Principles of Geometry
Chapter 1 to 4 Summary Sheet

Definitions

Chapter 1.2

- **Collinear points** are points that lie on the same line.

Chapter 1.3

- An **isosceles triangle** is a triangle that has two congruent sides.
- A **line segment** is the part of a line that consists of two points, known as endpoints, and all the points between them.
- The **distance** between two points A and B is the length of the line segment AB that joins the two points.
- **Congruent segments** are two segments that have the same length.
- The **midpoint** of a line segment is the point that separates the line segment into two congruent parts.
- **Ray** AB, is the union of $\overline{AB}$ and all points X on $\overleftrightarrow{AB}$ such that B is between A and X..
- **Parallel lines** are lines that lie in the same plane but do not intersect.

Chapter 1.4

- An **angle** is the union of two rays that share a common endpoint.
- A **right angle** is an angle whose measure is exactly 90°
- An **acute angle** is an angle whose measure is between 0° and 90°
- An **obtuse angle** is an angle whose measure is between 90° and 180°
- A **straight angle** is an angle whose measure is exactly 180°
- **Congruent angles** are two angles with the same measure.
- The **bisector** of an angle is the ray that separates the given angle into two congruent angles.
- Two angles are **complementary** if the sum of their measures is 90°. Each angle in the pair is known as the **complement** of the other angle.
- Two angles are **supplementary** if the sum of their measures is 180. Each angle in the pair is known as the **supplement** of the other angle.
- Vertical angles

Chapter 1.5

- **Addition Property of Equality**: If $a=b$, then $a+c=b+c$.
- **Subtraction Property of Equality**: If $a=b$, then $a-c=b-c$
- **Multiplication Property of Equality**: If $a=b$, then $a*c=b*c$.
- **Division Property of Equality**: If $a=b$, then $a/c=b/c$ and $c \neq 0$.
- **Distributive Property**: $a(b+c)= a*b + a*c$
- **Substitution Property**: If $a=b$, then a replaces b in any equation.
- **Transitive Property**: If $a=b$ and $b=c$, then $a=c$.

Chapter 1.6

- **Perpendicular lines** are two lines that meet to form congruent adjacent angles.
- **Reflexive property**: aRa
- **Symmetric property**: If aRb, then bRa.
- **Transitive property**: If aRb and bRc, then aRc.

Chapter 2.1

- **Corresponding angles** are two angles that lie in the same relative positions.
- **Alternating interior angles** are two interior angles that have different vertices and lie on opposite sides of a transversal
- **Alternating exterior angles** are two exterior angles that have different vertices and lie on opposite sides of the transversal

Chapter 2.4

- A **triangle** is the union of three line segments that are determined by three noncollinear points.
- A figure is **overdetermined** when it is impossible for all conditions described to be satisfied
- A figure is **underdetermined** when there is more than one possible figure
- An **acute triangle** has all acute angles.

- An **obtuse triangle** has one obtuse angle.
- A **right triangle** has one right angle.
- An **equiangular triangle** has all angles congruent.
- An **exterior angle** is found by extending a side of an object and creating an angle formed by a side and the extension of the adjacent side.

Chapter 2.5

- A **polygon** is a closed plan figure whose sides are line segments that intersect only at endpoints.
- A **concave polygon** has at least one reflex angle.
- A **convex polygon** has all angle measures between 0° and 90°
- A **diagonal** of a polygon is a line segment that joins two nonconsecutive vertices.
- A **regular polygon** is a polygon that is both equilateral and equiangular.

Chapter 2.6

- A figure has **symmetry with respect to a line** ℓ if for every point *A* on the figure, there is a second point *B* on the figure for which ℓ is the perpendicular bisector of $\overline{AB}$.
- A figure has **symmetry with respect to a point** *P* if for every point *A* on the figure, there is a second point *C* for which point *P* is the midpoint of $\overline{AC}$

Chapter 3.1

- Two triangles are **congruent** when the six parts of the first trangle are congruent to the six corresponding parts of the second triangle

Chapter 3.2

- **CPCTC** means corresponding parts of congruent triangles are congruent.
- The **hypotenuse** is the side opposite the right angle of a right triangle.

Chapter 3.3

- Angle bisector
- The **altitude** of an object is it's height.
- Perpendicular bisector
- The **perimeter** of a figure is the sum of the lengths of its sides.

Chapter 3.5

- Let a and b be real numbers. $a>b$ if and only if there is a positive number p for which $a=b+p$

Chapter 4.1

- A **quadrilateral** is a polygon that has four sides.
- A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.

Chapter 4.2

- A **kite** is a quadrilateral with two distinct pairs of congruent adjacent sides.

Chapter 4.3

- A **rectangle** is a parallelogram that has a right angle.
- A **square** is a rectangle that has two congruent adjacent sides.
- A **rhombus** is a parallelogram with two congruent adjacent sides.
- The **Pythagorean Theorem** in a right triangle with the hypotenuse of length *c* and the legs of lengths *a* and *b* is $c^2 = a^2 + b^2$.

Chapter 4.4

- A **trapezoid** is a quadrilateral with exactly two parallel sides.

Postulates

1 Through two distinct points, there is exactly one line.
2 (Ruler Postulate) The measure of any line segment is a unique positive number.
3 (Segment-Addition Postulate) If X is a point of $\overline{AB}$ and A-X-B, then AX + XB = AB.
4 If two lines intersect, they intersect at a point.
5 Through three noncollinear points, there is exactly one plane
6 If two distinct planes intersect, then their intersection is a line.
7 Given two distinct points in a plane, the line containing these points also lies in the plane
8 (Protractor Postulate) The measure of an angle is a unique positive number.
9 (Angle-Addition Postulate) If a point D lies in the interior of an angle ABC, then m$\angle ABD$ + m$\angle DBC$ = m$\angle ABC$.
10 (Parallel Postulate) Through a point not on a line, exactly one line is parallel to the given line.
11 If two parallel lines are cut by a transversal, then the corresponding angles are congruent.
12 (SSS) If the three sides of one triangle are congruent to the three sides of a second triangle, then the triangles are congruent.
13 (SAS) If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the triangles are congruent.
14 (ASA) If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the triangles are congruent.

Theorem

1.3.1 The midpoint of a line segment is unique.
1.4.1 There is one and only one angle bisector for a given angle.
1.6.1 If two lines are perpendicular, then they meet to form right angles.
1.6.2 If two lines intersect, then the vertical angles formed are congruent.
1.6.3 There is exactly one line perpendicular to a given line at any point on the line.
1.6.4 The perpendicular bisector of a line segment is unique.
1.7.1 If two lines meet to form a right angle, then these lines are perpendicular.
1.7.2 If two angles are complementary to the same angle (or to congruent angles), then these angles are congruent.
1.7.3 If two angles are supplementary to the same angle (or to congruent angles), then these angles are congruent.
1.7.4 Any two right angles are congruent.
1.7.5 If the exterior sides of two adjacent acute angles form perpendicular rays, then these angles are complementary.
1.7.6 If the exterior sides of two adjacent angles from a straight line, then these angles are supplementary.
1.7.7 If two segments are congruent, then their midpoints separate these segments into four congruent segments.
1.7.8 If two angles are congruent, then their bisectors separate these angles into four congruent angles.
2.1.1 From a point not on a given line, there is exactly one line perpendicular to the given line.
2.1.2 (Alternate interior angles) If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.
2.1.3 (Alternate exterior angles) If two parallel lines are cut by a transversal, then the alternate exterior angles are congruent.
2.1.4 (Interiors on same side) If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.
2.1.5 (Exteriors on same side) If two parallel lines are cut by a transversal then the exterior angles on the same side of the transversal are supplementary.
2.3.1 If two lines are cut by a transversal so that the corresponding angles are congruent, then these lines are parallel.
2.3.2 (Converse alternate interior angles) If two lines are cut by a transversal so that the alternate interior angles are congruent, then these lines are parallel

2.3.3 (Converse alternate exterior angles) If two lines are cut by a transversal so that the alternate exterior angles are congruent, then these lines are parallel.

2.3.4 (Converse interiors on same side) If two lines are cut by a transversal so that the interior angles on the same side of the transversal are supplementary, then these lines are parallel.

2.3.5 (Converse exteriors on same side) If two lines are cut by a transversal so that the exterior angles on the same side of the transversal are supplementary, then these lines are parallel.

2.3.6 If two lines are both parallel to a third line, then these lines are parallel to each other

2.3.7 If two coplanar lines are both perpendicular to a third line, then these lines are parallel to each other.

2.4.1 In a triangle, the sum of the measures of the interior angles is 180°

2.4.2 Each angle of an equiangular triangle measures 60°

2.4.3 The acute angles of a right triangle are complementary.

2.4.4 If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

2.4.5 The measures of an exterior angle of a triangle equals the sum of the measures of the two nonadjacent interior angles

2.5.1 The total number of diagonals *D* in a polygon of *n* sides is given by the formula $D = \frac{n(n-3)}{2}$

2.5.2 The sum *S* of the measures of the interior angles of a polygon with *n* sides is given by $D = (n-2)\cdot 180^\circ$ (Note that *n*>2 for any polygon.)

2.5.3 The measure I of each interior angle of a regular or equiangular polygon of n sides is $I = \frac{(n-2)\cdot 180^\circ}{n}$

2.5.4 The sum of the measures of the four interior angles of a quadrilateral is 360°

2.5.5 The sum of the measures of the exterior angles, one at each vertex, of a polygon is 360°

2.5.6 The measure *E* of each exterior angle of a regular or equiangular polygon of *n* sides is $E = \frac{360^\circ}{n}$

3.1.1 (AAS) If two angles and a nonincluded side of one triangle are congruent to two angles and a nonincluded side of a second triangle, then the triangles are congruent.

3.2.1 (HL) If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the triangles are congruent.

3.3.1 Corresponding altitudes of congruent triangles are congruent.

3.3.2 The bisector of the vertex angle of an isosceles triangle separates the triangle into two congruent triangles.

3.3.3 If two sides of a triangle are congruent, then the angles opposite these sides are also congruent.

3.3.4 If two angles of a triangle are congruent, then the sides opposite these angles are also congruent.

3.3.5 An equilateral triangle is also equiangular.

3.3.6 An equiangular triangle is also equilateral.

3.5.1 The measure of a line segment is greater than the measure of any of its parts.

3.5.2 The measure of an angle is greater than the measure of any of its parts.

3.5.3 The measure of an exterior angle of a triangle is greater than the measure of either nonadjacent interior angle.

3.5.4 If a triangle contains a right or obtuse angle, then the measure of this angle is greater than the measure of either of the remaining angles.

3.5.5 (Addition Property of Inequality) If $a > b$ and $c > d$, then $a + c > b + d$.

3.5.6 If one side of a triangle is longer than a second side, then the measure of the angle opposite the first side is greater than the measure of the angle opposite the second side.

3.5.7 If the measure of one angle of a triangle is greater than the measure of a second angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

3.5.8	The perpendicular segment from a point to a line is the shortest segment that can be drawn from the point to the line.
3.5.9	The perpendicular segment from a point to a plane is the shortest segment that can be drawn from the point to the plane.
3.5.10	(Triangle Inequality) The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
3.5.10	(Triangle Inequality) The length of one side of a triangle must be between the sum and the difference of the lengths of the other two sides.
4.1.1	A diagonal of a parallelogram separates it into two congruent triangles.
4.1.2	The opposite angles of a parallelogram are congruent.
4.1.3	The opposite sides of a parallelogram are congruent.
4.1.4	The diagonals of a parallelogram bisect each other.
4.1.5	Two consecutive angles of a parallelogram are supplementary.
4.1.6	Two parallel lines are everywhere equidistant.
4.1.7	If two sides of one triangle are congruent to two sides of a second triangle and the included angle of the first triangle is greater than the included angle of the second, then the length of the side opposite the included angle of the first triangle is greater than the length of the side opposite the included angle of the second.
4.1.8	In a parallelogram with unequal pairs of consecutive angles, the longer diagonal lies opposite the obtuse angle.
4.2.1	If two sides of a quadrilateral are both congruent and parallel, then the quadrilateral is a parallelogram.
4.2.2	If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram.
4.2.3	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
4.2.4	In a kite, one pair of opposite angles are congruent.
4.2.5	The segment that joins the midpoints of two sides of a triangle is parallel to the third side and has a length equal to one-half the length of the third side.
4.3.1	All angles of a rectangle are right angles.
4.3.2	The diagonals of a rectangle are congruent.
4.3.3	All sides of a square are congruent.
4.3.4	All sides of a rhombus are congruent.
4.3.5	The diagonals of a rhombus are perpendicular.
4.4.1	The base angles of an isosceles trapezoid are congruent.
4.4.2	The diagonals of an isosceles trapezoid are congruent.
4.4.3	The length of the median of a trapezoid equals one-half the sum of the lengths of the two bases.
4.4.4	The median of a trapezoid is parallel to each base.
4.4.5	If two base angles of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.
4.4.6	If the diagonals of a trapezoid are congruent, the trapezoid is an isosceles trapezoid.
4.4.7	If three (or more) parallel lines intercept congruent segments on one transversal, then they intercept congruent segments on any transversal.

<u>Constructions</u>

1. To construct a segment congruent to a given segment.
2. To construct the midpoint M of a given line segment AB.
3. To construct an angle congruent to a given angle.
4. To construct the angle bisector of a given angle.
5. To construct the line perpendicular to a given line at a specified point on the given line.
6. To construct the line that is perpendicular to a given line from a point not on the given line.
7. To construct the line parallel to a given line from a point not on that line.