Log Intuition: Length Function

\[ \log_{10}(888) \approx 3 \]
Log Intuition: Length Function

$$\log_{10}(888) \approx 3$$

err $\leq 1$
Log Intuition: Length Function

\[ \log_{10}(12345) \approx 5 \]

err $\leq 1$
Log Intuition: Length Function

\[ \log_{10}(12) \approx 2 \]

err \( \leq 1 \)
Log Intuition: Length Function

\[ \log_{10}(787878) \approx 6 \]

err \( \leq 1 \)
Log Intuition: Length Function

\[ \log_{10}(1234567890) \approx 10 \]

\[ \text{err} \leq 1 \]
Binary Examples

\[ \log_2(101) \approx 3 \]

\[ \text{err} \leq 1 \]
Binary Examples

\[ \log_2(11111) \approx 5 \]

err <= 1
Binary Examples

\[ \log_2(1001) \approx 4 \]

err \( \leq 1 \)
Binary Examples

\[ \log_2(1111100000) \approx 10 \]

\[ \text{err} \leq 1 \]
Assume:
\[ B^N = X \]

Then:
\[ \log_B(X) = N \]
Assume:
\[ B^N = X \]

Then:
\[ \log_B(X) = N \]

Use of logs in this class:
- \( B \) is always 2
- \( X \) is a power of 2
- \( N \) is a natural number
**Log Definition**

Assume:

\[ 2^N = X \]

Then:

\[ \log(X) = N \]

Use of logs in this class:
- \( B \) is always 2
- \( X \) is a power of 2
- \( N \) is a natural number
Intuition: Tree Leaves $\Rightarrow$ Height

leaves=$2$

height=$1$
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(2) = 1$

leaves = 2

height = 1
Intuition: Tree Leaves => Height

leaves=4

log(4) = 2

height=2
Intuition: Tree Leaves $\Rightarrow$ Height

leaves = 8

$log(8) = 3$
Intuition: Tree Leaves $\Rightarrow$ Height

leaves = 16

$\log(16) = 4$
Intuition: Tree Leaves $\Rightarrow$ Height

leaves = 32

$\log(32) = 5$
Intuition: Tree Leaves $\Rightarrow$ Height

$$\log(32) = 5$$
Intuition: Tree Leaves $\Rightarrow$ Height

$log(64) = 6$

$\times 2 \quad +1$

$log(64) = 6$
Intuition: Tree Leaves $\Rightarrow$ Height

$log(64) = 6$
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(128) = 7$

$\times 2 + 1$

$\log(128) = 7$
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(128) = 7$
Intuition: Tree Leaves $\implies$ Height

log(256) = 8

$\times 2 + 1$

log(256) = 8
Intuition: Tree Leaves $\Rightarrow$ Height

$log(256) = 8$
Intuition: Tree Leaves $\Rightarrow$ Height

\[ \log(512) = 9 \times 2 + 1 \]

\[ \log(512) = 9 \]
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(512) = 9$
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(1K) = 10$

$\times 2 \quad + 1$

\[
\log(1K) = 10
\]
Intuition: Tree Leaves $\Rightarrow$ Height

$log(1K) = 10$
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(1K) = 10$

$\times 4 \quad + \quad ?$

$\downarrow \quad \downarrow$

$log(1K) = 10$
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(4K) = 12$
Intuition: Tree Leaves $\Rightarrow$ Height

$log(4K) = 12$
Intuition: Tree Leaves => Height

\[
\log(4K) = 12
\]
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(32K) = 15$

$\times 8 \quad +3$

$\log(32K) = 15$
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(\ldots) = \ldots$
Intuition: Tree Leaves $\Rightarrow$ Height

$log(\ldots) = \ldots$

$\times 2^N + ?$

\[
\begin{align*}
\downarrow & \quad \downarrow \\
\log(\ldots) & = \ldots
\end{align*}
\]
Intuition: Tree Leaves $\Rightarrow$ Height

$x2^N + N$

$log(...)$ $\Rightarrow$ $\ldots$
Intuition: Tree Leaves => Height

\[ \log(\ldots) = \ldots \]
Intuition: Tree Leaves $\Rightarrow$ Height

$xN$ \quad +? \\
\downarrow \quad \downarrow \\
\log(\ldots) = \ldots
Intuition: Tree Leaves $\Rightarrow$ Height

$xN + \log(N) \Rightarrow \log(\ldots) = \ldots$
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(A) = \log(A) \times N + \log(N)$
Intuition: Tree Leaves $\Rightarrow$ Height

$log(A \times N) = log(A) + log(N)$
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(32) = 5$

leaves=32

height=5
Intuition: Tree Leaves $\Rightarrow$ Height

$\text{leaves} = 32$

$\log(32) = 5$
Intuition: Tree Leaves => Height

leaves = 32

log(32) = 5

height = 5
Intuition: Tree Leaves => Height

\begin{align*}
\log(4 \times 8) &= \log(4) + \log(8) = 2 + 3 \\
\log(32) &= 5
\end{align*}
Intuition: Tree Leaves $\Rightarrow$ Height

$\log(32) = 5$

$\log(4 \times 8) = \log(4) + \log(8) = 2 + 3$

$\log(32) = 5$
Intuition: Tree Leaves => Height

\[
\begin{align*}
\log(2 \times 16) &= \log(2) + \log(16) = 1 + 4 \\
\log(4 \times 8) &= \log(4) + \log(8) = 2 + 3 \\
\log(32) &= 5
\end{align*}
\]
Log of kilo, mega, giga

\[ \log(4K) = \log(4) + \log(K) \]
Log of kilo, mega, giga

\[ \log(4K) = \log(4) + \log(K) = 2 + 10 = 12 \]
Log of kilo, mega, giga

\[
\begin{align*}
\log(4K) &= \log(4) + \log(K) = 2 + 10 = 12 \\
\log(512M) &= \log(512) + \log(M)
\end{align*}
\]
Log of kilo, mega, giga

\[
\begin{align*}
\log(4K) &= \log(4) + \log(K) = 2 + 10 = 12 \\
\log(512M) &= \log(512) + \log(M) = 9 + 20 = 29
\end{align*}
\]
Log of kilo, mega, giga

\[ \log(4K) = \log(4) + \log(K) = 2 + 10 = 12 \]
\[ \log(512M) = \log(512) + \log(M) = 9 + 20 = 29 \]
\[ \log(8G) = \log(8) + \log(G) \]
Log of kilo, mega, giga

\[ \log(4K) = \log(4) + \log(K) = 2 + 10 = 12 \]
\[ \log(512M) = \log(512) + \log(M) = 9 + 20 = 29 \]
\[ \log(8G) = \log(8) + \log(G) = 3 + 30 = 33 \]
Log of kilo, mega, giga

\[
\begin{align*}
\log(4K) &= \log(4) + \log(K) = 2 + 10 = 12 \\
\log(512M) &= \log(512) + \log(M) = 9 + 20 = 29 \\
\log(8G) &= \log(8) + \log(G) = 3 + 30 = 33
\end{align*}
\]

Implication: only need to remember a few logs:
- \( \log(1) \), \( \log(2) \), \( \log(4) \), ..., \( \log(512) \)
- \( \log(K) = 10 \)
- \( \log(M) = 20 \)
- \( \log(G) = 30 \)
- \( \log(T) = 40 \)
Intuition: Bit Lengths

1 byte:

????????
Intuition: Bit Lengths

8 bytes:
Intuition: Bit Lengths

<table>
<thead>
<tr>
<th>addrs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>???????</td>
</tr>
<tr>
<td>001</td>
<td>???????</td>
</tr>
<tr>
<td>010</td>
<td>???????</td>
</tr>
<tr>
<td>011</td>
<td>???????</td>
</tr>
<tr>
<td>100</td>
<td>???????</td>
</tr>
<tr>
<td>101</td>
<td>???????</td>
</tr>
<tr>
<td>110</td>
<td>???????</td>
</tr>
<tr>
<td>111</td>
<td>???????</td>
</tr>
</tbody>
</table>
Intuition: Bit Lengths

```
  0 1 0 1 0 1 0 1
  0 0 1 0 0 1 0 0
  0 1 0 1 0 1 0 1
  0 0 1 0 0 1 0 0
  0 1 0 1 0 1 0 1
  0 0 1 0 0 1 0 0
  0 1 0 1 0 1 0 1
  0 0 1 0 0 1 0 0

addr
0 000
1 001
0 010
1 011
0 100
1 101
0 110
1 111
```
Intuition: Bit Lengths

access 101

addr

0 000
1 001
0 010
1 011
0 100
1 101
0 110
1 111
Intuition: Bit Lengths

access 000

addr

0 000
0 001
1 010
0 011
1 011
0 100
1 101
1 110
1 111
Intuition: Bit Lengths

access 011

addr

0

0 000

1 001

0 010

1 011

0 100

1 101

0 110

1 111
Intuition: Bit Lengths

```
0 000
0 001
1 001
0 010
1 010
0 011
1 011
1 100
0 100
1 101
1 101
0 110
1 110
1 111
1 111
```

Intuition: Bit Lengths

8-byte address space
Intuition: Bit Lengths

height: \( \log(8) = 3 \)

![Binary Tree Diagram]

- 8-byte address space
- Height: \( \log(8) = 3 \)
Intuition: Bit Lengths

height: $\log(8)=3$

3-bit addrs

8-byte address space

addr:
| 000 | ??????? |
| 001 | ??????? |
| 010 | ??????? |
| 011 | ??????? |
| 100 | ??????? |
| 101 | ??????? |
| 110 | ??????? |
| 111 | ??????? |
# Addresses

<table>
<thead>
<tr>
<th>address</th>
<th>combinations (binary)</th>
<th>log(combinations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>??</td>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>###</td>
<td>1000</td>
<td>3</td>
</tr>
<tr>
<td>???????</td>
<td>100000000000</td>
<td>10</td>
</tr>
</tbody>
</table>