Learning Bigrams from Unigrams

Andrew B. Goldberg
goldberg@cs.wisc.edu

University of Wisconsin–Madison

Joint work with Xiaojin Zhu, Michael Rabbat (McGill University), and Robert Nowak
Originally appeared at ACL 2008

Privacy attack through index file

your file

Privacy attack through index file

your file

NLP software

$\Longrightarrow$

bag-of-word
index

Privacy attack through index file

your file

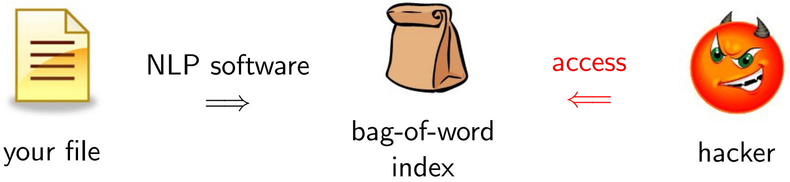
NLP software $\Longrightarrow$

bag-of-word index

access $\Longleftarrow$

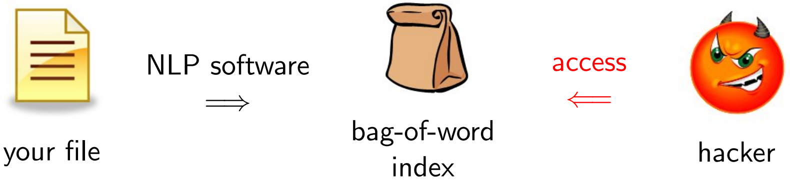
hacker

Privacy attack through index file



What can the hacker do?

Privacy attack through index file



What can the hacker do?

- Learn a bigram language model
- Reconstruct your original ordered documents

Bag-of-word (BOW) representation

- A document in its original order $\mathbf{z}_1 =$ "$\langle \mathrm{d} \rangle$ really really neat"
- Its BOW: unigram count vector

$$\mathbf{x}_1 = (x_{11}, \ldots, x_{1W}) = (1, 0, \ldots, 0, 1, 0, \ldots, 0, 2, 0, \ldots)$$

- Can the hacker recover word order from $\mathbf{x}_1$, without extra knowledge of the language?

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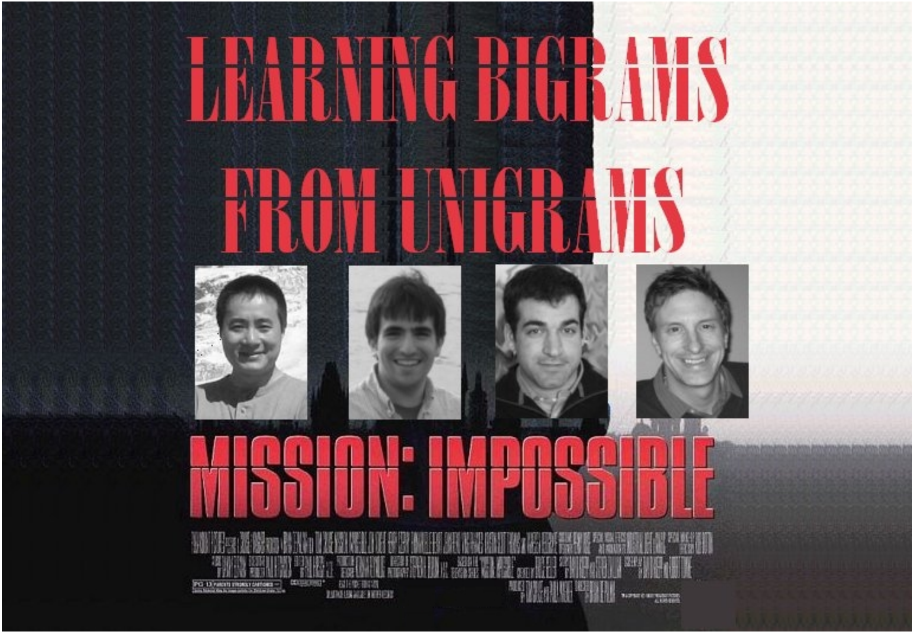
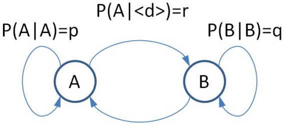
Perhaps surprisingly ...

We will learn a bigram LM from $\mathbf{x}_1, \ldots, \mathbf{x}_n$, as if we have the ordered documents $\mathbf{z}_1, \ldots, \mathbf{z}_n$.


LEARNING BIGRAMS
FROM UNIGRAMS
MISSION: IMPOSSIBLE

Mission: possible

An example of exact bigram LM recovery:



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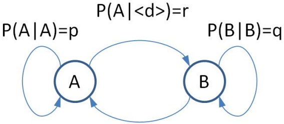

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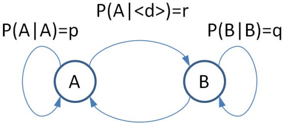

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1. $\mathbf{z} \sim \boldsymbol{\theta} = \{p, q, r\}$

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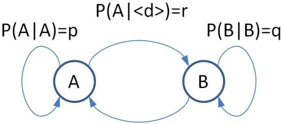

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1. $\mathbf{z} \sim \boldsymbol{\theta} = \{p, q, r\}$
2. $\mathbf{z} \rightarrow \mathbf{x}$ by removing word order

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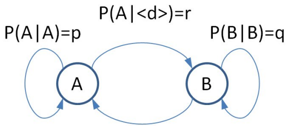

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Probability of a BOW vector:

$$P(\mathbf{x}|\boldsymbol{\theta}) = \sum_{\mathbf{z} \in \sigma(\mathbf{x})} P(\mathbf{z}|\boldsymbol{\theta}) = \sum_{\mathbf{z} \in \sigma(\mathbf{x})} \prod_{j=2}^{|\mathbf{x}|} P(z_j | z_{j-1})$$

$\sigma(\mathbf{x})$ is all unique orderings of $\mathbf{x}$

- $\sigma(\langle\mathrm{d}\rangle$:1, A:2, B:1)={"$\langle\mathrm{d}\rangle$ A A B", "$\langle\mathrm{d}\rangle$ A B A", "$\langle\mathrm{d}\rangle$ B A A"}

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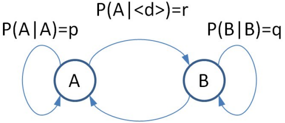

Assuming all docs have length $|\mathbf{x}| = 4$, then only 4 kinds of BOWs:

$\mathbf{x}$	$P(\mathbf{x}\|\boldsymbol{\theta}) = \sum_{\mathbf{z}\in\sigma(\mathbf{x})} \prod_{j=2}^{\|\mathbf{x}\|} P(z_j\|z_{j-1})$
($\langle$d$\rangle$:1, A:3, B:0)	rp^2
($\langle$d$\rangle$:1, A:2, B:1)	$rp(1-p) + r(1-p)(1-q) + (1-r)(1-q)p$
($\langle$d$\rangle$:1, A:0, B:3)	$(1-r)q^2$
($\langle$d$\rangle$:1, A:1, B:2)	1-above

Mission: possible

Let true $\boldsymbol{\theta} = \{r = 0.25, p = 0.9, q = 0.5\}$

Given $\mathbf{x}_1 \ldots \mathbf{x}_n, n \to \infty$, the observed frequency of BOWs will be:

($\langle$d$\rangle$:1, A:3, B:0)	20.25%
($\langle$d$\rangle$:1, A:2, B:1)	37.25%
($\langle$d$\rangle$:1, A:0, B:3)	18.75%
($\langle$d$\rangle$:1, A:1, B:2)	100%-above

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Matching probability with observed frequency

$$\begin{cases} rp^2 = 0.2025 \\ rp(1-p) + r(1-p)(1-q) \\ \qquad\qquad +(1-r)(1-q)p = 0.3725 \\ (1-r)q^2 = 0.1875 \end{cases}$$

exactly recovers $\boldsymbol{\theta}$.

Let's get real

Real documents are not generated from a bigram LM

Maximize log likelihood instead

Parameter $\boldsymbol{\theta} = [\theta_{uv} = P(v|u)]_{W \times W}$ (bigram LM matrix)

Normalized log likelihood under $\boldsymbol{\theta}$:

$$\ell(\boldsymbol{\theta}) \equiv \frac{1}{C} \sum_{i=1}^{n} \log P(\mathbf{x}_i|\boldsymbol{\theta}) = \frac{1}{C} \sum_{i=1}^{n} \log \sum_{\mathbf{z} \in \sigma(\mathbf{x})} \prod_{j=2}^{|\mathbf{x}|} P(z_j|z_{j-1})$$

$$C = \sum_{i=1}^{n} (|\mathbf{x}_i| - 1)$$

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But there are multiple local optima

Regularization

Normalized log likelihood under $\boldsymbol{\theta}$:

$$\ell(\boldsymbol{\theta}) \equiv \frac{1}{C} \sum_{i=1}^{n} \log P(\mathbf{x}_i|\boldsymbol{\theta})$$

Regularize with prior bigram LM ϕ (estimated from BOWs too)

- Average KL-divergence over all histories

$$\mathcal{D}(\boldsymbol{\phi}, \boldsymbol{\theta}) \equiv \frac{1}{W} \sum_{u=1}^{W} KL(\boldsymbol{\phi}_{u\cdot} \| \boldsymbol{\theta}_{u\cdot}).$$

- i.e., rows in $\boldsymbol{\theta}$ should be similar to rows in prior ϕ

$\mathcal{D}(\boldsymbol{\phi}, \boldsymbol{\theta})$ is convex (Cover and Thomas, 1991).

Regularized Optimization Problem

Our optimization problem:

$$\begin{array}{rl} \max\limits_{\boldsymbol{\theta}} & \ell(\boldsymbol{\theta}) - \lambda \mathcal{D}(\boldsymbol{\phi}, \boldsymbol{\theta}) \\ \text{subject to} & \boldsymbol{\theta}\mathbf{1} = \mathbf{1}, \quad \boldsymbol{\theta} \geq 0 \end{array}$$

Weight λ controls strength of prior
Constraints ensure valid bigram matrix

Regularized Optimization Problem

Our optimization problem is non-concave:

$$\max_{\boldsymbol{\theta}} \quad \frac{1}{C}\sum_{i=1}^{n} \log \sum_{\mathbf{z}\in\sigma(\mathbf{x})} \prod_{j=2}^{|\mathbf{x}|} P(z_j|z_{j-1}) - \lambda\mathcal{D}(\boldsymbol{\phi},\boldsymbol{\theta})$$

$$\text{subject to} \quad \boldsymbol{\theta}\mathbf{1} = \mathbf{1}, \quad \boldsymbol{\theta} \geq 0$$

Summation over hidden ordered documents couples the variables.
It's non-concave, so we use an EM algorithm.

Derivation of EM algorithm

Let $\mathcal{O}(\boldsymbol{\theta}) \equiv \ell(\boldsymbol{\theta}) - \lambda \mathcal{D}(\boldsymbol{\phi}, \boldsymbol{\theta})$

By Jensen's inequality, lower-bound $\mathcal{O}$:

$$
\begin{aligned}
\mathcal{O}(\boldsymbol{\theta}) &= \frac{1}{C} \sum_{i=1}^{n} \log \sum_{\mathbf{z} \in \sigma(\mathbf{x}_i)} P(\mathbf{z}|\boldsymbol{\theta}^{(t-1)}, \mathbf{x}) \frac{P(\mathbf{z}|\boldsymbol{\theta})}{P(\mathbf{z}|\boldsymbol{\theta}^{(t-1)}, \mathbf{x})} - \lambda \mathcal{D}(\boldsymbol{\phi}, \boldsymbol{\theta}) \\
&\geq \frac{1}{C} \sum_{i=1}^{n} \sum_{\mathbf{z} \in \sigma(\mathbf{x}_i)} P(\mathbf{z}|\boldsymbol{\theta}^{(t-1)}, \mathbf{x}) \log \frac{P(\mathbf{z}|\boldsymbol{\theta})}{P(\mathbf{z}|\boldsymbol{\theta}^{(t-1)}, \mathbf{x})} - \lambda \mathcal{D}(\boldsymbol{\phi}, \boldsymbol{\theta}) \\
&\equiv \mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\theta}^{(t-1)})
\end{aligned}
$$

Lower bound $\mathcal{L}$ involves $P(\mathbf{z}|\boldsymbol{\theta}^{(t-1)}, \mathbf{x})$

- Probability of hidden ordering under previous iteration's model
- Computed in the E-step

Derivation of EM algorithm

EM iteratively maximizes lower bound

$$\begin{aligned} \max_{\boldsymbol{\theta}} \quad & \mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\theta}^{(t-1)}) \\ \text{subject to} \quad & \boldsymbol{\theta}\mathbf{1} = \mathbf{1} \end{aligned}$$

Setting the partial derivatives of the Lagrangian to zero leads to the following update:

$$\theta_{uv}^{(t)} \equiv P(v|u; \boldsymbol{\theta}^{(t)}) \propto \sum_{i=1}^{n} \sum_{\mathbf{z} \in \sigma(\mathbf{x}_i)} P(\mathbf{z}|\mathbf{x}_i, \boldsymbol{\theta}^{(t-1)}) c_{uv}(\mathbf{z}) + \lambda \frac{C}{W} \phi_{uv}$$

This is the M-step.

- First term: expected count of "uv"; Second term: pulls toward prior
- $c_{uv}(\mathbf{z})$ is count of "uv" in $\mathbf{z}$
- Normalize over $v = 1 \ldots W$

Approximate E-step

Update equation has a computational problem:

$$\theta_{uv}^{(t)} \propto \sum_{i=1}^{n} \sum_{\mathbf{z} \in \sigma(\mathbf{x}_i)} P(\mathbf{z}|\mathbf{x}_i, \boldsymbol{\theta}^{(t-1)}) c_{uv}(\mathbf{z}) + \frac{C}{W} \phi_{uv}$$

$\sigma(\mathbf{x})$ can be huge

- Estimate $\sum_{\mathbf{z} \in \sigma(\mathbf{x}_i)} P(\mathbf{z}|\mathbf{x}_i, \boldsymbol{\theta}^{(t-1)}) c_{uv}(\mathbf{z})$ with importance sampling
- Monte Carlo approximation using re-weighted samples from an easy-to-sample distribution
- Based on sampling/weighting scheme in [Rabbat et al. NIPS 2007]

EM algorithm recap

Input:

- BOW documents $\{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$
- a prior bigram LM ϕ
- weight λ

1. $t = 1$. Initialize $\boldsymbol{\theta}^{(0)} = \phi$.
2. Repeat until the objective $\mathcal{O}(\boldsymbol{\theta})$ converges:
    1. (E-step) Approximate $\sum_{\mathbf{z} \in \sigma(\mathbf{x}_i)} P(\mathbf{z}|\mathbf{x}_i, \boldsymbol{\theta}^{(t-1)}) c_{uv}(\mathbf{z}), i = 1, \ldots, n$
    2. (M-step) Compute $\boldsymbol{\theta}^{(t)}$ using update equation. Let $t = t + 1$.

Output: The recovered bigram LM $\boldsymbol{\theta}$

A prior bigram LM ϕ

Prior LM used to avoid local optima

- Our prior uses no extra language knowledge
  - (can and should be included for specific domains)
- Frequency of document co-occurrence

$$\phi_{uv} \equiv P(v|u;\boldsymbol{\phi}) \propto \sum_{i=1}^{n} \delta(u,v|\mathbf{x})$$

- $\delta(u,v|\mathbf{x}) =$
  - 1, if words u and v both occur in BOW $\mathbf{x}$
  - 0, otherwise
- Asymmetric because of normalization

Recovering documents using $\boldsymbol{\theta}$

Formulate document recovery as:

$$\mathbf{z}^* = \text{argmax}_{\mathbf{z} \in \sigma(\mathbf{x})} P(\mathbf{z}|\boldsymbol{\theta})$$

Use memory-bounded A* search with an admissible heuristic

Search space

- State = ordered, partial document
- Successor state = append one more unused word in $\mathbf{x}$
- Goal = any complete document using all the words in $\mathbf{x}$

In our case, A* chooses the successor with the highest $f = g + h$

- g = log probability of the successor state
- h = heuristic value estimating probability from successor state to goal

A* heuristic

Requirement: upper bound the remaining log probability

Main idea: choose the "best history" for each word

- Let $(c_1, \ldots, c_W)$ = count vector for the remaining BOW
- One admissible heuristic is

$$h = \log \prod_{u=1}^{W} (\max_v \theta_{vu})^{c_u}$$

- "Best history" v chosen from unused words or last word in partial doc
- Upper bound because, in reality, u usually contributes less than θ_{vu}

Any questions so far?

So far

- Problem formulation
- EM algorithm for bigram LM recovery
- Prior bigram LM based on document co-occurrence
- A^* search for document recovery

Next: experimental results

Corpora

SVitchboard 1 [King et al. 2005]

- Small vocabulary Switchboard, phone conversation transcripts, 6 versions
- "okay i enjoyed talking to you"
- "take a twenty two and go out"
- "you know you can't you can't make it"

SumTime-Meteo [Sripada et al. 2003]

- Weather forecasts for offshore oil rigs in the North Sea
- "Low over the Norwegian Coast will move slowly NNW"
- "A weak ridge will move East across the North Sea during Friday"

Data sets

Smallish, due to efficiency issues

Corpus	$W-1$	# Docs	# Tokens	$\lvert\mathbf{x}\rvert-1$
SV10	10	6775	7792	1.2
SV25	25	9778	13324	1.4
SV50	50	12442	20914	1.7
SV100	100	14602	28611	2.0
SV250	250	18933	51950	2.7
SV500	500	23669	89413	3.8
SumTime	882	3341	68815	20.6

- SV10–SV500: SVitchboard using different vocabulary sizes

We recover sensible bigrams in $\boldsymbol{\theta}$

Most demoted and promoted bigrams in $\boldsymbol{\theta}$ compared to prior ϕ (sorted by the ratio θ_{hw}/ϕ_{hw} on SV500)

h	$w \downarrow$	$w \uparrow$
i	yep, bye-bye, ah, goodness, ahead	mean, guess, think, bet, agree
you	let's, us, fact, such, deal	thank, bet, know, can, do
right	as, lot, going, years, were	that's, all, right, now, you're
oh	thing, here, could, were, doing	boy, really, absolutely, gosh, great
that's	talking, home, haven't, than, care	funny, wonderful, true, interesting, amazing
really	now, more, yep, work, you're	sad, neat, not, good, it's

Our $\boldsymbol{\theta}$ has good test set perplexity

- Train on $\mathbf{x}_1 \ldots \mathbf{x}_n$, test on ordered documents $\mathbf{z}_{n+1} \ldots \mathbf{z}_m$ (5-fold cross validation, all differences statistically significant)
- "Oracle" bigram trained on $\mathbf{z}_1 \ldots \mathbf{z}_n$ to provide lower bound (Good-Turing)

Corpus	unigram	prior ϕ	$\boldsymbol{\theta}$	oracle	1 EM iter
SV10	7.48	6.52	6.47	6.28	$<$1s
SV25	16.4	12.3	11.8	10.6	0.1s
SV50	29.1	19.6	17.8	14.9	4s
SV100	45.4	29.5	25.3	20.1	11s
SV250	91.8	60.0	47.3	33.7	8m
SV500	149.1	104.8	80.1	50.9	3h
SumTime	129.7	103.2	77.7	10.5	4h

Our $\boldsymbol{\theta}$ reconstructs $\mathbf{z}$ from $\mathbf{x}$ better

- Recall $\mathbf{z}^* = \text{argmax}_{\mathbf{z}\in\sigma(\mathbf{x})} P(\mathbf{z}|\boldsymbol{\theta} \text{ or } \phi)$
- Document recovery results using memory-bounded A* search

Accuracy %	whole doc	word pair	word triple
ϕ	30.2	33.0	11.4
$\boldsymbol{\theta}$	31.0	35.1	13.3

(SV500, 5-fold CV, all differences statistically significant)

$\mathbf{z}$ by ϕ	$\mathbf{z}$ by $\boldsymbol{\theta}$
just it's it's it's just going	**it's just it's just it's going**
it's probably out there else something	**it's probably something else out there**
the the have but it doesn't	**but it doesn't have the the**
you to talking nice was it yes	**yes it was nice talking to you**
that's well that's what i'm saying	**well that's that's what i'm saying**
a little more here home take	**a little more take home here**
and they can very be nice too	**and they can be very nice too**
i think well that's great i'm	**well i think that's great i'm**
but was he because only always	**but only because he was always**

Discussion

- Also experimented with two other priors
- Unigram prior, ignores word history $\phi_{uv}^{unigram} \propto \sum_{i=1}^{n} x_{iv}$
- Permutation-based prior
  - Based on expected bigram count in all unique orderings of BOWs

$$\phi_{uv}^{perm} \equiv P(v|u; \boldsymbol{\phi}^{perm}) \propto \sum_{i=1}^{n} \mathsf{E}_{\mathbf{z} \in \sigma(\mathbf{x}_i)}[c_{uv}(\mathbf{z})]$$

  - Assumes uniform probability over permutations $\mathbf{z}$; Closed form solution

Detailed experimental comparisons in

Jerry Zhu, Andrew B. Goldberg, Michael Rabbat, and Robert Nowak. *Learning bigrams from unigrams.* (ACL 2008).

Conclusions

Summary

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- EM algorithm
- Prior bigram models
- A^* search for document recovery
- Recovered bigram LMs outperform other naïve models

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Future work

- Improve efficiency
- Extend to trigram and higher-order models
- Include some ordered documents / phrases if available
- Adapt a general English LM using only BOWs from a special domain

We thank

Wisconsin Alumni Research Foundation
NSF CCF-0353079, CCF-0728767
NSERC of Canada

and you.