Question 1: Simple Optimizations [8]

\[ f(x) = -x^5 + \frac{2}{3}x^4 + \frac{75}{3}x^3 + 5 \]

\[ f'(x) = -5x^4 + 10x^3 + 75x^2 \]
\[ = -5x^2(x^2 - 2x - 15) \]
\[ = -5x^2(x - 5)(x + 3) \]

So, \( x = -3, 0, 5 \) for \( f'(x) = 0 \). Now,

\[ f(-3) = -224.5, \quad f(0) = 5, \quad f(-3) = -166.5, \quad f(4) = 1221 \]

\( x = 5 \) is not in the interval \([-3.5, 4]\).
The minimum value of \( f(x) \) when \( x \in [-3.5, 4] \) is -224.5.

Question 2: 3-NN Classification [20]

a) The following table shows the distance between the given points with the training set points and the predicted class. The 3 nearest neighbors are given in bold.

<table>
<thead>
<tr>
<th>Points</th>
<th>(1, 1)</th>
<th>(2.2)</th>
<th>(1.2)</th>
<th>(3.25)</th>
<th>(2.7, 2.8)</th>
<th>(1.8, 3.2)</th>
<th>(0.9, 4)</th>
<th>(2.8, 4.1)</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.5, 1.5)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2.5</td>
<td>2.5</td>
<td>2</td>
<td>3.1</td>
<td>3.9</td>
<td>class1</td>
</tr>
<tr>
<td>(3, 2.2)</td>
<td>3.2</td>
<td>1.2</td>
<td>1.2</td>
<td>0.3</td>
<td>0.9</td>
<td>2.2</td>
<td>3.9</td>
<td>2.3</td>
<td>class2</td>
</tr>
<tr>
<td>(1.3, 3)</td>
<td>2.3</td>
<td>1.7</td>
<td>1.3</td>
<td>2.2</td>
<td>1.6</td>
<td>0.7</td>
<td>1.4</td>
<td>2.6</td>
<td>class3</td>
</tr>
</tbody>
</table>

b) The following table shows the distance between the given points and the new training point. If this distance is shorter than any within the 3 nearest neighbors from 2a), then it is shown in bold. The table also shows the predicted class.

<table>
<thead>
<tr>
<th>Points</th>
<th>(2.5, 2.7)</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.5, 1.5)</td>
<td>2.2</td>
<td>class1</td>
</tr>
<tr>
<td>(3, 2.2)</td>
<td>1</td>
<td>class2</td>
</tr>
<tr>
<td>(1.3, 3)</td>
<td>1.5</td>
<td>class3</td>
</tr>
</tbody>
</table>
Question 3: From Rules to Decision Trees [12]

- a) Accuracy on the tune set = \( \frac{2}{5} \times 100\% = 40\% \)
- b) Accuracy on the tune set after pruning node width \( \leq 0.6 \) is \( \frac{3}{5} \times 100\% = 60\% \) 
  Accuracy on the tune set after pruning node width \( \leq 1.7 \) is \( \frac{3}{5} \times 100\% = 60\% \) 
  Accuracy on the tune set after pruning node length \( \leq 1.7 \) is \( \frac{2}{5} \times 100\% = 40\% \) 
  Accuracy on the tune set after pruning node width \( \leq 1.5 \) is \( \frac{4}{5} \times 100\% = 80\% \)
- c) So, the node that needs to be pruned is width \( \leq 1.5 \)? 
  Decision tree after node is pruned will be:

Question 4: Pruning a Decision Tree by Hand [20]

- a) Accuracy on the tune set = \( \frac{2}{5} \times 100\% = 40\% \)
- b) Accuracy on the tune set after pruning node width \( \leq 0.6 \) is \( \frac{3}{5} \times 100\% = 60\% \) 
  Accuracy on the tune set after pruning node width \( \leq 1.7 \) is \( \frac{3}{5} \times 100\% = 60\% \) 
  Accuracy on the tune set after pruning node length \( \leq 1.7 \) is \( \frac{2}{5} \times 100\% = 40\% \) 
  Accuracy on the tune set after pruning node width \( \leq 1.5 \) is \( \frac{4}{5} \times 100\% = 80\% \)
- c) So, the node that needs to be pruned is width \( \leq 1.5 \)? 
  Decision tree after node is pruned will be: