
Hyunwoo J. Kim, Nagesh Adluru, Heemanshu Suri, Baba C. Vermuri, Sterling C. Johnson, Vikas Singh

http://pages.cs.wisc.edu/~hwkim/projects/riem-mem/

THE MOTIVATING PROBLEM
- Develop nonlinear mixed effects models for responses (Y) that lie on curved spaces such as the manifold of symmetric positive definite (SPD) matrices.
- Characterize complex morphological longitudinal brain changes using Cauchy deformation tensors (CDTs) derived from MRI data.
- Capture subject-specific progression rate and disease onset time using nonlinear mixed effects models on CDTs.

EUCLIDEAN MIXED EFFECTS MODELS

- Euclidean linear mixed effects model (mean shift), and intercept (spatial shift).
- Euclidean mixed effects model: \( E(U) = \mu + \sum_{i} \alpha_i \bar{U}^i + \sum_{i} \sum_{j} \beta_{ij} \bar{U}^i \bar{U}^j + \epsilon \), where \( \bar{U}^i \) and \( \bar{U}^j \) are normally distributed.

RIEMANNIAN MIXED EFFECTS MODELS

- Riemannian linear mixed effects model (RLME): \( y_{ij} = f_0 + f_1 U_{ij} + f_2 U_{ij}^2 + \epsilon \), where \( f_0, f_1, f_2 \) are parameters.
- Riemannian mixed effects model (RMLE): \( y_{ij} = f_0 + f_1 U_{ij} + f_2 U_{ij}^2 + \epsilon \), where \( f_0, f_1, f_2 \) are parameters.

REGRESSION ON MANIFOLDS: BASIC OPERATIONS

Operation Translation Addition Distance Mean Covariance
Euclidean \( \bar{X} + \alpha \) \( \bar{X} + \bar{V} \) \( \| \bar{X} - \bar{Y} \| \) \( \bar{X} \odot \bar{Y} \) \( \bar{X} \) \( \bar{X} \odot \bar{X} \)
Riemannian \( \bar{X} + \log \alpha \) \( \bar{X} + \bar{V} \) \( \| \bar{X} - \bar{Y} \| \) \( \bar{X} \odot \bar{Y} \) \( \bar{X} \) \( \bar{X} \odot \bar{X} \)

FEATURES FOR MORPHOMETRIC CHANGES

- CDT estimation: \( \dot{X} = \frac{d}{dt} X = V(X) \), where \( V \) is the deformation field.
- CDT differentiation: \( \frac{d}{dt} \dot{X} = \ddot{X} = \frac{d}{dt} V(X) \).
- CDT integration: \( X(t) = X(t_0) + \int_{t_0}^{t} \dot{X} dt \).

CAUCHY DEFORMATION TENSORS vs. DET(J)

- CDT: \( \det(J) \), where \( J \) is the Jacobian matrix.

RELATIVE SPEED MAPS OF MORPHOMETRIC CHANGES

GROUP DIFFERENCE ON SUBJECT-SPECIFIC SPATIAL SHIFTS

GROUP (GENDER) DIFFERENCE IN \( \bar{U} \) AVERAGED IN ROIs

REFERENCE