Human Memory Search as Initial–Visit Emitting Random Walk

Kwang–Sung Jun† (kjun@discovery.wisc.edu), Xiaojin Zhu†, Timothy Rogers‡, Zhuan Yang§, Ming Yuan§,
Wisconsin Institute for Discovery, Department of Computer Sciences, Information, Statistics, University of Wisconsin–Madison, Department of Mathematical Sciences, Tsinghua University

**INITIAL–VISIT EMITTING (INVITE) RANDOM WALK**

***KEY***: Output the state only when visiting it for the first time.

- **n states, m initial distribution, P**: Markov chain (row stochastic)
- **ξ = cow, N = horse**,...
- **censored list is not a permutation of n items or a prefix of it.**
- **Does it produce every permutation? Or every prefix?**

*Transient state*: state that has nonzero probability of not coming back to itself in any finite time.

*Recurrent state*: state that is not transient. A is closed if i ∈ A and i implies a walk from i cannot reach j.

**The random walk runs indefinitely.**

**A censored list is not Markovian anymore.**

**GOOD NEWS**: captures important human behavior in a cognitive task (see below)

**BAD NEWS**: Parameter estimation is HARD!

**Main Contribution**

1. First tractable model for the maximum likelihood estimate (MLE) of INVITE
2. Consistency of the INVITE MLE

**Verbal Fluency: A Human Memory Search Task**

**TASK**: List examples of animals in 60 seconds without repetition

- Different categories possible: e.g., vehicles
- A "generic" task where participants must remember past productions, inhibit this, and focus on the task.

1. Clinical application: different neurological syndromes have different patterns in lists (e.g. repeated, more, less, irrelevant items).
2. Study of human memory: responses are runs of semantically related items.
3. Related structure in semantic representation
4. Our focus is on the second application, i.e., repetition is general, but can be allowed by a reduction (see future work).

**Consistency of INVITE MLE**

- **Arg**: α, γ, θ – INVITE (P)
- **Which**: (m, P), (m, Π), log Pr(α = m, P)

**Question**: Does (m, P) converge to (m, Π) ?

**A necessary condition: identifiability**

4. INVITE is not identifiable (adjusting self-transitions does not change the log likelihood).

5. INVITE with m > 0 (element-wise) and without self-transitions in m is identifiable.

**Challenge**: a common strategy is to show uniform convergence of the log likelihood, which is not true in INVITE MLE.

**Solution**: show the local uniform convergence

-uniformly convergent in an intersection of a max-norm ball and a subspace of “equivalent chain decomposition” around the true parameter (m, Π).

**Consistency**: if m > 0 (element-wise), INVITE MLE is consistent.

**Parameter Estimation: Regularized MLE**

- **Data**: Dnm = (d(1), ..., d(n))
- **Relax**: assumption true time-evolving random walk terminates after finite number of steps.
- **Initial distribution m**: MAP estimation (easy)
- **Transition Matrix P**: constrained (nonnegative, sum to 1)
- **Easier**: unconstrained parameterization

**Optimization problem**

**Experiment**: Toy

**Goal** (1): confirm the consistency result (2) compare with baselines

- Naive Random Walk (NRW)


**Use INVITE** to generate censored lists

**RESULTS**

- **Data**: Name m n Leash Min Max Mean Median
- **Fixed**: 452 4622 1 47 30.73 21

**Result**: negative log likelihood on holdout set (smaller is better)

- **Without**: some neg. bias

**References**


