Name ____ Email ____________________

CS540-1 F18 midterm exam. Write down the answer only (no intermediate steps please).

1. (Search) A search tree has 3 levels (the root is at level 1), and every internal node has 4 children. Suppose there is no goal node. How many goal checks will depth first search perform?

> There is 1 root node. It has 4 children. Each child has 4 children. $1 + 4 + 16 = 21$.

2. (Search) Let the search space be integers. Each state n has two successors: $2n, 2n+1$. Write down the shortest path (i.e. the sequence of states) from the initial state 0 to the goal state 540.

> 0,1,2,4,8,16,33,67,135,270,540

3. (Search) Consider a 3-puzzle where, like in the usual 8-puzzle game, a tile can only move to an adjacent empty space. Tiles cannot move diagonally. Given the initial state

3	2
	1

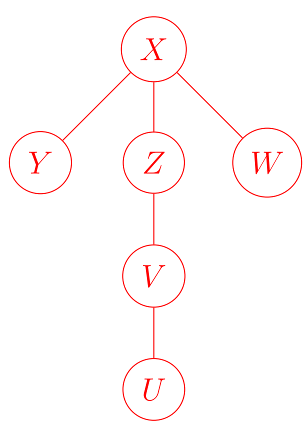
, how many moves does it take to reach the goal state

3	1
	2

?

> The puzzle cannot reach the goal state from the initial state. A proof might involve enumerating the entire search space, which is finite (and fairly small) here.

4. (Iterative deepening) Consider Iterative Deepening Search on a tree, where the nodes are denoted by letters. Iterative deepening search visited the following sequence of node: XYZWXYZVWXYZVUW. Reconstruct *one* possible tree.



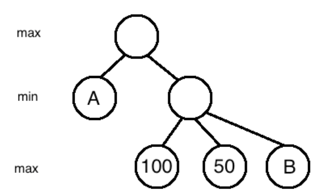
5. (A^* search) If h and h' are both admissible heuristic functions, is $\max(h, h')$ an admissible heuristic function?

Yes, if $h(s) \leq h^*(s)$ and $h'(s) \leq h^*(s)$, then $\max(h(s), h'(s)) \leq h^*(s)$, and so $\max(h, h')$ is an admissible heuristic.

6. (A^* search) If on every state s a heuristic function $h(s) = h^*(s) + 100$, where $h^*(s)$ the true cost from that state s to the optimal goal, will A^* search with this $h()$ function be guaranteed to find the optimal goal?

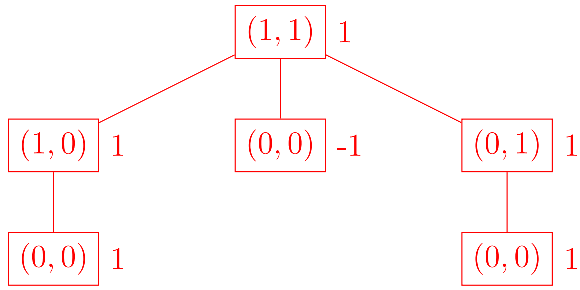
Yes, while admissibility is sufficient to guarantee optimality, it is not necessary. In particular, shifting all the values of the true cost by some constant will still guide the search to the optimal, as both $f(s)+h(s)$ and $f(s)+h^*(s)$ are minimized by the same state. Therefore, the search will explore the state space in the same manner as it would for an admissible heuristic (h^*).

7. (Alpha-beta pruning) What is the most general condition on A's value such that B will be alpha-beta pruned?



The parent of B receives $\alpha = A, \beta = \infty$. It then updates β values, first to 100, then to 50. If A $\geq$ 50, we have $\alpha \geq \beta$, and node B is pruned.

8. (Game tree) Consider the following game. There are two piles, each pile has one stick. A player can take one stick from a single pile, or she may take two sticks, one from each pile. The player who takes the last stick loses. Let the game theoretical value be 1 if the first player wins. Draw the game tree, and mark the game theoretical value at each node.



9. (Hill climbing) Let the states be 2D integer points with integer coordinates (i, j) with boundary constraints $-10 \leq i \leq 10, -10 \leq j \leq 10$. Each state (i, j) has four successors $(i-1, j), (i+1, j), (i, j-1), (i, j+1)$ or a subset thereof subject to the boundary constraints. The score of state (i, j) is $s(i, j) = ai + bj$ with $a, b \neq 0$. a, b are fixed positive or negative constants. How many initial states, if hill climbing start from them, will lead to a local maximum worse than the global maximum?

The problem is symmetric and $s(i, j)$ has a maximum on one of the corners of the 2D grid depending on the signs of $a, b \neq 0$. That maximum is both the only local maximum and a global maximum. Therefore, there are no initial initial states which can lead to a local maximum worse than the global maximum.

10. (Simulated annealing) In simulated annealing one accepts a transition from s to an inferior neighbor t with probability $\exp\left(-\frac{|f(s)-f(t)|}{T}\right)$, where T is the temperature parameter. Suppose $f(s) = 2, f(t) = 1$. At what temperature is the transition probability $\frac{1}{2}$?

Solve for T in $\frac{1}{2} = \exp\left(-\frac{|f(s)-f(t)|}{T}\right)$ to get $T = \frac{1}{\ln(2)}$.

11. (Probability) There are two biased coins in my pocket: coin A has $P(Heads) = \frac{1}{5}$, coin B has $P(Heads) = \frac{3}{4}$. I took out a coin from the pocket at random with equal probability. I flipped it twice: both outcomes were Heads. What is the probability that the coin was A?

Let A be the event that the coin was A. Let B be the event that the coin was B. Let H be the event that we received 2 heads.

$$P(A|H) = \frac{P(H,A)}{P(H)} = \frac{P(H|A)P(A)}{P(H|A)P(A) + P(H|B)P(B)} = \frac{\frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{2}}{\frac{1}{5}\cdot\frac{1}{5}\cdot\frac{1}{2}+\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{1}{2}} = \frac{16}{241}$$

12. (Smoothing) You have a vocabulary with 1,000,000 word types. You want to estimate the unigram probability $p(w)$ for each word type w in the vocabulary. In your corpus the total word token count $\sum_w c_w$ is 10^{12}, and $c_{\text{“zoodles”}} = 3$. Using add-one smoothing, write down the formula for $p(\text{“zoodles”})$ (your answer should not contain variables, but do not numerically compute the decimal number either: just give the fraction).

$$p(\text{“zoodles”}) = \frac{3+1}{10^{12}+10^6}$$

13. (PCA) You performed PCA in R^3. If the first principal component is $v_1 = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})^\top$, and the second principal component is $v_2 = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)^\top$, what is the new 2D coordinates for the point $x = (1,1,1)^\top$?

$$(v_1^\top x, v_2^\top x) = (\sqrt{3}, 0)$$

14. (Linear algebra) Let $x, v \in \mathbb{R}^d$, where the norm $\|v\| = 1$. The projection of x onto v is the d dimensional point y on the direction of v such that the line connecting x, y is perpendicular to v. Express y using x, v (hint: you will need inner product).

$$y = (x^\top v)v$$

15. (Entailment) We define a new Boolean logic connective $\heartsuit$ as follows:

P	Q	P ♡ Q
F	F	T
F	T	F
T	F	F
T	T	T

Does $P \wedge Q$ entail $P \heartsuit Q$?

Yes, we see from extending the table

P	Q	P ∧ Q	P ♡ Q
F	F	F	T
F	T	F	F
T	F	F	F
T	T	T	T

that there is no assignment to P, Q for which P ∧ Q is true while P ♡ Q is false. Therefore P ∧ Q entails P ♡ Q.

16. (Conjunctive normal form) Write down the CNF for $\neg(P \vee Q) \vee R$.

$$\neg(P \vee Q) \vee R$$
$$(\neg P \wedge \neg Q) \vee R$$
$$(\neg P \vee R) \wedge (\neg Q \vee R)$$