Homework 3
Introduction to Numerical Methods

Problem 1

Let A be a square matrix of order n, and let it satisfy

$a_{ij} = 0$ for $i > j$

Such a matrix is called *upper triangular*.

Part A Show that the sum and products of such matrices are also upper triangular. Is the inverse upper triangular?

Part B A square matrix A is called *lower triangular* if its transpose A^T is upper triangular. Use this definition, results of part A, and properties of matrix transposition to show that the sum and product of lower triangular matrices are also lower triangular.

Problem 2

Given the following system of equations

$$2x_1 - 6x_2 - x_3 = -38$$
$$-3x_1 - x_2 + 7x_3 = -34$$
$$-8x_1 + x_2 - 2x_3 = -20$$

Part A Compute the determinant.
Part B Solve using Cramer's rule.
Part C Solve using Naïve Gauss.
Part D Solve using Gauss elimination with partial pivoting.
Part E Calculate A inverse using LU decomposition.
Part F Solve using LU decomposition. Also solve the system for an alternative right-hand-side vector b^T=[-76 -68 -40].

Note: Show all steps of the computation for each part.

Problem 3

Part A What is the operation count for a matrix-matrix product, AB? You should set up the problem using summation notation as discussed in class and then solve to obtain the final counts. Specify the counts for addition/subtraction and multiplication/division.
Part B Repeat part A, assuming that A is tridiagonal.
Part C Repeat part A, assuming that both A and B are tridiagonal.
Part D Calculate the total number of cycles it would take to execute **Part A-C** on a computer that has an ALU that takes 1 cycle for addition/subtraction and 10 cycles for multiplication/division.

Problem 4

When A is a symmetric, positive definite matrix, it can be factored in the form

$$A=LL^T$$

For some lower triangular matrix L that has nonzero diagonal elements, usually taken to be positive. This is called the Cholesky Factorization. Recall that a symmetric matrix A is said to be positive definite if for any $x \neq 0, x^T Ax > 0$.

Part A

For

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 5 \end{bmatrix}$$

let

$$L = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix}$$

Find L such that $LL^T=A$.

Part B

Repeat this process for

$$A = \begin{bmatrix} 2.25 & -3 & 4.5 \\ -3 & 5 & -10 \\ 4.5 & -10 & 34 \end{bmatrix}$$

finding a lower triangular matrix L for which $LL^T=A$.

Problem 5

Calculate the cond(A) for

$$A = \begin{bmatrix} 1 & c \\ c & 1 \end{bmatrix}, \quad |c| \neq 1$$

When does A become ill-conditioned? What does this say about the linear system $Ax=b$? How is cond(A) related to det(A)?