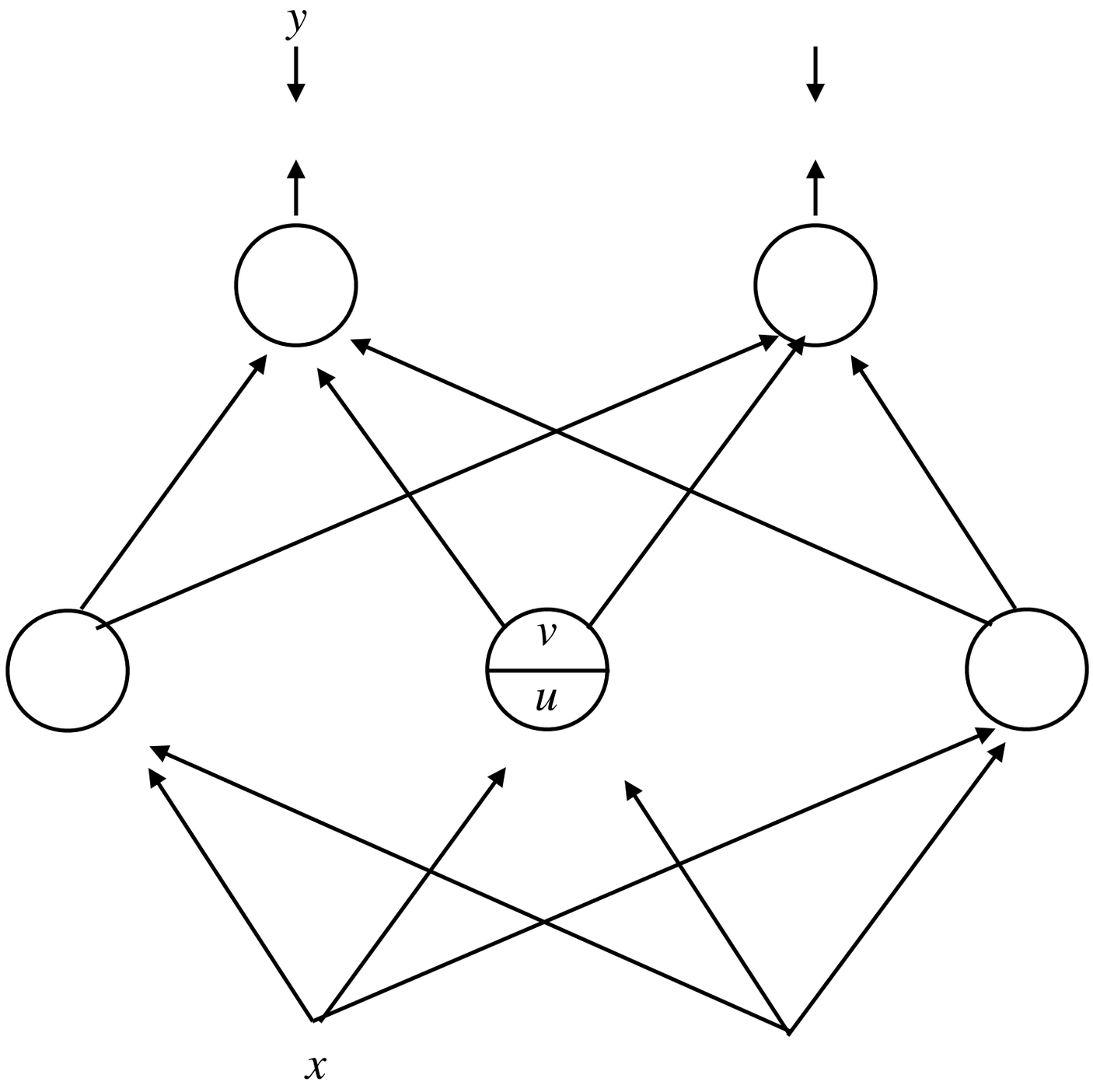
Backpropagation



Forward

- input layer:

$$v_j = x_j$$

- hidden and output layers:

$$u_j = \sum_{i:i \to j} w_{ij} v_i$$

$$v_j = f_j(u_j)$$

- output errors

$$E_j = \frac{1}{2}(v_j - y_j)^2$$

$$E = \sum_{j \in \textbf{output}} E_j$$

Backward

- output

$$\frac{\partial E}{\partial v_j} = v_j - y_j$$

$$\frac{\partial E}{\partial u_j} = \frac{\partial E}{\partial v_j}\frac{df_j(u_j)}{du_j}$$

- Hidden

$$\frac{\partial E}{\partial v_j} = \sum_{k:j\to k} w_{jk}\frac{\partial E}{\partial u_k}$$

$$\frac{\partial E}{\partial u_j} = \frac{\partial E}{\partial v_j}\frac{df_j(u_j)}{du_j}$$

$$\frac{\partial E}{\partial w_{ij}} = v_i\frac{\partial E}{\partial u_j}$$