OPTION VALUATION OF CLAIMS ON REAL ASSETS: THE CASE OF OFFSHORE PETROLEUM LEASES*

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This paper extends financial option theory by developing a methodology for the valuation of claims on a real asset: an offshore petroleum lease. Several theoretical and practical problems, not present in applying option pricing theory to financial assets, are addressed. Most importantly, we show the necessity of combining option pricing techniques with a model of equilibrium in the market for the underlying asset (petroleum reserves). The advantages of this approach over conventional discounted cash flow techniques are emphasized. The methodological development provides important insights for both company behavior and government policy. Promising empirical results are reported.

I. INTRODUCTION

One of the most fruitful areas of research in financial economics has been the development of the theory of valuing stock options. Since the seminal work of Black and Scholes [1973] and Merton [1973] appeared, many papers have used this method of analysis to value other financial assets with "option-like" characteristics. See Smith [1976] for a survey of this literature.

More recently, it has been observed that there are contractual claims to real assets that also display option-like characteristics, which suggests that the Black-Scholes-Merton analysis might be useful in valuing such claims. (See Brealey and Myers [1984] and Mason and Merton [1985].) Examples include the following: McDonald and Siegel [1985], who study project valuation where the firm has the option to shut down production; McDonald and Siegel [1986] and Myers and Majd [1983], who study the valuation of investment and scrapping opportunities; and Brennan and Schwartz [1985], who study natural resource investments.

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This paper uses option valuation theory to develop a new approach to valuing leases for offshore petroleum. Our treatment makes several contributions to the literature on valuing "real options." First, we demonstrate how to integrate an explicit model of equilibrium in the market for the underlying real asset (developed petroleum reserves) with option-pricing theory to derive the value of a real option. The necessity of this type of integration follows from McDonald and Siegel's [1984] point that valuing real options may require a deeper understanding of equilibrium in the market for the underlying asset than valuing options on financial assets. Second, by using oil leases as an example, we specify a valuation problem in sufficient detail to allow close examination of the many theoretical and practical issues involved in extending financial option valuation theory to real options. Finally, the detail of the valuation problem allows us to consider informational and computational economies of the option valuation methodology relative to conventionally applied discounted cash flow techniques. In particular, we show the efficient use made by option valuation of market data, which mitigates the need to use (among other things) expected future commodity prices or risk-adjusted discount rates.

The valuation of offshore leases is an important issue in itself. Firms perform valuations as inputs to their bidding decisions. The government uses valuations to establish presale reservation prices and to study the effect of policy changes on revenues it expects to receive from lease sales. Because the bidding process involves billions of dollars, it is important to obtain accurate valuations. Government valuations have tended to underestimate industry bids. Using the same geological and cost data as the government, our option valuations are closer to industry bids.

Embedded in any approach to valuing petroleum leases is a rule specifying when and if a firm should explore and develop a particular leased property (i.e., exercise its options). Deriving the optimal rule is often difficult, especially using conventional discounted cash flow techniques. The option valuation approach we develop, however, leads to a straightforward form for this rule, which depends only upon observable variables. Using this analysis, we are able to study the effects of exploration and development costs and lags, and relinquishment requirements[1] on exploration

1. Relinquishment requirements put a limit on the time a firm can hold a petroleum tract before exploring and developing it.

and development investment-timing decisions. We also demonstrate empirically the form of the decision rule.

The paper is organized as follows. Section II discusses relevant economic and technological characteristics of offshore petroleum leases. Section III discusses discounted cash flow valuation techniques as currently used by firms and the government, and points out their weaknesses. Section IV develops the option valuation approach, along with the investment-timing rule. Section V presents empirical results drawn from a sample of offshore petroleum leases. We compare valuations based upon the discounted cash flow approach (from government calculations) and the option valuation approach (using government cost and geological data) with actual industry bids. Finally, we explore the empirical effect of lessor policy and economic variables on tract value, and characteristics of the investment-timing rule. Section VI summarizes and discusses extensions of this research.

II. Economic and Technological Characteristics of Offshore Petroleum Leases

The holder of an offshore petroleum lease must pass through three stages before he can obtain hydrocarbons above the ground: exploration, development, and extraction. The exploration stage involves seismic and drilling activity to obtain information on the quantities of hydrocarbon reserves present in the tract, as well as the costs of bringing them out. If the exploration results are favorable, the firm may then proceed to the development stage, which involves putting the equipment in place to extract the oil: for example, constructing platforms and drilling production wells. The development expenditures convert *undeveloped* reserves into *developed* reserves; the latter are defined as reserves with productive capacity. The government subjects the leaseholder to relinquishment requirements that dictate how long a company can wait before beginning exploration and development. Often, holders of leases will relinquish the lease by deciding not to explore or develop a tract before the lease runs out. Thus both the exploration and development stages represent options of the leaseholder. If he does explore and develop the tract, the extraction stage involves using the installed capacity to take the hydrocarbons out of the ground. The proper valuation of a petroleum lease involves valuing the cash flows from this multistage process.

III. Tract Valuation by the Discounted Cash Flow Approach

In the discounted cash flow (DCF) approach, expected future cash flows to leaseholders are determined, discounted to the present, and summed to yield the lease value. To determine expected cash flows and proper discount rates, it is first necessary to specify statistical distributions (not necessarily independent) for exploration costs, quantities of hydrocarbon reserves, development costs, hydrocarbon prices, and operating costs. For each set of realizations from these distributions, an analyst must determine whether it is optimal for the firm to explore, develop, and extract. To complicate matters further, the analyst must also make assumptions about the *timing* of exploration and development, as well as the rate of extraction. Then, using the prices, costs, quantities, and timing decisions from a particular set of realizations, the time path of cash flows is determined. The path of expected cash flows is found by integrating over all possible sets of realizations from the statistical distributions. As typically applied, this DCF analysis involves multivariate Monte Carlo simulations. A set of risk-adjusted discount rates is derived in principle by determining the covariance of these respective cash flows with other assets in the economy and using a pricing model such as the Capital Asset Pricing Model.

The popularity of the DCF approach derives from its sound theoretical foundations. If the set of discounted cash flows is correctly determined, then the sum of these flows (net of the acquisition cost) yields the market value addition to the firm acquiring the lease. Performing these calculations correctly, however, is very difficult, and the DCF approach as applied has five major weaknesses that inhibit correct lease value determination.

1. The proper timing of exploration and development is not transparent. The choice of timing for the DCF calculations is therefore typically arbitrary and subject to error. This problem leads to valuations that are divergent between companies, the government, and the capital markets.

2. Different companies, as well as the government, may have different assessments of future statistical distributions, and thus expected paths, of hydrocarbon prices, none of which need conform to the aggregate expectations held by capital markets. This also leads to divergent valuations.

3. The process of choosing the correct set of risk-adjusted discount rates in the presence of the complex statistical structure of

the cash flows is a difficult task, which is also subject to a great deal of subjectivity and error. For example, the investment-timing rules used by the firm will affect the risk of the cash flows in complicated ways. Thus, the optimal investment-timing rule will need to take account of this relationship. Companies, as well as the government, often resort to simple rules of thumb such as "use 20 percent for the exploration phase and 10 percent thereafter." The choice of discount rates is crucial, however, because the DCF valuations are very sensitive to the rates chosen.

4. The DCF calculations, particularly Monte Carlo applications, are very complex and costly.

5. Because tract information is often relatively sparse at the bidding stage, the assessments of geological and cost distributions can vary, perhaps widely, across companies and the government. This also causes large discrepancies among respective valuations.

The next section develops the option valuation methodology and shows how it is not subject to the first four of these problems. Because it is purely a financial valuation tool, however, the problems associated with number 5 above remain.

IV. Tract Valuation by the Option Valuation Approach

As was discussed in Section II, valuing a lease involves valuing the cash flows from a three-stage process. These stages form a nested set of options and each of these stages has distinct characteristics relevant to the option valuation (OV) approach. By making plausible assumptions about underlying price processes, we shall be able to use option-pricing techniques to value this nested set of options.

A. Characteristics of the Stages

Exploration. The exploration stage consists of the option to make the exploration expenditures and to receive undeveloped reserves. This is analogous to a stock option, which confers the right to pay the exercise price and receive the stock. Just as a stock option has an expiration date, the leaseholder is subject to relinquishment requirements which stipulate that it must give up the lease if it does not explore and develop by a certain date. There are important differences, however, related to uncertainties in the exploration process.

The primary uncertainty surrounding the exploration stage is the quantity of hydrocarbons. This uncertainty is resolved by

exploring. Because offshore development costs are primarily driven by factors related to economies of scale, exploration also resolves uncertainty about development costs. We can represent the exploration stage as the option to spend the expected exploration costs $\bar{E}$, and receive the expected value of undeveloped reserves,

$$X^*(V) = \int QX(V,T\text{-}t;D(Q))dF(Q), \tag{1}$$

where

Q = random quantity (possibly zero) of recoverable hydrocarbons in the tract

$D(Q)$ = per unit development cost (in real dollars), a function of quantity[2]

V = *current* value of a unit of developed hydrocarbon reserves

$F(Q)$ = probability distribution over the quantity of hydrocarbons

$X(V,T\text{-}t;D(Q))$ = *current* per unit value of undeveloped reserves given the *current* per unit value of a developed reserve and per unit development cost

t = current date

T = expiration date.

We can represent the current value of the reserves obtained after exploration as the expectation over the value of the undeveloped reserves, because the quantity risk is almost entirely technological and geological. The risk is therefore nonsystematic and requires no risk premium. Thus, this "risk-neutral" technique is appropriate.[3] We assume here that exploration is instantaneous; later, we show how to relax this assumption.

Development. Once exploration has provided an indication of the quantity of hydrocarbons and the magnitude of development costs, the leaseholder has the option to pay the development costs and install productive capacity. Therefore, ownership of an undeveloped reserve is an option to obtain developed reserves by paying

2. As McDonald and Siegel [1986] discuss, $X(\cdot)$ is first degree homogeneous in V and D.

3. There are certainly systematic components to development cost uncertainty. Costs of steel, concrete, platform crews, ships, and other factors of production will all move somewhat systematically. However, these systematic sources of variability are quantitatively unimportant when compared with geological uncertainty and tract-specific cost factors. The only reason for a systematic component to geological risk is the unlikely event that a field could be large enough such that the realization of Q could affect all other assets in the economy in a perceptible way. Certainly, movement in other assets will not affect geological risk.

the development cost. This option has value $X(V,T\text{-}t;D(Q))$. As with exploration, we shall for now treat development expenditures as occurring in one instantaneous lump sum. We again relax this assumption below.[4]

Extraction. Once the leaseholder has exercised its development option, he owns developed reserves. He then has the option to extract the hydrocarbons if he chooses. Valuation of the developed reserves requires assumption about oil quality, future extraction rates and costs, tax and royalty regimes, and hydrocarbon prices. Fortunately, a firm can observe the value that competitive asset markets place on similar developed reserves. There are active secondary markets in properties containing developed reserves, so that a firm knows or can determine within a reasonable tolerance, the market value of a given quantity and type of developed reserves. This market value reflects the value of reserves with similar extraction rates and operating costs, quality of hydrocarbons and tax regime as for the tract being valued. Of these, extraction rates and operating costs for a particular tract are the most difficult to predict, ex ante. Fortunately, extraction rates and operating costs do not vary as much as exploration and development costs across tracts. For a given hydrocarbon quality and tax regime, this leads to a relative homogeneity in the market value of developed reserves. The option valuation technique we develop uses this market information about the value of developed reserves in an explicit and straightforward manner.[5] Our use of market values for developed reserves mitigates possible errors in explicitly modeling extraction, as in Brennan and Schwartz [1985].

4. See Adelman and Paddock [1980] for a discussion and justification of this "collapsing" technique. This sum is essentially the present value of development expenditures. There are two possible objections to this approach. First, the developed reserves are not obtained until after a lag equal to the development time. We shall explicitly account for this lag. Second, the firm has latitude over the speed and quantity of development, which it can vary as new information arrives. We discuss this problem in Section VI.

5. In current practice the DCF method does not typically use this market information. While this information could be incorporated into a DCF valuation, it would be much more difficult than in the OV approach. See below for a detailed discussion of this point.

Tourinho [1979] suggests looking at petroleum reserves as options. However, he lumps development and extraction costs together as the exercise price and considers the option to extract the petroleum. This formulation does not lead to a usable valuation scheme because it does not separately address the critical development option, and does not make efficient use of market information. Brennan and Schwartz [1985] also discuss natural resource extraction in an option framework. They do not, however, address the important issues arising out of our discussion of equilibrium in the market for developed reserves.

B. Valuation

The above discussion indicates that a hydrocarbon lease can be modeled as a compound option, where the unexplored tract is an option on the development option. The extraction option is already incorporated in the current market value of a developed reserve. Valuing compound options has been explored by Geske [1979] in the context of financial options. As we now show, extending this theory to valuing real options requires some important modifications. One important feature of valuing stock options is that, other than specifying a stochastic process for the underlying stock price, it is not necessary to understand equilibrium in the market for the stock itself. As we now show, this is not true for valuing unexplored tracts or undeveloped reserves. To see this, we first characterize the behavior of developed reserve prices, using a model of equilibrium in the market for petroleum reserves. We then demonstrate how to integrate this model with option pricing techniques, valuing first the development option and then the option to explore. (The equilibrium model that follows is based upon the model in McDonald and Siegel [1983].)

Petroleum Reserve Market Equilibrium. In equilibrium the expected net payoff from holding a developed reserve (payouts plus capital gains) must compensate the owner for the opportunity cost of investing in that reserve. Let B_t be the number of units of petroleum in a developed reserve, V_t be the value of a unit of developed reserves, and R_t be the instantaneous per unit time net payoff from holding the reserve, all at time t. Assume that the rate of return to the owner follows the diffusion process:

$$R_t dt / B_t V_t = \alpha_v^* dt + \sigma_v dz_v, \tag{2}$$

where α_v^* is the required (expected) rate of return to the owner, σ_v is the instantaneous per unit time standard deviation of the rate of return, and dz_v is an increment to a Wiener (diffusion) process. If the owner is to be compensated for the opportunity cost of investing in the reserve, α_v^* must equal the expected rate of return on a stock with risk $\sigma_v dz_v$.

The assumption that the total rate of return to holding a developed reserve follows a diffusion process is as plausible as the assumption that stock rates of return follow such a process.[6] Like a

6. The finance literature has modeled stock rates of return by a continuous-time random walk because if information flows into the market continuously, then in an efficient market participants must update their expectations and valuations continuously.

stock price, B_tV_t represents the market value of an asset whose owners expect to be compensated for their investment. In fact, there are several companies listed on the New York Stock Exchange (Permian Basin Royalty Trust, for example) and the London Stock Exchange (LASMO, for example) whose assets consist largely of producing developed reserves.

The net payoff R_t comes from two sources: (1) the profits from production; and (2) the capital gain on holding the remaining petroleum. Suppose that production from a developed reserve follows an exponential decline:[7]

$$dB_t = -\gamma B_t dt. \tag{3}$$

Then the net payoff can be written as

$$R_t dt = \{\gamma B_t P_t dt\} + \{(1 - \gamma dt)B_t(V_t + dV_t) - B_t V_t\}, \tag{4}$$

where the net payoff is over a short interval dt. P_t is the after-tax operating profit from selling a unit of petroleum. Substituting (4) into (2) yields the process for the value of a producing developed reserve:[8]

$$\begin{aligned} \frac{dV}{V} &= (\alpha_v^* - \delta_t)dt + \sigma_v dz_v \\ &\equiv \alpha_v dt + \sigma_v dz_v, \end{aligned} \tag{5}$$

where

$$\alpha_v = \alpha_v^* - \delta_t,$$
$$\delta_t = \gamma[P_t - V_t]/V_t.$$

δ_t is the payout rate of the producing developed reserve,[9] and α_v is the expected rate of capital gain. Therefore, V follows a diffusion process.

It is clear from (5) that in equilibrium no agent will hold a nonproducing developed reserve.[10] The expected rate of return from a strategy of holding nonproducing developed reserves, α_v, is less than the required rate of return, α_v^*, on an asset with risk $\sigma_v dz_v$. The

7. This is a standard assumption in the literature on petroleum extraction (see Adelman and Jacoby [1979]) and reflects geological constraints on the extraction rate.

8. Note that $dtdV_t = 0$. α_v^* and α_v can vary with time.

9. This is similar to the payout rate in Myers and Majd [1983].

10. Note that P will exceed V because one would prefer a barrel of oil above the ground to one in the ground due to extraction costs and the time it takes to extract the oil. This will be true as long as storage costs above the ground are small relative to extraction costs and the time value costs associated with waiting until a barrel of oil can be extracted.

TABLE I
COMPARISON OF VARIABLES FOR PRICING MODELS OF STOCK CALL OPTIONS AND UNDEVELOPED PETROLEUM RESERVES

Stock option	Undeveloped reserve
• Current stock price	• Value of developed reserve discounted for development lag
• Variance of rate of return on the stock	• Variance of rate of change of the value of a developed reserve
• Exercise price	• Per unit development cost
• Time to expiration	• Relinquishment requirement
• Riskless rate of interest	• Riskless rate of interest
• Dividend	• Net production revenue less depletion

Note. This table is modified for unexplored tracts by replacing the development lag by the combined exploration and development lag, and the per unit development cost by the combined per unit expected exploration and development cost.

rate of return shortfall to the strategy would be the payout rate δ_t. Note that δ_t can be estimated using observable variables. (See Section V for a detailed discussion of how to estimate δ_t.) The behavior of V has important implications for the proper valuation equation for both undeveloped reserves and unexplored tracts, which we now discuss.

Valuing Undeveloped Reserves. We now turn to the problem of determining the value of an undeveloped reserve on a tract that has already been explored, $X(V,T\text{-}t;D)$. This is of interest in its own right, as firms often wish to value these reserves. It is also the first stage in the valuation of unexplored tracts.

Table I summarizes the analogy between an undeveloped reserve and a stock call option. Because the firm can begin development at any time before expiration of the lease, the analogy is with an American option.

For simplicity, assume that α_v^*, σ_v, and δ_t in (5) are constant over the life of the lease.[11] Then V follows geometric Brownian motion. One way to find the value of the undeveloped reserve, $X(V,T\text{-}t;D)$, would be to invoke standard arbitrage arguments that rely on replicating the undeveloped reserve's payoff by holding a portfolio of developed reserves and riskless bonds [Merton, 1973]. There are, however, two ways to accomplish this replication: (1) by

11. As is true with standard option pricing, α_v^* can vary with the state variables of the problem without changing the valuations. In Section V below, we justify the assumption that δ_t is constant. If σ_v or δ_t is a function of state variables, more complex numerical techniques than those we use can be employed.

holding nonproducing developed reserves; and (2) by holding producing developed reserves. The equilibrium model above demonstrated that $\alpha_v < \alpha_v^*$. McDonald and Siegel [1984] have shown that in this case the first replication strategy is inefficient because it entails holding an asset which carries a rate of return shortfall δ. The resulting undeveloped reserve value will be too high. The second replication strategy is efficient, because the holder of a developed reserve who produces from it earns a fair rate of return. In this case, the payout (at rate δ) is identical to a proportional dividend on a stock, and the partial differential equation characterizing the value of an option on such a stock is appropriate for valuing an undeveloped reserve.

Alternatively, because effecting the actual arbitrage would be difficult, one can value the undeveloped reserve using the equilibrium analysis in Constantinides [1978]. This approach yields the same partial differential equation as the arbitrage analysis:

$$\frac{\partial X}{\partial t} = rX - (r - \delta)\frac{\partial X}{\partial V} - \frac{1}{2}\sigma_v^2 V^2 \frac{\partial^2 X}{\partial V^2}, \tag{6}$$

where r is the riskless rate of interest, assumed constant over the life of the lease (on the undeveloped reserve). The link between the equilibrium model of petroleum reserves and option pricing comes in a straightforward way through the parameter δ. Note that there is no measure of the systematic risk of V in equation (6).

Following Merton [1973] and McDonald and Siegel [1986], the main boundary condition of equation (6) arises from a stopping rule that says the reserve should be developed when the ratio $C_t = V_t/D$ strikes a hitting boundary $\{C_t^*\}$ ($t \in [0,T]$) from below for the first time, or

$$X(V_t, T\text{-}t; D) = V_t - D \quad \text{if } C_t = C_t^* \text{ and } C_s < C_s^* \text{ for all } s < t. \tag{7}$$

$\{C_t^*\}$ is determined as the boundary that maximizes the solution to (6) and is independent of V and D. Therefore, this boundary will apply to *all* leases with an expiration date of T. To illustrate, Figure I shows the values of $\{C_t^*\}$ for a sample of offshore petroleum leases that will be discussed below. The hitting boundary declines toward unity as we move though calendar time because the option value implicit in the undeveloped reserve declines with time. With no time left, there is no option value, and it is optimal to develop if and only if the value of the developed reserve exceeds the development cost.

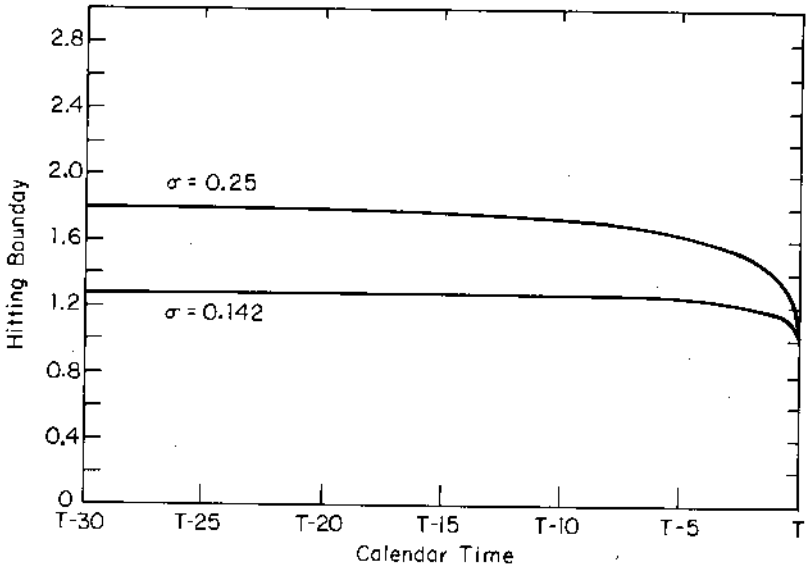

FIGURE I
Hitting Boundaries

Two other boundary conditions are

$$X(0,T\text{-}t;D) = 0 \quad \text{for all } t, \tag{8}$$

and

$$X(V_T,0;D) = \max[0,V_T - D] \text{ if } C_s < C_s^* \text{ for every } s < T. \tag{9}$$

For $T < \infty$, there are no closed forms for the solution to (6) or for $\{C_t^*\}$, but numerical solutions are easy to obtain. We provide examples in Section V.

Valuing Unexplored Tracts. We now determine the value of an unexplored tract, which is the same as valuing the option to make the expected exploration expenditures $\overline{E}$ and receive the expected value of undeveloped reserves $X^*(V)$, defined in (1). In general, valuing the unexplored tract involves complications arising out of the properties of the development option and optimal development timing. To avoid these problems, we make the simplifying assumption that it is optimal to begin development immediately after successful exploration has occurred. We discuss the appropriateness of this assumption below. Using our simplifying assumption, we can "collapse" the development option into the exploration

option. Formally, if the development option will always be exercised immediately, then it will have value,

$$X(V,T\text{-}t;D(Q)) = V - D(Q). \tag{10}$$

Combining (1) and (10), it is clear that exercising the exploration option requires paying $\bar{E}$ and receiving

$$\begin{aligned} X^*(V) &= \int Q[V - D(Q)]dF(Q) \\ &= V\int QdF(Q) - \int QD(Q)dF(Q). \end{aligned} \tag{11}$$

Alternatively, we can view exercising the exploration option as paying

$$\int QD(Q)dF(Q) + \bar{E} = \bar{D} + \bar{E} \tag{12}$$

and receiving

$$V\int QdF(Q) = V\bar{Q},$$

where $\bar{D}$ and $\bar{Q}$ are the expected total development cost and expected reserve quantity, respectively. Using the homogeneity of the valuation discussed above (see footnote 2), we can represent the value of an unexplored tract as

$$\bar{Q}W(V,T\text{-}t;S); \qquad S = (\bar{D} + \bar{E})/\bar{Q}, \tag{13}$$

where $W(V,T\text{-}t;S)$ is the current value of an option to receive a unit of developed reserves by paying the per unit combined expected exploration and development cost S.

The option value $W(V,T\text{-}t;S)$ can be solved in the same way as the value of an undeveloped reserve $X(V,T\text{-}t;D)$ can be solved, $W(V,T\text{-}t;S)$ must satisfy the partial differential equation (6) and meet the boundary conditions (7), (8), and (9), with the development cost D replaced by the expected combined exploration and development cost, S. The hitting boundary will be the same because the underlying asset, developed reserves, is the same and because the boundary only depends upon the ratio of the developed reserve to the (expected) expenditure required to obtain the developed reserve (D for undeveloped reserves and S for unexplored leases).

With no geological uncertainty, collapsing together the development and exploration options is always appropriate. If V/S exceeds the hitting boundary, then so will V/D because $S > D$. However, with geological uncertainty and economies of scale to development of reserves, it is quite possible that V/S can exceed the hitting boundary, but that $V/D(Q)$ will be below the hitting boundary for Q sufficiently below $\bar{Q}$. Thus, one may explore and

find that the quantity of reserves found is small enough that it is optimal to wait to develop because the size of the per unit development costs is large. Thus, the collapsing technique gives a lower bound to the true option value. We believe that this does not represent a significant problem for our analysis. First, we show below that the economies of scale for development are moderate. Second, exploration costs are an important component of total investment costs in our sample. Finally, this problem is present only for small values of Q, and this will have a small impact on the valuation.

Exploration and Development Lags. In the above analysis we have assumed that the holder of the undeveloped reserve receives the developed reserve immediately upon beginning development and that the holder of the unexplored tract receives the developed reserve immediately upon beginning exploration (and therefore development). In fact, he receives it only after a lag equal to the development time for the undeveloped reserve and the combined exploration and development time for the unexplored lease. Let $\hat{t}$ be the length of this lag. The value of a claim at time t to receive a developed reserve at $t + \hat{t}$ is simply the present value,[12]

$$\hat{V}_t = e^{-\alpha_0^* \hat{t}} E_t[V_{t+\hat{t}}] = e^{-\alpha_0^* \hat{t}} V_t e^{(\alpha_0^* - \delta)} = e^{-\delta \hat{t}} V_t. \tag{14}$$

Since by beginning development (or exploration and development) at t, the firm receives such a claim (rather than the developed reserve itself), the actual asset underlying either the development or the exploration option is the present value with price $\hat{V}_t$. Notice, however, that $\hat{V}_t$ also follows (5), so that we can simply replace V_t everywhere by $\hat{V}_t$, for both the exploration and development option.

Optimal Investment Timing. As discussed above, the optimal hitting boundary will be the same for both the exploration and the development options. This boundary provides an investment rule for the firm: begin development or exploration the first time the $C_t (= \hat{V}/D$ for development and $\hat{V}/S$ for exploration) hits $\{C_t^*\}$ from below. Notice that C_t depends only upon the observable variables V_t, δ, $\hat{t}$, and D or S.

Two interesting insights about investment timing arise from this analysis. First, for a given $\hat{t}$, C_t is a decreasing function of per unit investment costs. Thus, reserves with low investment costs will hit the boundary before those with high costs and will be explored

12. If α_0^* varies with the state variables of the problem, then $\hat{V}_t$ can be priced using (6). The resulting value of the claim will again be (14).

or developed first. This is consistent with Herfindahl's [1967] equilibrium. Second, for a given S or D, C_t is a decreasing function of $\hat{t}$. Thus, properties with shorter investment lags will be explored or developed before those with longer lags (see (14)).

Comparative Statics. The comparative statics of the solution to (6) are the same as those for a stock call option. The value of the unexplored tract or undeveloped reserve is increasing, ceteris paribus, in the time to relinquishment, riskless rate of interest, and the standard deviation of the rate of change in the value of a developed reserve.

C. Comparison of Option Valuation and Discounted Cash Flow Approaches

One of the most important features of models used to price stock options is the small number of input parameters needed. These same advantages are present in pricing an unexplored tract or an undeveloped reserve using our option valuation approach, particularly when compared with DCF analysis. Table I provides a list of parameters needed to solve equation (6). Because exploration and development costs are in real dollars, all other parameters will also be in real terms. Of these parameters, only the standard deviation of the rate of change in developed reserve value and the real riskless rate are not directly observable. We shall discuss how to estimate them below. As with stock option pricing, the most important parameters not on the list are risk-adjusted discount rates or expected future prices (e.g., of petroleum or developed reserves). Therefore, as with stock option pricing, one does not need to know the systematic risk of the underlying asset. Comparing these information requirements with the substantial requirements for standard discounted cash flow analysis discussed in Section III demonstrates the power of the option valuation approach to reduce information requirements.

There are several ways in which the option valuation approach reduces information requirements relative to standard discounted cash flow analysis. The discounted cash flow approach typically explicitly models the extraction stage. This requires the analyst to make assumptions about expected future oil prices and optimal extraction-timing. As discussed above, the option valuation approach lets the market place a value on developed reserves, by finding market prices of developed reserves similar to those that the firm would acquire after exploration and development. In fact, the

DCF approach could also use the market value of developed reserves to avoid modeling the extraction stage.

There are, however, two important advantages of the OV approach over DCF that are present, even if the DCF analyst uses the market value of developed reserves. First, the OV approach reduces information requirements by eliminating the need to estimate future developed reserve values. Even using the market value of developed reserves, the DCF analyst would still need to make assumptions about the expected rate of appreciation of the value of a developed reserve, α_D. Second, the OV approach eliminates the need to determine risk-adjusted discount rates. As discussed in Section III, this is an important consideration, because the optimal investment-timing decision must take account of the feedbacks between the investment-timing rule and the risk of the resulting cash flows. In practical applications this is nearly an impossible task. This problem is not present with the OV approach.

V. Empirical Results

In this section we use the option valuation approach to arrive at estimates of the market value of selected offshore petroleum tracts awarded to the industry in federal lease sale No. 62. The option valuation estimates we generate are then compared with value estimates prepared by the U.S. Geological Survey (USGS) using the DCF method. Both sets of estimates are compared with industry bids on the same tracts. Finally, we demonstrate empirically the comparative statics results discussed in the preceding section.

A. Tract Sample and Data Sources

Federal lease sale No. 62 was held on November 18, 1980, and covered western and central portions of the Gulf of Mexico. In total, 67 tracts were awarded in the sale; however, we have been able to gather consistent data on only 21 of these tracts. All of these are one-sixth royalty tracts.[13] The tract-specific data we do have were provided by the USGS. Data elements provided by the USGS include the following items for each tract:

(1) mean and variance for quantities of recoverable oil reserves

13. A more complete specification of contract terms appears in the *Federal Register*, Vol. 45, no. 202, pp. 68866–883. We have chosen to look only at one-sixth royalty tracts because our developed reserve values are most appropriate for this type of tract.

(2) mean and variance for quantities of recoverable condensate reserves
(3) mean and variance for quantities of recoverable gas reserves
(4) probability that the tract is dry
(5) expected exploration cost
(6) expected development cost
(7) USGS estimate of tract value.

Items (1) through (6) are a subset of the input parameters used by the USGS in their DCF analysis of tract values (results of which are given by item (7)).

The means and variances of reserve quantities reported by the USGS (items (1)–(3)) are conditional on the tract not being dry (i.e., not devoid of recoverable hydrocarbons). The USGS provides separately its (subjective) probability that each tract is in fact dry (item (4) above). We assume that, conditional on a tract being wet, the statistical distribution of oil[14] and gas reserves for a given tract is joint lognormal.[15] Therefore, the joint distribution of oil and gas reserves for each tract has a spike at the origin equal to the probability that the tract is dry, and a continuous distribution in the positive quadrant equal to the wet tract probability times the appropriate joint lognormal density.[16]

B. Inputs into the Valuation Equation

Using the data provided by the USGS, along with the market data described below, we construct each of the inputs to the valuation equation (6), which are summarized in Table I and are discussed below.

Developed Reserve Value. It is necessary for our purposes to establish the market value of developed reserves of oil and gas as of November 1980. Gruy et al. [1982] analyzed a number of private sales of developed oil reserves that occurred around this time and their results indicate that a value of approximately $12 per barrel of oil reserves is appropriate. This value is also supported by an analysis (see Paddock [1982]) of the Oil Production Stocks of the London and Scottish Marine Oil Co., Ltd., which are traded on the London Stock Exchange, as well as of similar securities traded on

14. We combine oil and condensate by adding means and variances. While oil and condensate may not be independently distributed, we have no estimate of the covariance to work with. Because the quantities of condensate are very small, this procedure should not affect our results in a significant way.

15. This is the usual distributional assumption (empirically supported) for hydrocarbon quantities. See Reece [1978, p. 371] for a discussion.

16. See Press [1982] for a discussion of the joint log-normal distribution.

exchanges in the United States, such as the Permian Basin Royalty Trust (which is traded on the New York Stock Exchange). These securities are financial claims to the net revenues of developed oil reserves, and are valued and marketable in developed capital markets.

Gas is a bit more problematic. Strictly on a BTU-equilibrating basis, the value of an mcf of gas (at burner tip) would be approximately one sixth the value of a barrel of oil, i.e., $2. However, there are many reasons to doubt that such a direct relationship links the in-situ values of the two fuels: e.g., natural gas is a preferred fuel in many applications, and is also subject to different extraction, transportation and storage costs, and taxes. Consequently, $2 per mcf provides a benchmark, but not the best estimate of the value of developed gas reserves. Private correspondence with investment bankers who were actively involved in the market for developed gas reserves indicates that a figure closer to $3 per mcf would be a better reflection of market values for the latter part of 1980.[17] Due to the guesswork in our estimate of this parameter, and to illustrate the sensitivity of our results to it, we report two sets of results based alternatively on gas reserve values of $2 and $3 per mcf.[18]

The values for developed oil and gas reserves that we use here are for illustrative purposes. In applying the OV method to a particular property, the analyst must be careful to choose the market value of a reserve that has the same hydrocarbon quality, cost structure, and tax regime, as discussed above. We do not have this kind of detailed information for the tracts we study. However, given the active markets in developed reserves, firms that are in the market for reserves would have access to these values.

Variance. The variance of the rate of change in the value of developed reserves is an input parameter that must be estimated. One technique would be to estimate this variance from past data on market values of developed reserves. Unfortunately, while devel-

17. Government forecasts made in 1981 indicated that delivered prices for new gas would jump to nearly $7 per mcf (1980 dollars) when scheduled decontrol measures took effect in 1985 [U. S. DOE, 1981]. This was the expected price level supporting in-situ values of $2 to $3 per mcf in 1980. The dramatic fall that has since occurred was not foreseen in 1980.

18. Notice that the oil and gas reserve market values include deductions for expected taxes and royalties, as well as allowances for depreciation of tangible development costs associated with the producing reserve. When a company purchases a developed reserve, it receives any unused depreciation allowances associated with the reserve. The depreciation allowance it receives will depend upon the magnitude of the exploration and development costs. It is not likely, however, that this will cause great variability in the market value of developed reserves across tracts with different exploration and development costs.

oped reserves are traded in competitive markets, market value data are not publicly available at regular enough intervals to estimate this variance directly. We can, however, get a reasonable estimate using a result in Gruy et al. [1982] (which is also commonly used by industry participants) that developed reserve prices tend to be about one third of crude oil prices. This approximate relationship has held for a number of years, so (at least for illustrative purposes) we can use the variance of the rate of change of crude oil prices as a proxy for the variance of the rate of change of developed reserve prices. Using monthly data for the period 1974–1980, the annualized variance of the real (CPI deflated) refiner cost of imported crude oil is about $\sigma^2 = 0.02019$, implying that $\sigma = 0.142$.[19]

The period 1974–1980 is probably representative of the type of period that market participants might have expected to occur from 1980 on. It includes periods of crisis, as well as periods of relative tranquility. To further validate this variance estimate, we have constructed 95 percent confidence intervals for future crude oil prices implicit in the variance estimate. To capture a possible increase in perceived uncertainty, we also examine a variance of $\sigma^2 = 0.0625$ ($\sigma = 0.250$). The ranges of future prices shown in Table II are 95 percent confidence intervals for years 1 through 10. We assume that the expected rate of increase in value is 3 percent for $\sigma = 0.142$ and 5 percent for $\sigma = 0.250$.[20] While recent experience has demonstrated that crude prices can fall well outside of the confidence interval for $\sigma = 0.142$, we are interested in market expectations in late 1980. We believe that this confidence interval better captures those expectations than that for the higher standard deviation. Therefore, we have adopted the value $\sigma = 0.142$ as our base case, but we also report results based on the assumption that $\sigma = 0.250$.

Expected Exploration and Development Costs. Expected exploration costs reported by the USGS are before tax. About 90 percent of these expenditures are "intangible" and can be expensed for tax purposes.[21] The remaining 10 percent are depreciated. Because such a high proportion of these costs can be expensed, we simply multiply these costs by $(1 - \tau)$, where τ is the corporate income tax rate, taken to be 46 percent.[22]

19. Imported crude is more appropriate than domestic crude, because domestic crude was controlled during this period and decontrolled soon after.

20. See Jacoby and Paddock [1983] for a discussion of these price forecasts. Note, however, that the confidence intervals are not very sensitive to these forecasts.

21. This proportion is taken from the National Petroleum Council [1981].

22. We use undiscounted expected exploration costs because most drilling lags are short.

TABLE II
PER BARREL CRUDE OIL WELLHEAD PRICE RANGES IMPLICIT IN STANDARD DEVIATIONS; YEAR 0 IS 1980 WITH A PRICE OF $36 PER BARREL

	$\sigma = 0.142$		$\sigma = 0.250$	
Year	Lower	Upper	Lower	Upper
1	27.37	48.30	21.56	58.62
2	24.57	54.86	17.31	71.21
3	22.67	60.63	14.58	82.44
5	20.04	71.36	11.05	103.45
10	16.18	97.51	6.54	154.42

Note. The ranges of future prices are 95 percent confidence intervals for years 1 through 10, assuming that oil prices are distributed log-normally:

$$\ln\left(\frac{S_t}{S_0}\right) \sim N\left(\left(\alpha_o - \frac{1}{2}\sigma_o^2\right)t, \sigma^2 t\right),$$

where S_t = crude oil price at t.

Whereas the USGS reports only the expected development cost for each tract, it is necessary for our purposes to derive a development cost function that relates development cost to reserve size. Assume that (real) total development costs are of the deterministic form:

$$D_j = A_j[6Q_{oj} + Q_{gj}]^{\beta}, \tag{15}$$

where Q_{oj} and Q_{gj} are quantities of recoverable oil and gas reserves on the jth tract, and A_j is a tract-specific scaling parameter that might vary with parameters such as water depth and drilling depth.[23] The term in square brackets in equation (15) represents total reserve volume measured in terms of cubic feet of gas equivalent.[24] We have set the economy of scale parameter, β, equal to ⅔, which is consistent with at least one recent study of development costs in the Gulf of Mexico.[25] To arrive at the tract-specific

23. In Section IV we did not distinguish between oil and gas. It is necessary to view the option to develop as the option to obtain $V_oQ_o + V_gQ_g$, where V_o and V_g are the values of developed reserves of oil and gas, respectively. The exercise price is now the *total* development cost. As mentioned above, homogeneity of the value of an undeveloped reserve allows this transformation.

24. We have converted oil to equivalent gas quantities on a BTU basis, using the conversion factor: 1 barrel = 6 mcf.

25. Mansvelt Beck and Wiig [1977] tabulate total development expenditures in the Gulf of Mexico as a function of field size, for both gas and oil fields (see their tables C3 and C4). Their data imply a gas-field scale parameter (β) precisely equal to 0.66, and an oil-field scale parameter equal to 0.80. Because federal sale No. 62 consisted predominantly of gas prospects, we have used the lower value. Experimentation with higher values, however, shows that our results are not affected appreciably by the presumed magnitude of scale economies.

parameters A_j, we use the following fitting procedure. First, we take the expectation of a second-order Taylor Expansion of (15) to yield

$$(16)\quad \overline{D}_j = A_j(6\overline{Q}_{oj} + \overline{Q}_{gj})^{\beta} + (18\sigma_{oj}^2 + \sigma_{gj}^2/2 + 6\sigma_{ogj})A_j\beta(\beta - 1)(6\overline{Q}_{oj} + \overline{Q}_{gj})^{\beta-2},$$

where σ_{oj}^2 and σ_{gj}^2 are variances of oil and gas quantities, σ_{ogj} is the covariance between them, and bars represent expected values. We then make the arbitrary assumption that $\sigma_{ogj} = 0.5\sigma_{oj}\sigma_{gj}$, and solve (16) for the equilibrating value of A_j for each tract using the tract-specific means of the distributions for D_j, Q_{oj}, and Q_{gj} provided by the USGS.[26] The resulting set of tract-specific development cost functions is an approximation to the true development cost functions used by the USGS, which are not available to us.

Approximately 50 percent of development expenditures in the Gulf of Mexico are intangible and can be expensed for tax purposes.[27] The remainder are tangible expenses that are depreciated.[28] Therefore, after-tax development costs will be about 77 percent $(1 - (0.46)(0.50))$ of actual development costs.

Relinquishment Requirement. Contract provisions for all leases issued in sale 62 set the term to relinquishment (time to expiration) at five years.

Riskless Rate. Constantinides [1978] shows that the riskless rate appears in (6) because it is the certainty-equivalent of the required rate of return on the underlying asset. Because investors are interested in after-tax rates of return, the appropriate certainty-equivalent is the return on riskless tax-free bonds or the after-tax return on treasury securities for an investor who is indifferent between taxable and tax-free riskless debt. Skelton [1983] estimates that this marginal tax rate varies from the low 20s for long-term bonds to about 50 percent for short-term bonds. Until recently, a common estimate of the average (pretax) real riskless rate was about 2 percent. We therefore use a real riskless rate of 1.25 percent. Note that using a real model as we do gives more plausibility to the assumption of a constant riskless rate than using a nominal model.

Delta. We derive an estimate for δ from equation (5). for a

26. Because of Jensen's Inequality, it is not proper to simply insert expectations in (15) to arrive at A_j.

27. See the National Petroleum Council [1981].

28. Depreciation allowances can only be taken once production has begun. As we discussed above, depreciation allowances associated with the producing developed reserve will be reflected in the market value of the reserve.

producing reserve, define at time t

M_t = market price of crude oil, per barrel
OC_t = operating cost, per barrel (including royalty)
DA_t = depreciation allowance, per barrel
$x = OC_t/M_t$, assumed constant over time
$y = DA_t/M_t$, assumed constant over time.

After-tax barrel profit from production is

$$P_t = M_t - OC_t - \tau(M_t - OC_t - DA_t) = M_t(1 - x) - 0.46M_t(1 - x - y). \tag{17}$$

With the "one-third" rule discussed above, $M_t/V_t = 3$, and (17) becomes

$$P_t = 3V_t(1 - x) - (0.46)(3)V_t(1 - x - y) = 1.62V_t(1 - x + 0.85y). \tag{17'}$$

Substituting (17′) into (5) yields

$$\delta_t = \gamma[1.62V_t(1 - x + 0.85y) - V_t]/V_t = \gamma[0.62 - 1.62x + 1.377y]. \tag{18}$$

We assume that $x = 0.30$, $y = 0.20$, and $\gamma = 0.10$, which are consistent with data from late 1980. Using these parameter values to solve (18) yields $\delta = 0.041$. Given our assumptions, δ will be constant over time. Alternative assumptions can easily be incorporated into the analysis. For example, x and y can be made functions of M_t, and therefore of V_t. δ will then be a function of V_t. Similarly, δ can be made to vary deterministically with time. In either case (6) can be solved in a straightforward manner numerically.

Combined exploration and development activities in the Gulf of Mexico take place in about one year. Therefore, given this value of delta, $\hat{V}/V = e^{-\delta t} = e^{-0.041} = 0.959829$. The present values of the developed reserves are therefore \$11.52 for oil, and \$1.92 and \$2.88 for gas. These values will be used in the option valuation below.

C. Option Valuation Comparisons

Based on the input data described above, we have computed option valuation estimates using equation (6). We provide summary statistics for comparisons between our option valuation estimates, USGS estimates and industry bids. There are several considerations in evaluating these comparisons.

Comparison with USGS Estimates a. Because our underlying geological and cost data are provided by the USGS, differences

between the option valuation estimates and USGS estimates should be due primarily to differences in the financial valuation techniques.

b. By statute, the USGS must assign a small positive value to tracts that they estimate to have a zero or negative value. To increase the fairness of the comparisons, we have assigned a zero value to these tracts.

c. The USGS's values are examples of values derived using discounted cash flow techniques. As was discussed above, other analysts might derive significantly different discounted cash flow values using the same geological and cost data.

Comparison with Industry Bids. The comparison of option valuation estimates to industry bids is less straightforward.

a. The cost and geological data used by the USGS (and therefore by the option valuation) may deviate from industry expectations on these particular tracts, for both quantities of hydrocarbons and development costs.

b. Even if the underlying USGS data match industry expectations, we still do not observe industry valuations directly. The bids that we observe are not simple relevations of bidders' internal valuations or reservation prices. Instead, they are the outcome of a strategic bidding process. Theoretically, the high bid tendered for an item of uncertain value should be strictly lower than the item's expected value, but should converge to it as the number of bidders grows large (see Wilson [1977] and Milgrom and Weber [1982]). Some practitioners argue, however, that winning bids in OCS lease sales appear to systematically exceed true expected underlying tract values—the "winner's curse" (see Capen, Clapp, and Campbell [1971] and Lohrenz and Dougherty [1983]). Recent experimental evidence (see Kagel and Levin [1986]) gathered in controlled bidding environments, where experienced subjects put real money at stake, also shows that high bids tend to exceed true expected underlying values. Because of these contrary views on what high bids do represent, we present both high bids and geometric mean bids as indicators of industry valuations for the OCS tracts in our sample.[29]

Results. To preserve confidentiality of USGS data, we are not able to provide tract-specific results. Tables III and IV provide summary measures for the comparisons between option valuation,

29. Because the distribution of industry bids is so markedly skewed, it is customary in the literature to use the geometric mean as a measure of central tendency.

TABLE III
SIMPLE CORRELATION COEFFICIENTS BETWEEN VALUATIONS
(N = 21)

	$2 per mcf gas				$3 per mcf gas			
	OV	USGS	GB	HB	OV	USGS	GB	HB
OV	1.00				1.00			
USGS	0.99	1.00			0.98	1.00		
GB	0.39	0.39	1.00		0.38	0.39	1.00	
HB	0.21	0.18	0.55	1.00	0.24	0.18	0.55	1.00

TABLE IV
MEAN AND STANDARD DEVIATIONS FOR VALUATIONS
(N = 21) (MILLIONS $)

	$2 per mcf of gas		
Valuation methodology	Sample mean	Sample standard deviation	Standard error of the mean
OV	4.13	5.56	1.21
USGS	4.93	6.32	1.38
GB	6.03	3.58	0.78
HB	18.95	16.07	3.51
OV-USGS	−0.79	1.26	0.27
GB-OV	1.90	5.33	1.16
HB-OV	14.81	15.88	3.47
GB-USGS	1.11	5.94	1.30
HB-USGS	14.02	16.19	3.53
	$3 per mcf of gas		
Valuation methodology	Sample mean	Sample standard deviation	Standard error of the mean
OV	8.20	9.42	2.06
USGS	4.93	6.32	1.38
GB	6.03	3.58	0.78
HB	18.95	16.07	3.51
OV-USGS	3.27	3.50	0.76
GB-OV	−2.17	8.69	1.90
HB-OV	13.22	16.52	3.60
GB-USGS	1.11	5.94	1.30
HB-USGS	14.02	16.19	3.53

Note. OV-USGS represents the tract-by-tract difference in OV and USGS valuations, etc.

USGS and industry bid values. We use the following notation:

OV = option valuation
USGS = USGS discounted cash flow valuation
GB = geometric mean of industry bids
HG = high (winning) industry bid.

Table III presents simple correlation coefficients that measure the degree of linear association between the alternative measures of tract value. It is immediately apparent that the OV and USGS estimates are very highly correlated. In view of the common data inputs (cost and geological) they share, this is not surprising. In spite of the high correlation, there are significant disparities between the OV and USGS estimates that will be pointed out shortly.

It is also apparent from Table III that the USGS estimates are not highly correlated with the level of industry bids. This has been a recurrent problem for the USGS, and undoubtedly reflects the well-known fact that USGS appraisals of geological potential differ markedly in many cases from the individual assessments of private firms. The OV estimates also correlate poorly with industry bids, since the OV estimates rely on the same underlying geological data used by the USGS.

We turn next to a comparison of average tract values, which are recorded in Table IV. The results are sensitive to the value chosen for gas. For a $2 per mcf value for gas (implied by strict BTU pricing parity), the OV estimates are, on average, below the values from the USGS and both industry measures. For the $3 per mcf value (drawn from market sources), the OV estimates are, on average, above both the USGS values and the average of mean industry bids. They are, however, well below the mean high bid. As this experiment is meant to be a broad test of the plausibility of the OV approach, these results are quite promising. Uman et al. [1979] find that there is no apparent bias in USGS ex ante geological estimates, even though they are subject to large errors. That the OV approach with the preferred $3 per mcf value for gas falls between the two industry measures provides room for optimism.

We can only speculate about how the OV technique might perform in conjunction with geological assessments more in line with industry expectations. However, on the basis of the present results, we are greatly encouraged to try such an experiment. The most valid comparison would be an OV estimate compared with industry valuations based on the same geological and cost assumptions. This is not possible at present due to data restrictions.

D. Comparative Statics

In Section III we discussed how increases in reserve price variability and relinquishment time limits unambiguously increase the OV value of undeveloped reserves and unexplored tracts. However, neither extending relinquishment time limits nor increasing the variance has significant effects for the tracts in sale 62. The variance effect arises because of the possibility of not exploring and developing. In our data set, nearly all of the tracts are very much in the money (conditional on the presence of hydrocarbons), meaning that the developed reserve value greatly exceeds combined exploration and development costs. Therefore, given that oil and gas have been found, there is little likelihood that exploration and development will *not* occur immediately. Hence the variance effect is not important. Relaxing the relinquishment requirement does not have much effect for reasons we discuss below.

To demonstrate how variance and time limits would affect tract value in areas subject to *higher* unit investment costs, we value a set of hypothetical undeveloped reserves that have cost structures representative of those found in Alaska and the North Atlantic region. These tracts have the following: (a) 100 million barrels of oil are proven; and (b) $\hat{V}/D$ ratios varying from 0.7 to 1.0. We increase the standard deviation of the rate of change in the value of a developed reserve from 0.142 to 0.250 and increase the relinquishment time limit from five to both ten and fifteen years. (The USGS has granted ten-year limits on certain high cost tracts.) Table V presents valuations for these tracts. Clearly, increasing volatility from $\sigma = 0.142$ to $\sigma = 0.250$ has a large impact on high-cost undeveloped reserve values. The effect dampens as $\hat{V}/D$ increases, because the tract becomes more likely to be developed. For example, for $\overline{T} = T - t = 10$, increasing the volatility increases the reserve value by 435 percent for $\hat{V}/D = 0.70$ and by 115 percent for $\hat{V}/D = 1.0$. Relaxing relinquishment requirements can also have a significant effect on tract values. For example, for $\sigma = 0.142$, increasing the time to relinquishment from five to fifteen years increases the reserve value by 160 percent for $\hat{V}/D = 0.70$ and by 23 percent of $\hat{V}/D = 1.0$. Again, the effect diminishes as $\hat{V}/D$ increases. Thus, the option of waiting to explore and develop is quite valuable to companies leasing tracts in high-cost areas, especially during periods of great uncertainty about future hydrocarbon prices. Since the government appears to capture at least all residual economic rents on OCS lease sales (see Mead et al. [1983]), the private option to wait is then also valuable to the government, which could expect

TABLE V

VALUATIONS FOR OV FOR DIFFERENT VARIANCES AND EXPIRATION TIMES: HYPOTHETICAL HIGH-DEVELOPMENT COST UNDEVELOPED RESERVES (MILLIONS $)

Variance and expiration times	Value to development cost ratio $(\bar{V}/D)$ =			
	0.70	0.80	0.90	1.00
$\sigma = 0.142, \overline{T} = 5$	10.78	26.06	51.51	88.25
$\sigma = 0.142, \overline{T} = 10$	21.76	40.49	67.14	102.41
$\sigma = 0.142, \overline{T} = 15$	28.04	47.65	74.28	108.65
$\sigma = 0.250, \overline{T} = 5$	73.74	106.47	142.96	182.06
$\sigma = 0.250, \overline{T} = 10$	116.50	149.64	184.19	219.70
$\sigma = 0.250, \overline{T} = 15$	139.67	171.84	204.76	238.19

Hypothetical High Cost Parameters
Reserve value = $11.52
Hydrocarbon reserves = 100 million barrels
$\overline{T} = T - t$.

to receive commensurately larger bonus bids.[30] The only exception arises if some distortive element of the tax or royalty system would somehow be aggravated by lengthening the term to relinquishment, resulting in less efficient extraction.[31]

E. Exploration and Development Timing

As discussed above, embedded in the option valuation is the solution of the optimal investment timing problem. Figure I gives hitting boundaries based on the same parameters that were used in the valuations above; i.e., $\sigma = 0.142$, and $\sigma = 0.250$. As discussed

30. By extending the term to relinquishment, the government does reduce the present value of its option to resell the tract if the tract is unexplored when relinquishment occurs. However, to the extent that a relinquishment requirement forces a company to explore and develop earlier than is optimal, extending the term to relinquishment should have an overall positive effect on total rents. However, we have not estimated this effect explicitly. See footnote 31 for a discussion of when this may not be true.

31. Royalties and excise taxes generally cause the developer to postpone production. If longer terms to relinquishment accommodate this, they also would reduce the total net social value of the lease. We do not propose, however, that the government attempt to neutralize the tax distortion by manipulating the relinquishment terms. In any event, the effect of extending relinquishment on bonus bids would be as described in the text. The developer postpones production only if doing so increases the net private value of the tract, and this increment in value is captured by the government via the competitive bidding process.

For a given number of years to expiration at date T, if $C = V/D\ (V/S)$ is less than C^*, the firm defers development (exploration and development). If C exceeds C^*, investment should proceed immediately. σ represents the standard deviation of the rate of change of V. To account for investment lags, replace V by $\bar{V}$.

above, this boundary is appropriate for either valuing an unexplored tract (assuming that immediate development is always optimal) or an undeveloped reserve. Each curve represents the boundary that $C = \hat{V}/S$ or $\hat{V}/D$ must hit for immediate investment to be optimal. As would be expected, the boundary for the higher volatility is above that for the lower volatility. With greater volatility the firm wants to allow for the possibility that prices may become very high.

The most interesting feature of both hitting boundaries is that they are relatively flat before diving to 1.0 at expiration. This indicates that tracts with certain cost and geological characteristics should be explored or developed immediately, while others should be held either until close to the expiration of the relinquishment time limit or until developed reserve prices increase sufficiently to induce explorations or development. In particular, low-cost tracts will be explored or developed immediately and higher cost tracts will be held from exploration or development. This is a particularly simple form of an investment rule, given that the hitting boundary will be the same for all tracts. The firm need only calculate $C = \hat{V}/S$ or $\hat{V}/D$ to decide whether a tract should be explored or developed immediately.

VI. Summary and Extensions

This paper has extended financial option theory by developing an approach to valuing a claim on a real asset: an offshore petroleum lease. We have addressed several theoretical and practical problems, not present in applying option pricing theory to financial assets. Most importantly, we show the necessity of combining option pricing techniques with a model of equilibrium in the market for the underlying asset. Our new approach has several advantages over currently used discounted cash flow techniques. First, it requires significantly less data because it efficiently uses market information. Second, it has less computational cost and is less subject to error. Third, it provides a guide for the optimal timing of development. Finally, it provides important insights for both government policy and company behavior. Empirical application of the approach suggests that the approach has significant promise.

There are three important extensions of this work. The first involves obtaining good measures of the market value of developed reserves. While companies have access to these values from their

dealings in active secondary markets, it would be useful to be able to obtain these values from more public sources. One possibility is to try to extract these values from the (traded) stock prices of companies whose assets consist solely of developed reserves (producing properties). These companies exist both in the United States and in the United Kingdom. Controlling for variables such as differing tax regimes, price controls, cost structures, and oil quality will be an important consideration.

The second extension involves a more complex specification of the development decision. We have modeled development as a lump instantaneous expenditure. Actually, companies have latitude in varying the rate of development. It may, for example, be optimal to develop slowly to take advantage of new information. This gives the company the *option* to discontinue further development if it becomes unprofitable. This option effect must be weighed against possible economies of proceeding more rapidly. The resulting analysis would be similar to that in Majd and Pindyck [1987].

Finally, there are many other real assets with option-like characteristics. The kinds of informational economies, insights, and problems discussed here in relation to valuing petroleum leases should be present in valuing claims on other real assets as well.

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