1. (Population and Parameter/Sample and Statistic)
   (a) (TRUE/FALSE): Parameters are random variables
   (b) (TRUE/FALSE): The sample mean and the sample variance are random variables
   (c) Suppose we have data listed as \((X_1, \ldots, X_n)\). Which of the following statistics are sensitive to outliers? Circle all that apply.
   i. Sample mean, \(\bar{X}\)
   ii. Sample median, \(X_{0.5}\)
   iii. Sample variance, \(\hat{\sigma}^2\)
   iv. Interquartile range, \(X_{0.75} - X_{0.25}\)
   v. The maximum, \(\max(X_1, \ldots, X_n)\)

2. Suppose we have \((X_1, \ldots, X_n)\) where \(X_i \sim F(\mu, \sigma^2)\) for some arbitrary \(F\). In elementary statistics classes, there is no mathematical justification as to why \(\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2\) is used instead of \(\frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2\) in estimating the population variance, \(\sigma^2\). In these series of (hopefully) short questions, we’ll provide one mathematical justification as why \(\frac{1}{n-1}\) is preferred over \(\frac{1}{n}\)
   (a) Is \(\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2\) an unbiased estimator for \(\sigma^2\)? A simple Yes/No will suffice.
   (b) Is \(\frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2\) an unbiased estimator for \(\sigma^2\)? You must provide mathematical justification for your answer.

3. Suppose we have \((X_1, \ldots, X_n)\) where \(X_i \sim F(\mu, \sigma^2)\) for some arbitrary \(F\) and we know what the population mean, \(\mu\), is. Consider the following estimators for the population variance

   Estimator 1: \(\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \mu)^2\)

   Estimator 2: \(\frac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^2\)

   (a) Which estimator is an unbiased estimate for the population variance? You must provide mathematical justification for your answer
(b) If $F$ is a normal distribution, what is the sampling distribution of the unbiased estimator? 
*Hint: You may need some constants to be multiplied to your estimator and use the definition of Chi-square discussed in class*

(c) If $F$ is any arbitrary distribution, what is the limiting distribution of the unbiased estimator? 
*Hint: You may need some constants to be multiplied and added to your estimator before you use the Central Limit Theorem. The answer is NOT Chi-Squared, $\chi^2$. 