Reading Assignment

- Section 14.5 - 14.7 of CaC
- Pages 31 - 63 of "Automatic Program Optimization"
- Assignment 2

Dominance Frontiers

Dominators and postdominators tell us which basic block must be executed prior to, of after, a block N.

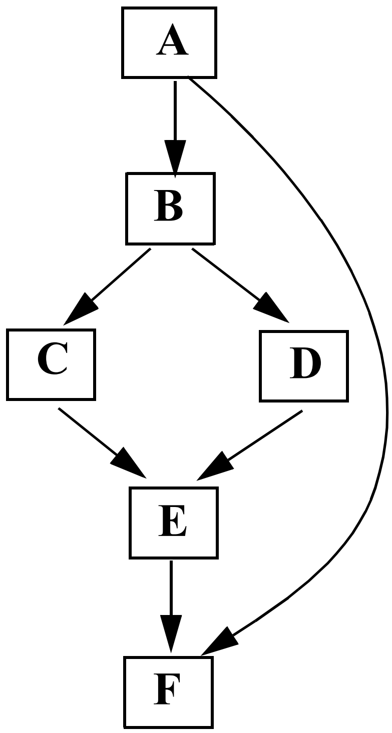
It is interesting to consider blocks "just before" or "just after" blocks we're dominated by, or blocks we dominate.

The Dominance Frontier of a basic block N, DF(N), is the set of all blocks that are immediate successors to blocks dominated by N, but which aren't themselves strictly dominated by N.

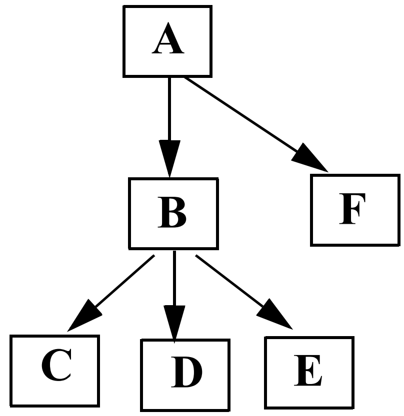
DF(N) =
{Z | M$\rightarrow$Z & **(N dom M)** &
$\neg$(N sdom Z)}

The dominance frontier of N is the set of blocks that are not dominated N and which are "first reached" on paths from N.

Example



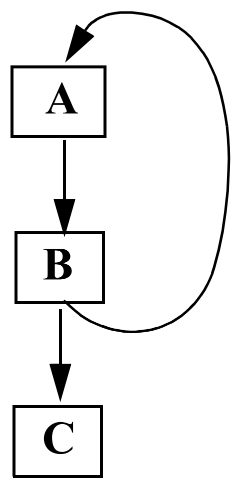
Control Flow Graph



Dominator Tree

Block	**A**	**B**	**C**	**D**	**E**	**F**
Domi-nance Frontier	ϕ	**{F}**	**{E}**	**{E}**	**{F}**	ϕ

A block can be in its own Dominance Frontier:



Here, DF(A) = {A}

Why? Reconsider the definition:

DF(N) =
$\{Z \mid M \rightarrow Z \ \& \ (N \text{ dom } M) \ \& \ \neg(N \text{ sdom } Z)\}$

Now B is dominated by A and B→A.

Moreover, A does not *strictly* dominate itself. So, it meets the definition.

Postdominance Frontiers

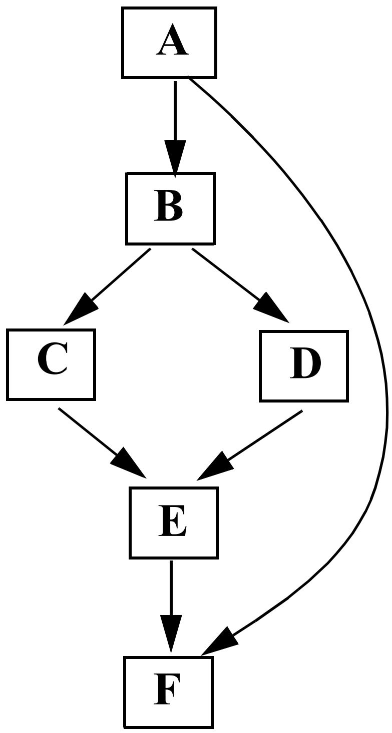
The Postdominance Frontier of a basic block N, PDF(N), is the set of all blocks that are immediate predecessors to blocks postdominated by N, but which aren’t themselves postdominated by N.

PDF(N) =

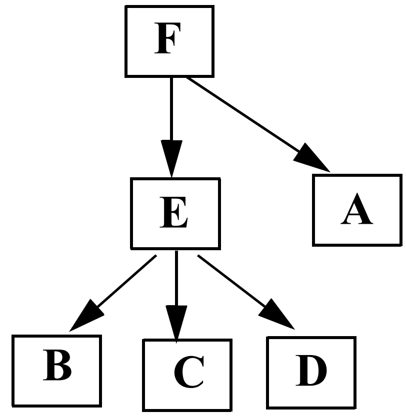
$\{Z \mid Z \rightarrow M \ \& \ (N \text{ pdom } M) \ \& \ \neg(N \text{ pdom } Z)\}$

The postdominance frontier of N is the set of blocks closest to N where a choice was made of whether to reach N or not.

Example



Control Flow Graph



Postominator Tree

Block	A	B	C	D	E	F
Postdomi-nance Frontier	ϕ	{A}	{B}	{B}	{A}	ϕ

Control Dependence

Since CFGs model flow of control, it is useful to identify those basic blocks whose execution is controlled by a branch decision made by a predecessor.

We say Y is *control dependent* on X if, reaching X, choosing one out arc will force Y to be reached, while choosing another arc out of X allows Y to be avoided.

Formally, Y is control dependent on X if and only if,

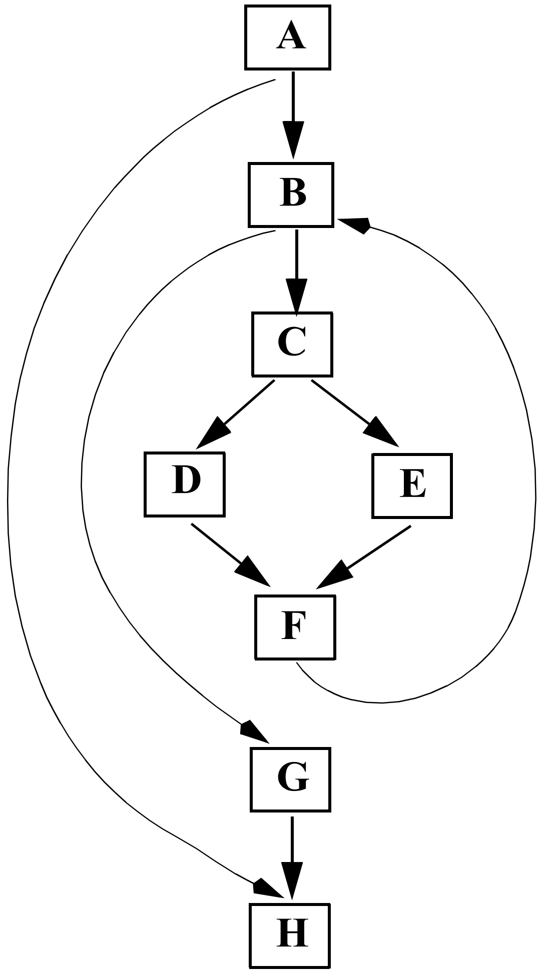
(a) Y postdominates a successor of X.

(b) Y does not postdominate all successors of X.

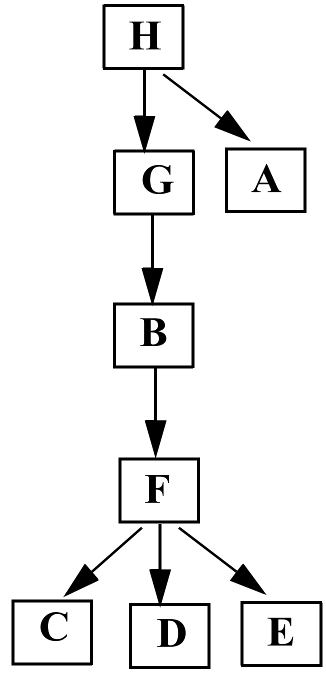
X is the most recent block where a choice was made to reach Y or not.

Control Dependence Graph

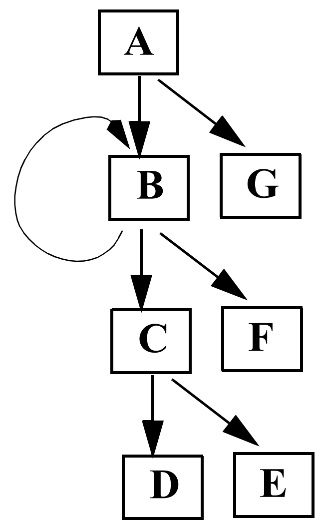
We can build a *Control Dependence Graph* **that shows (in graphical form) all Control Dependence relations.**
(A Block *can be* **Control Dependent on itself.)**



Control Flow Graph



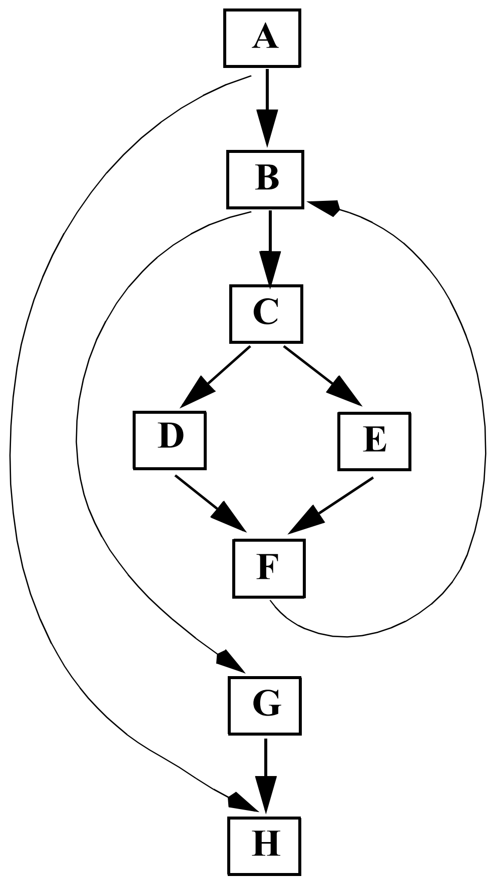
Postominator Tree



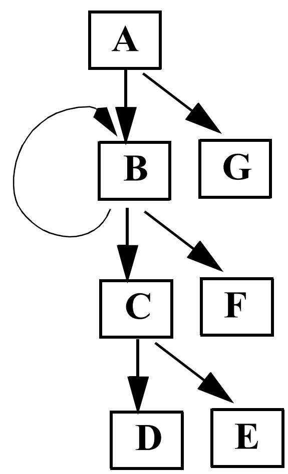
Control Dependence Graph

What happened to H in the CD Graph?

Let’s reconsider the CD Graph:



Control Flow Graph



Control Dependence Graph

Blocks C and F, as well as D and E, seem to have the same control dependence relations with their parent. But this isn’t so!

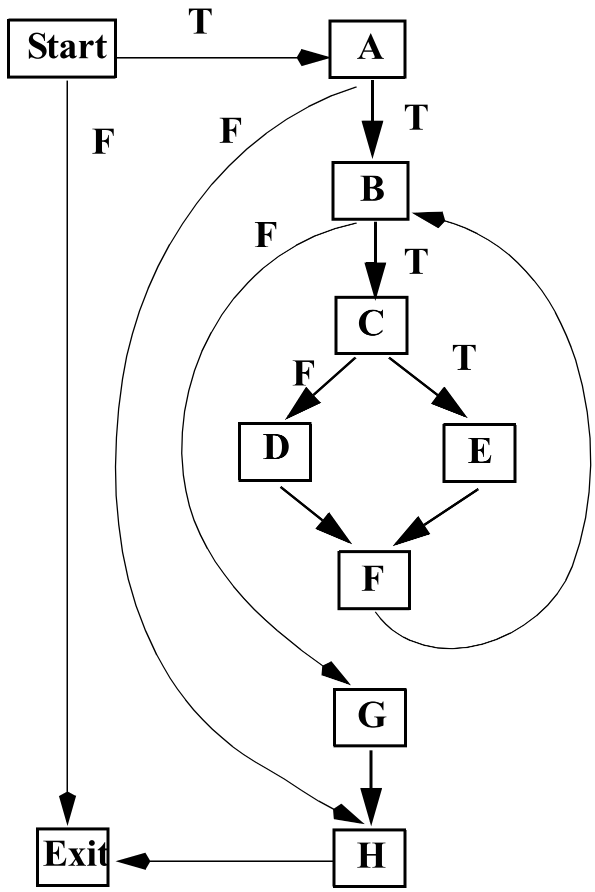
C and F *are* **control equivalent, but D and E are** *mutually exclusive*!

Improving the Representation of Control Dependence

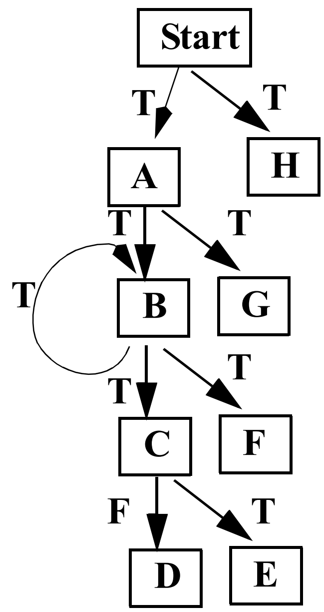
We can label arcs in the CFG and the CD Graph with the condition (T or F or some switch value) that caused the arc to be selected for execution.

This labeling then shows the conditions that lead to the execution of a given block.

To allow the exit block to appear in the CD Graph, we can also add "artificial" start and exit blocks, linked together.



Control Flow Graph



Control Dependence Graph

C and F have the same Control Dependence relations. They are part of the same extended basic block.

But D and E aren’t identically control dependent. A and H are control equivalent, as are B and G.

Data Flow Frameworks Revisited

Recall that a Data Flow problem is characterized as:

(a) A Control Flow Graph

(b) A Lattice of Data Flow values

(c) A Meet operator to join solutions from Predecessors or Successors

(d) A Transfer Function
$Out = f_b(In)$ or $In = f_b(Out)$

Value Lattice

The lattice of values is usually a *meet semilattice* **defined by:**

A: a set of values

T and $\perp$ ("top" and "bottom"): distinguished values in the lattice

$\leq$: **A reflexive partial order relating values in the lattice**

$\wedge$: **An associative and commutative meet operator on lattice values**

Lattice Axioms

The following axioms apply to the lattice defined by A, T, $\perp$, $\leq$ and $\wedge$:

$$a \leq b \Leftrightarrow a \wedge b = a$$

$$a \wedge a = a$$

$$(a \wedge b) \leq a$$

$$(a \wedge b) \leq b$$

$$(a \wedge T) = a$$

$$(a \wedge \perp) = \perp$$

Monotone Transfer Function

Transfer Functions, f_b:L $\rightarrow$ L (where L is the Data Flow Lattice) are normally required to be monotone.

That is $x \le y \Rightarrow f_b(x) \le f_b(y)$.

This rule states that a "worse" input can't produce a "better" output.

Monotone transfer functions allow us to guarantee that data flow solutions are stable.

If we had $f_b(T) = \perp$ and $f_b(\perp)=T$, then solutions might oscillate between T and $\perp$ indefinitely.

Since $\perp \le T$, $f_b(\perp)$ should be $\le f_b(T)$. But $f_b(\perp) = T$ which is not $\le f_b(T) = \perp$. Thus f_b isn't monotone.

Dominators fit the Data Flow Framework

Given a set of Basic Blocks, N, we have:

A is 2^N (all subsets of Basic Blocks).

T is N.

$\perp$ is ϕ.

$a \le b \equiv a \subseteq b$.

$f_Z(in) = In \cup \{Z\}$

$\wedge$ is $\cap$ (set intersection).

The required axioms are satisfied:

$$a \subseteq b \Leftrightarrow a \cap b = a$$

$$a \cap a = a$$

$$(a \cap b) \subseteq a$$

$$(a \cap b) \subseteq b$$

$$(a \cap N) = a$$

$$(a \cap \phi) = \phi$$

Also f_Z is monotone since

$$a \subseteq b \Rightarrow a \cup \{Z\} \subseteq b \cup \{Z\} \Rightarrow f_Z(a) \subseteq f_Z(b)$$