Question 1: Game Playing [25]

a) [10] See the following tree.

```
   4
  / \
min 11 4
 /   \
min -15 12 4
    /   \
min 15 5 13
```

b) [15] See the following tree. Each node shows the alpha and beta values during the searching and the return value for the corresponding sub-tree.

```
   4
  / \
max [-∞, ∞], [-13, ∞], [2, ∞], 2
 /   \
min [-∞, ∞], [-∞, -13], [-∞, -13], -13
 /       \
min [-13, ∞], [-13, ∞], [-13, -13], [-13, 2], [-13, 2], [-13, -13], [-13, 2], [-13, -13], 2
```

Question 2: Game Theory [25]

a) [10] The following two tables show the pure strategies for player A and B.
(a) A’s Strategy | state a | state d \\
| I | L | L \\
| II | L | R \\
| III | R | L \\
| IV | R | R \\

B’s Strategy | state b | state c \\
| I | L | L \\
| II | L | R \\
| III | R | L \\
| IV | R | R \\

(b) Matrix normal form:

<table>
<thead>
<tr>
<th>A \ B</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-1</td>
<td>-1</td>
<td>+2</td>
<td>+2</td>
</tr>
<tr>
<td>II</td>
<td>+4</td>
<td>+4</td>
<td>+2</td>
<td>=2</td>
</tr>
<tr>
<td>III</td>
<td>+5</td>
<td>-2</td>
<td>+5</td>
<td>-2</td>
</tr>
<tr>
<td>IV</td>
<td>+5</td>
<td>-2</td>
<td>+5</td>
<td>-2</td>
</tr>
</tbody>
</table>

(a) First B-II and B-IV are strictly dominated by B-I, thus eliminate B-II and B-IV. Then A-I and A-IV are strictly dominated by A-II, thus eliminated A-I and A-IV. After the elimination, the matrix normal form becomes

<table>
<thead>
<tr>
<th>A \ B</th>
<th>I</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>II</td>
<td>5.5</td>
<td>4.8</td>
</tr>
<tr>
<td>III</td>
<td>7.9</td>
<td>2.6</td>
</tr>
</tbody>
</table>

(b) After the elimination, there are two Nash Equilibriums, which are (7,9) and (4,8). Since each player both know the complete information about the game, and they both are rational, therefore they will only play the strategies at Nash Equilibriums. Moreover, (7,9) is consistently better than (4,8) for both A and B. Thus, they will choose (7,9) rather than (4,8), namely, A will choose III and B will choose I.