

# Analysis of Electrical Characteristics using a Lambert W-Function Technique and MATLAB Simulation for Dye Sensitised ZnO Solar Cell

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**Abstract:** Present article deals with analysis of output characteristics in ZnO based Dye Sensitised Solar Cell (DSSC) using a Lambert W-function technique and MATLAB Simulation. We have used an analytical approach having no approximation for extracting various parameters of DSSC, a well known Lambert W-function technique. The model used in present work is single diode model including the capacitance in equivalent circuit and hence study its effect on current-voltage characteristics and other DSSC parameters. Results hence obtained are discussed and validated using MATLAB Simulation.

**Keywords:** Dye Sensitised Solar Cell, Lambert W-Function, MATLAB, Electrochemical Impedance Spectroscopy, Simulation.

## 1. INTRODUCTION

DSSC an electrochemical photovoltaic device consist of mesoporous semiconductor wetted with photoactive dye and plunged into a liquid electrolyte [1]. It differs from the conventional solar cell as it separates the function of light absorption from charge carrier transport. So far study of solar cell characteristics was dominated by their static behaviour [2], few attempts have been made to observe the dynamic behaviour of solar cells [3-5]. Capacitance characteristics which give rise to dynamic behavior are unique components of DSSCs arising due to oxide/dye/electrolyte interface of the order of mF/cm<sup>2</sup> [5], while in Si solar cells capacitance of the order of  $\mu\text{F}/\text{cm}^2$  [6] is due to junction capacitance. Han and Dai [3,4] gave a qualitative approach to study effect of capacitance on DSSC which was further improved by Hanmin *et al.* [7] when they introduced both qualitative and quantitative analysis for capacitive effect. Han *et al.* gave detailed impedance analysis using EIS and predicted the capacitive part in internal impedance of DSSC [8, 9] Merhej *et al.* [10] provided a computational simulation technique using MATLAB to study the effect of capacitance on current-voltage characteristics of solar cells.

Present work takes into account both resistive and capacitive effect while studying DSSCs. Both qualitative and quantitative analysis has been performed to conclude the dominance of capacitance in DSSCs. Capacitive effects in DSSC are investigated and hence a new single diode model having both resistance and capacitance is proposed in DSSC for improving the existing mathematical model [11]. Equivalent circuit of DSSC has been used for analytical analysis

and statistical analysis of the experimental data. For verification of our model we used the accurate solar cell simulator developed by Ishaque *et al.* [12]. The use of this simulator had an advantage over developing our own as it used a different basic model for the thin film solar cells, hence giving proof of accuracy for our values calculated using our DSSC mathematical model and the statistical analysis of the experimental data.

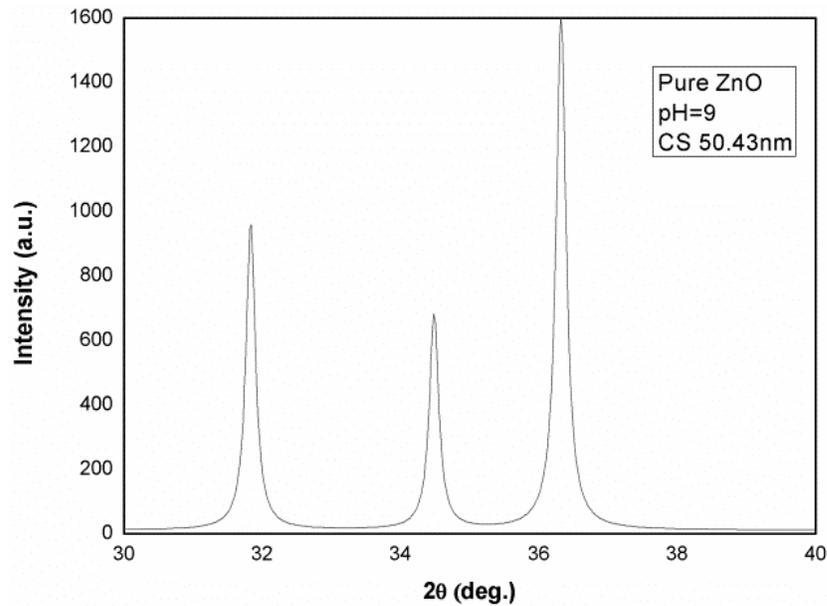
## 2. STATISTICAL ANALYSIS OF THE EXPERIMENTAL DATA AND EQUIVALENT CIRCUIT

### 2.1. Preparation of DSSC

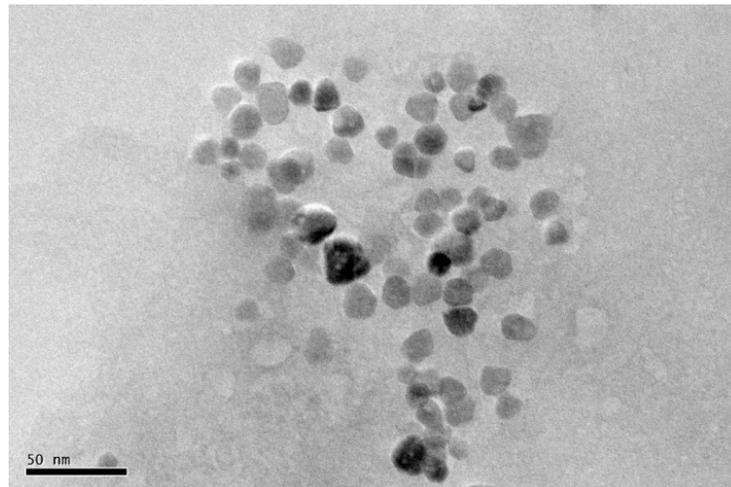
ZnO nano particles were prepared using Sol-gel technique. Nanoparticles were prepared by dissolving 0.2M Zinc acetate dehydrate [ $\text{Zn}(\text{CH}_3\text{COO})_2 \cdot 2\text{H}_2\text{O}$ ] in methanol at room temperature. Then this solution was mixed ultrasonically for 2 hours at 25°C. Transparent and clear solution was obtained. NaOH was then added for the desired pH level (pH 9) in the solution. The solution was again stirred ultrasonically for 60 min. The resulting solution was kept undisturbed. Precipitates collected at the bottom were filtered and washed with excess methanol to remove the starting material. Precipitates were dried for 15 min at 80°C on hot plate [13]. The structural and optical properties of synthesized ZnO nanoparticles were studied. The XRD measurements were carried out using Bruker AXS – D8 discover diffractometer as shown in Fig. (1). XRD showed that average size of ZnO crystalline particles was 50 nm and was confirmed by using TECNAI G<sup>2</sup> T30, u-TWIN TEM, as shown in Fig. (2).

DSSCs were fabricated using the synthesized powder. The porous ZnO film was deposited on Indium-tin-oxide (ITO)-coated glass substrate by doctor blade technique. The substrate was cleaned using acetone, methanol and distilled water in sequence in an ultrasonic bath. ZnO powder(1.2 gm)

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**Fig. (1).** XRD of ZnO nanoparticles.



**Fig. (2).** TEM of ZnO particles.

was grounded by a mortar and pestle with 4ml deionised water, and polyethylene glycol(PEG<sub>20,000</sub>, .0.5gm) to break up the aggregated particles into a dispersed paste [14]. The paste was spread on the surface of conducting substrate with a glass rod. After drying in the air the films were sintered for 30 minutes at 400<sup>0</sup> C in air. The resulting film was immersed in a 3.0x10<sup>-4</sup> M solution of acridine orange dye in ethanol for 24 hours. To minimise adsorption of impurities from moisture in the ambient air, the electrode was dipped in the dye solution while they were still warm (80<sup>0</sup>C).The dye covered electrodes were then rinsed with ethanol to remove excess dye on the surface and dried at room temperature. A sandwich-type DSSC was fabricated with dye sensitised electrode, a thin platinum sheet as counter electrode and an electrolyte as spacer.

## 2.2. Electrochemical Impedance Spectroscopy (EIS) & Equivalent Circuit

Electrochemical impedance spectroscopy (EIS) has been used to analyze internal resistance of DSSC and three internal resistances have been found. [8, 9]. In Plot of DSSC

three semicircles are observed in the frequency range of 10<sup>-1</sup> – 10<sup>6</sup> Hz which correspond to three types of impedance namely Z<sub>1</sub>, Z<sub>2</sub> and Z<sub>3</sub>. Z<sub>1</sub> is related to charge transport at the Pt counter electrode in the high frequency (KHz range) region. Z<sub>2</sub> is the impedance observed in middle frequency (10- 100 Hz) region at TiO<sub>2</sub>/ dye/ electrolyte interface. Impedance Z<sub>3</sub> due to Nernstian diffusion is prominent in low frequency (m Hz) region. [8]. Origin of Z<sub>1</sub> and Z<sub>3</sub> are similar to those reported by Kern *et al.* [5]. Han L *et al.* [8] proved that Z<sub>2</sub> is impedance of diode.

EIS of a DSSC prepared at our centre having ZnO as electrode and platinum as counter electrode using potentiostat (Gamry potentiostat and analyzer) also showed similar results as shown in Fig. (3).

Hence it can be concluded that the DSSC model consists of series, shunt resistive and capacitive parts. Z<sub>1</sub> is parallel combination of R<sub>1</sub> and C<sub>1</sub>, Z<sub>2</sub> is parallel combination of R<sub>2</sub> and C<sub>2</sub>, and Z<sub>3</sub> is parallel combination of R<sub>3</sub> and C<sub>3</sub>. From EIS plot order of C<sub>1</sub> is estimated to be 2.0 to 14.0 μF and C<sub>2</sub> from 0.3 to 70 mF.

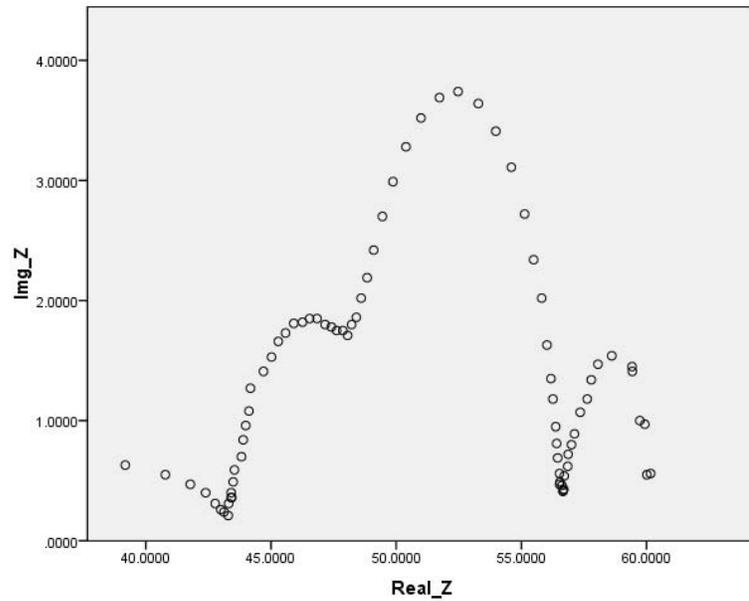


Fig. (3). EIS of DSSC.

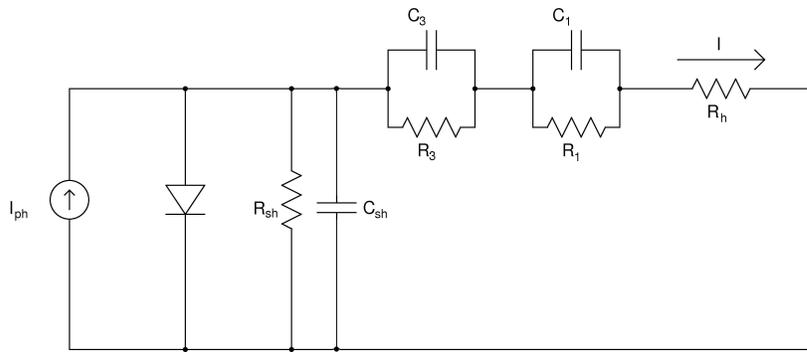


Fig. (4a). Equivalent circuit of DSSC.

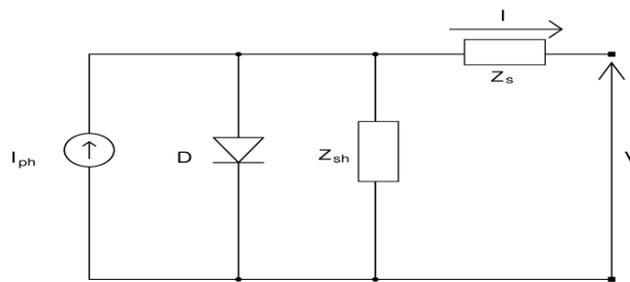


Fig. (4b). simplified equivalent circuit of DSSC.

Equivalent circuit of dye sensitized solar cell (DSSC) is shown in Fig. (4a, 4b).

**2.3. Theoretical Analysis**

Single diode model for DSSC including capacitance was taken into consideration to derive the current voltage relation. The equation of equivalent circuit can be written as:

$$i = i_{ph} - i_0 \left( e^{\frac{V + iZ_s}{nK}} - 1 \right) - \frac{V + iZ_s}{Z_{sh}} \tag{1}$$

Where  $K = kT/q$ .

In this equation  $i_{ph}$  represents photocurrent,  $i_0$  Initial current,  $Z_s$  series resistance  $Z_{sh}$  shunt resistance,  $n$  ideality factor,  $q$  elementary electric charge,  $k$  Boltzmann constant and  $T$  temperature,  $Z_s$  and  $Z_{sh}$  contains both resistance and capacitance.

Mathematical expressions for  $Z_s$  and  $Z_{sh}$  are:

$$Z_s = \frac{R_s(1 - I\omega C_s R_s)}{1 + \omega^2 R_s^2 C_s^2}$$

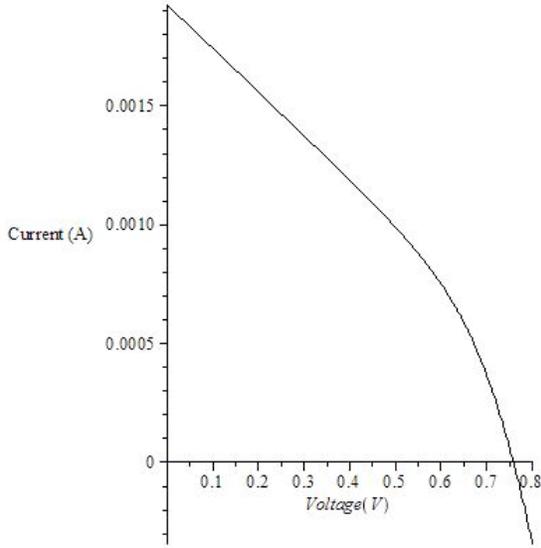


Fig. (5). I-V curve of DSSC.

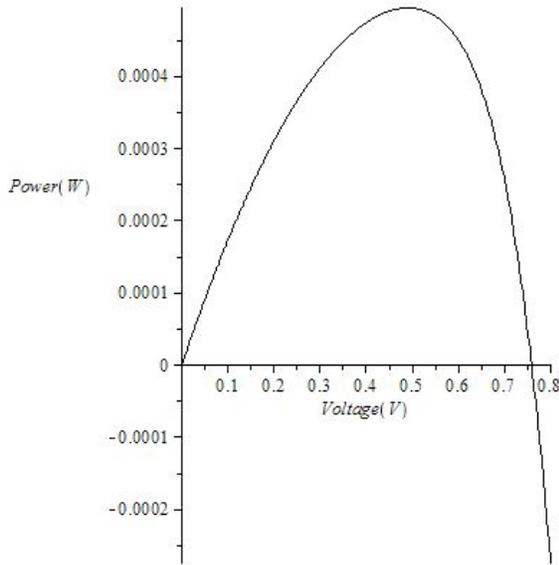


Fig. (6). P-V curve of DSSC.

$$Z_{sh} = \frac{R_{sh}(1 - I\omega R_{sh} C_{sh})}{1 + \omega^2 R_{sh}^2 C_{sh}^2}$$

Equation (1) is transcendental in nature hence it is not possible to solve it explicitly. The explicit solution for current and voltage can be expressed using Lambert W function [11, 15].

$$V = -iZ_s - iZ_{sh} + i_{ph}Z_{sh} - nK \text{ LambertW} \left( \frac{i_o Z_{sh} e^{\frac{Z_{sh}(i - i_{ph} - i_o)}{nK}}}{nK} \right) + i_o Z_{sh} \quad (2)$$

$$i = -\frac{V}{Z_s} + \frac{1}{Z_s} \left( \left( -\text{LambertW} \left( \frac{Z_{sh}(Z_s i_{ph} + Z_s i_o + V)}{nK(Z_{sh} + Z_s)} \right) \right) \frac{Z_s i_o Z_{sh} e^{\frac{Z_{sh}(Z_s i_{ph} + Z_s i_o + V)}{nK(Z_{sh} + Z_s)}}}{Z_s nK + Z_{sh} nK} \right) + \frac{Z_{sh}(Z_s i_{ph} + Z_s i_o + V)}{nK(Z_{sh} + Z_s)} \Big) nK \quad (3)$$

Expression for open circuit voltage and short circuit current can be obtained by substituting  $i=0$  in eq. 2 and  $V=0$  in eq.3:

$$V_{oc} = i_{ph} Z_{sh} - nK \text{ LambertW} \left( \frac{Z_{sh}(i_{ph} - i_o)}{nK} \right) + i_o Z_{sh} \quad (4)$$

$$i_{sc} = \frac{\left( -\text{LambertW} \left( \frac{Z_{sh}(Z_s i_{ph} + Z_s i_o)}{nK(Z_{sh} + Z_s)} \right) \right) \frac{Z_s i_o Z_{sh} e^{\frac{Z_{sh}(Z_s i_{ph} + Z_s i_o)}{nK(Z_{sh} + Z_s)}}}{Z_s nK + Z_{sh} nK} + \frac{Z_{sh}(Z_s i_{ph} + Z_s i_o)}{nK(Z_{sh} + Z_s)} \Big) nK}{Z_s} \quad (5)$$

Expressions for calculating maximum power point are:

$$P = i \left( -iZ_s - iZ_{sh} + i_{ph}Z_{sh} - nK \text{ LambertW} \left( \frac{i_o Z_{sh} e^{\frac{Z_{sh}(i - i_{ph} - i_o)}{nK}}}{nK} \right) + i_o Z_{sh} \right) \quad (6)$$

and

$$P = \frac{1}{Z_s} \left( -V^2 + \left( -\text{LambertW} \left( \frac{Z_{sh}(Z_s i_{ph} + Z_s i_o + V)}{nK(Z_{sh} + Z_s)} \right) \right) \frac{Z_s i_o Z_{sh} e^{\frac{Z_{sh}(Z_s i_{ph} + Z_s i_o + V)}{nK(Z_{sh} + Z_s)}}}{Z_s nK + Z_{sh} nK} \right) + \frac{Z_{sh}(Z_s i_{ph} + Z_s i_o + V)}{nK(Z_{sh} + Z_s)} \Big) nK \quad (7)$$

To achieve maximum power points the first partial derivatives of power w.r.t current and voltage should be set equal to zero [11, 15]: Using explicit solutions of current voltage relations of DSSC (that includes capacitance) different parameters were extracted viz.  $I_{sc}$ ,  $V_{oc}$ ,  $I_{max}$ ,  $V_{max}$ . The respective values of extracted parameters are;

$I_{sc} = 0.0019238$ ,  $V_{oc} = 0.758651$ ,  $I_{max} = 0.00101$ ,  $V_{max} = 0.490194$ . I-V and P-V curves were also plotted using W-function results (Fig. 5 and 6).

### 3. MATLAB SIMULATION

Availability of PV module-simulator in MATLAB platform is a tool of immense advantage for researchers as MATLAB has become a de-facto standard in various researches and engineering disciplines. Besides having better accuracy this simulator is simple and fast, making it a perfect choice for analyzing behaviour of different solar cells including DSSC. The input parameters for the simulation in present work are  $I_{sc}$ ,  $V_{oc}$ ,  $I_m$  and  $V_m$ . Using the values of these parameters obtained from W-function technique, static and dynamic I-V, P-V, curves are plotted using PV module-simulator in MATLAB Fig. (7, 8). The dynamic behaviour in the Fig. (7, 8) is expressing the I-V characteristics taking capacitive effects of DSSC while static one are without considering them. It is from the curves that maximum power point (PV) changes with taking capacitive effects in mathematical model of DSSC. The same has been observed analytically using Maple and by simulation using MATLAB. The effect can be explained by charging, discharging of various capacitances present in the DSSC (equivalent circuit).

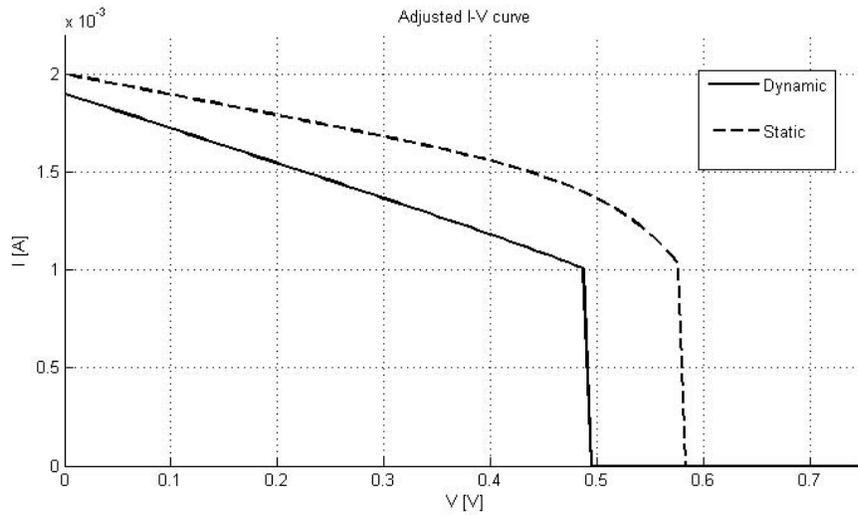


Fig. (7). Simulated I-V curves of DSSC.

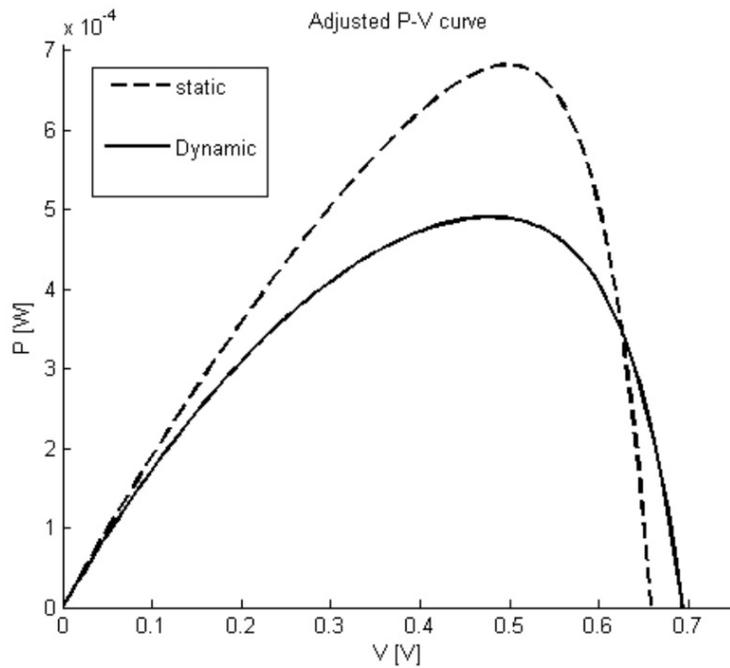


Fig. (8). Simulated P-V curves of DSSC.

#### 4. DISCUSSION AND CONCLUSION

The existence of double charged layer at semiconductor and electrolyte interface leads to capacitance characteristics. The presence of capacitance in DSSC was confirmed by EIS. The traditional IPCE measurement method of solar cell is not applicable to DSSC because of its capacitance characteristics. The effect of capacitance on output characteristic is analytically proved by Lambert W-function technique and validated by MATLAB Simulation as reflected in I-V and P-V curves with good accuracy. Present article can be extended to study detailed analysis of capacitive effects of capacitance in various solar cells and hence can lead to better understanding of internal mechanisms in solar cells.

#### CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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