.

This rule states that a "worse" input can't produce a "better" output.

Monotone transfer functions allow us to guarantee that data flow solutions are stable.

If we had $f_b(T) = \perp$ and $f_b(\perp)=T$, then solutions might oscillate between T and $\perp$ indefinitely.

Since $\perp \leq T$, $f_b(\perp)$ should be $\leq f_b(T)$. But $f_b(\perp) = T$ which is not $\leq f_b(T) = \perp$. Thus f_b isn't monotone.

Dominators fit the Data Flow Framework

Given a set of Basic Blocks, N, we have:

A is 2^N (all subsets of Basic Blocks).

T is N.

$\perp$ is ϕ.

$a \leq b \equiv a \subseteq b$.

$f_Z(in) = In \cup \{Z\}$

$\wedge$ is $\cap$ (set intersection).

The required axioms are satisfied:

$a \subseteq b \Leftrightarrow a \cap b = a$

$a \cap a = a$

$(a \cap b) \subseteq a$

$(a \cap b) \subseteq b$

$(a \cap N) = a$

$(a \cap \phi) = \phi$

Also f_Z is monotone since

$a \subseteq b \Rightarrow a \cup \{Z\} \subseteq b \cup \{Z\} \Rightarrow f_Z(a) \subseteq f_Z(b)$