Informed Search

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[Based on slides from Andrew Moore http://www.cs.cmu.edu/~awm/tutorials]

Main messages

- A*. Always be optimistic.

Uninformed vs. informed search

- **Uninformed search** (BFS, uniform-cost, DFS, ID etc.)
  - Knows the actual path cost ***g(s)*** from start to a node ***s*** in the fringe, but that's it.

- **Informed search**

  - also has a heuristic ***h(s)*** of the cost from ***s*** to goal. ('h'= heuristic, non-negative)
  - Can be much faster than uninformed search.

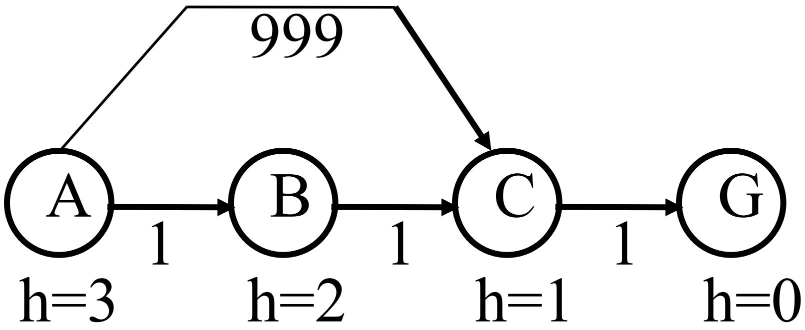
Recall: Uniform-cost search

- Uniform-cost search: uninformed search when edge costs are not the same.
- Complete (will find a goal). Optimal (will find the least-cost goal).
- Always expand the node with the least ***g(s)***
  - Use a priority queue:
    - Push in states with their first-half-cost ***g(s)***
    - Pop out the state with the least ***g(s)*** first.
- Now we have an estimate of the second-half-cost ***h(s)***, how to use it?

First attempt: Best-first greedy search

- Idea 1: use ***h(s)*** instead of ***g(s)***
- Always expand the node with the least ***h(s)***
  - Use a priority queue:
    - Push in states with their second-half-cost ***h(s)***
    - Pop out the state with the least ***h(s)*** first.
- Known as "best first greedy" search
- How's this idea?

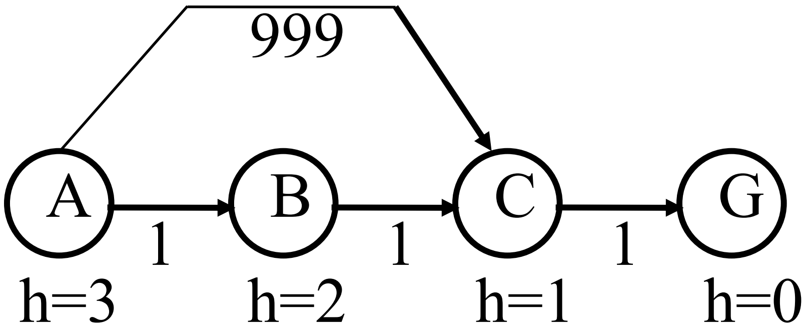
Best-first greedy search looking stupid



- It will follow the path *A*→*C*→*G* (why?)
- Obviously not optimal

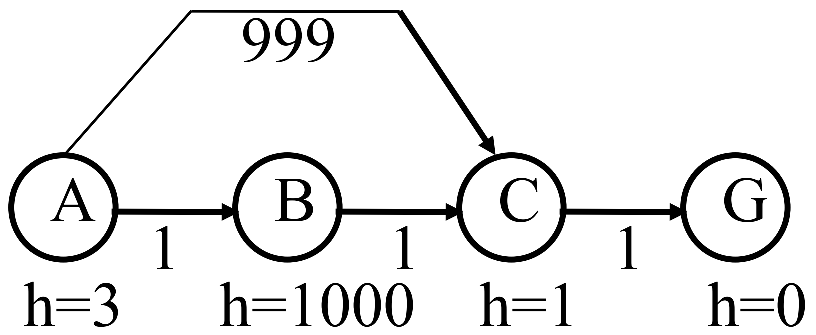
Second attempt: A search

- Idea 2: use $g(s)+h(s)$
- Always expand the node with the least $g(s)+h(s)$
  - Use a priority queue:
    - Push in states with their first-half-cost $g(s)+h(s)$
    - Pop out the state with the least $g(s)+h(s)$ first.
- Known as "A" search
- How's this idea?



- Works for this example

A search still not quite right



- A search is not optimal.

Third attempt: A* search

- Same as A search, but the heuristic function ***h()*** has to satisfy $h(s) \leq h^*(s)$, where $h^*(s)$ is the true cost from node *s* to the goal.
- Such heuristic function ***h()*** is called admissible.
  - An admissible heuristic never over-estimates

It is always optimistic

- A search with admissible ***h()*** is called A* search.

Admissible heuristic functions *h*

- 8-puzzle example

Example State

1		5
2	6	3
7	4	8

Goal State

1	2	3
4	5	6
7	8	

- Which of the following are admissible heuristics?
  - •h(n)=number of tiles in wrong position
  - •h(n)=0
  - •h(n)=1
  - •h(n)=sum of Manhattan distance between each tile and its goal location

Admissible heuristic functions *h*

- 8-puzzle example

Example State

1		5
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Goal State

1	2	3
4	5	6
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- Which of the following are admissible heuristics?
  - h(n)=number of tiles in wrong position YES
  - h(n)=0 YES, uninformed uniform cost search
  - h(n)=1 NO, goal state
  - h(n)=sum of Manhattan distance between each tile and its goal location YES

Admissible heuristic functions *h*

- In general, which of the following are admissible heuristics? h*(n) is the true optimal cost from n to goal.
  - h(n)=h*(n)
  - h(n)=max(2,h*(n))
  - h(n)=min(2,h*(n))
  - h(n)=h*(n)-2
  - h(n)=sqrt(h*(n))

Admissible heuristic functions *h*

- In general, which of the following are admissible heuristics? h*(n) is the true optimal cost from n to goal.
  - h(n)=h*(n) YES
  - h(n)=max(2,h*(n)) NO
  - h(n)=min(2,h*(n)) YES
  - h(n)=h*(n)-2 NO, possibly negative
  - h(n)=sqrt(h*(n)) NO if h*(n)<1

Heuristics for Admissible heuristics

- How to construct heuristic functions?

Example State

1		5
2	6	3
7	4	8

Goal State

1	2	3
4	5	6
7	8	

- Often by relaxing the constraints
  - h(n)=number of tiles in wrong position
    Allow tiles to fly to their destination in one step
  - h(n)=sum of Manhattan distance between each tile and its goal location
    Allow tiles to move on top of other tiles

“my heuristic is better than yours”

- A heuristic function h2 **dominates** h1 if for all s

  $h1(s) \leq h2(s) \leq h^*(s)$

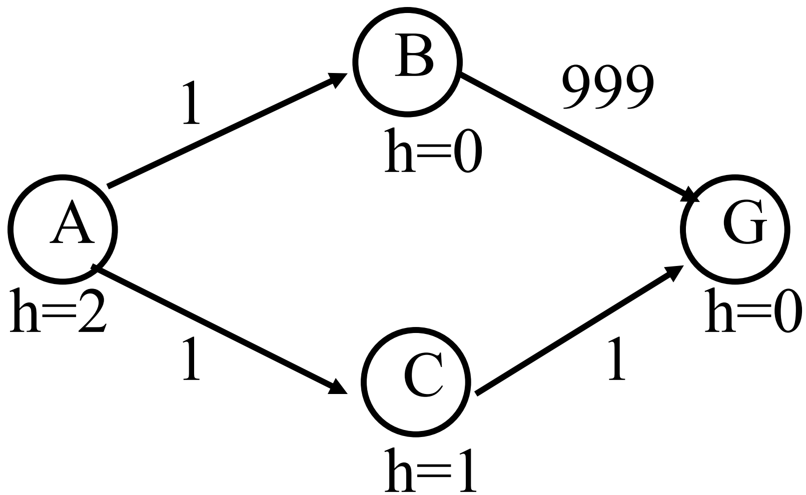
- We prefer heuristic functions as close to h* as possible, but not over h*.

But

- Good heuristic function might need complex computation
- Time may be better spent, if we use a faster, simpler heuristic function and expand more nodes

Q1: When should A* stop?

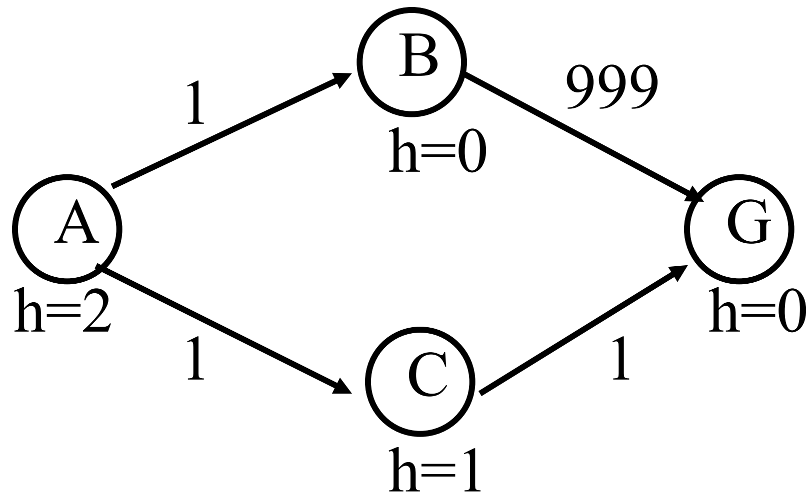
- Idea: as soon as it generates the goal state?



- h() is admissible
- The goal *G* will be generated as path *A*→*B*→*G*, with cost 1000.

Q1: The correct A* stop rule

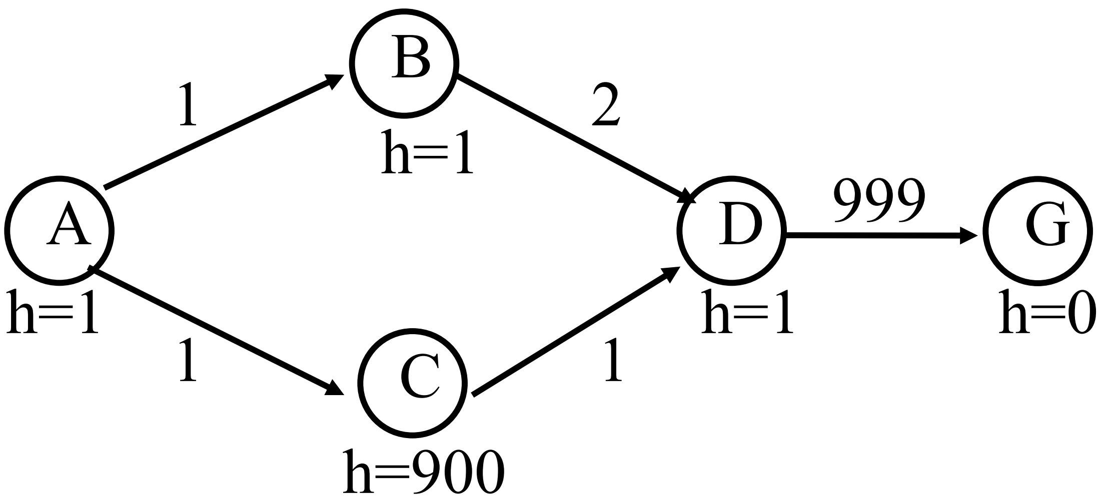
- A* should terminate only when a goal is popped from the priority queue



- If you have exceedingly good memory, you'll remember this is the same rule for uniform cost search on cyclic graphs.
- Indeed A* with h()≡0 is exactly uniform cost search!

Q2: A* revisiting expanded states

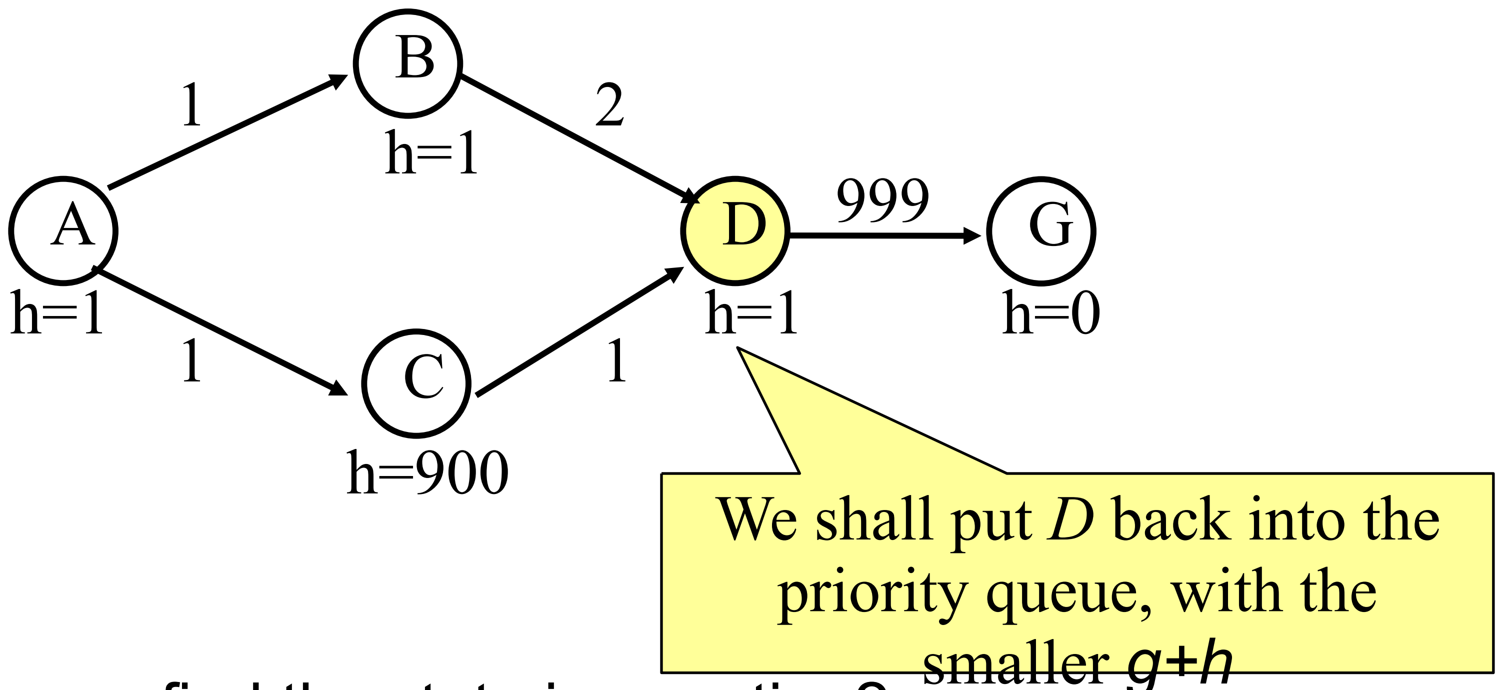
- One more complication: A* can revisit an expanded state, and discover a shorter path



- Can you find the state in question?

Q2: A* revisiting expanded states

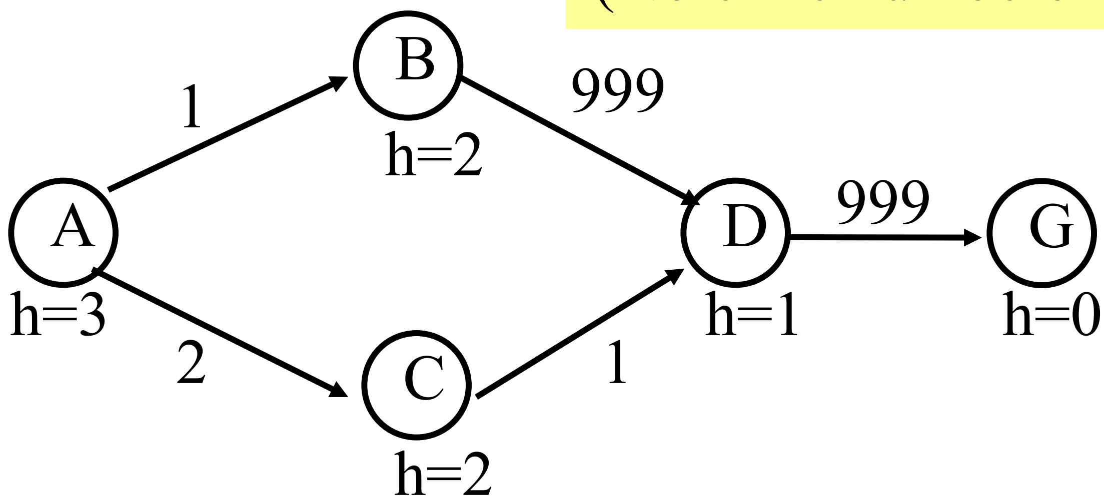
- One more complication: A* can revisit an expanded state, and discover a shorter path



- Can you find the state in question?

Q3: What if A* revisits a state in the PQ?

(Note the numbers are different)



- We've seen this before, with uniform cost search
- 'promote' *D* in the queue with the smaller cost

The A* algorithm

1. Put the start node S on the priority queue, called OPEN
2. If OPEN is empty, exit with failure
3. Remove from OPEN and place on CLOSED a node n for which f(n) is minimum
4. If n is a goal node, exit (trace back pointers from n to S)
5. Expand n, generating all its successors and attach to them pointers back to n. For each successor n' of n
    1. If n' is not already on OPEN or CLOSED estimate h(n'),g(n')=g(n)+ c(n,n'), f(n')=g(n')+h(n'), and place it on OPEN.
    2. If n' is already on OPEN or CLOSED, then check if g(n') is lower for the new version of n'. If so, then:
        1. Redirect pointers backward from n' along path yielding lower g(n').
        2. Put n' on OPEN.
    3. If g(n') is not lower for the new version, do nothing.
6. Goto 2.

A*: the dark side

- A* can use lots of memory.

  O(number of states)

- For large problems A* will run out of memory
- We'll look at two alternatives:
  - IDA*
  - Beam search

IDA*: iterative deepening A*

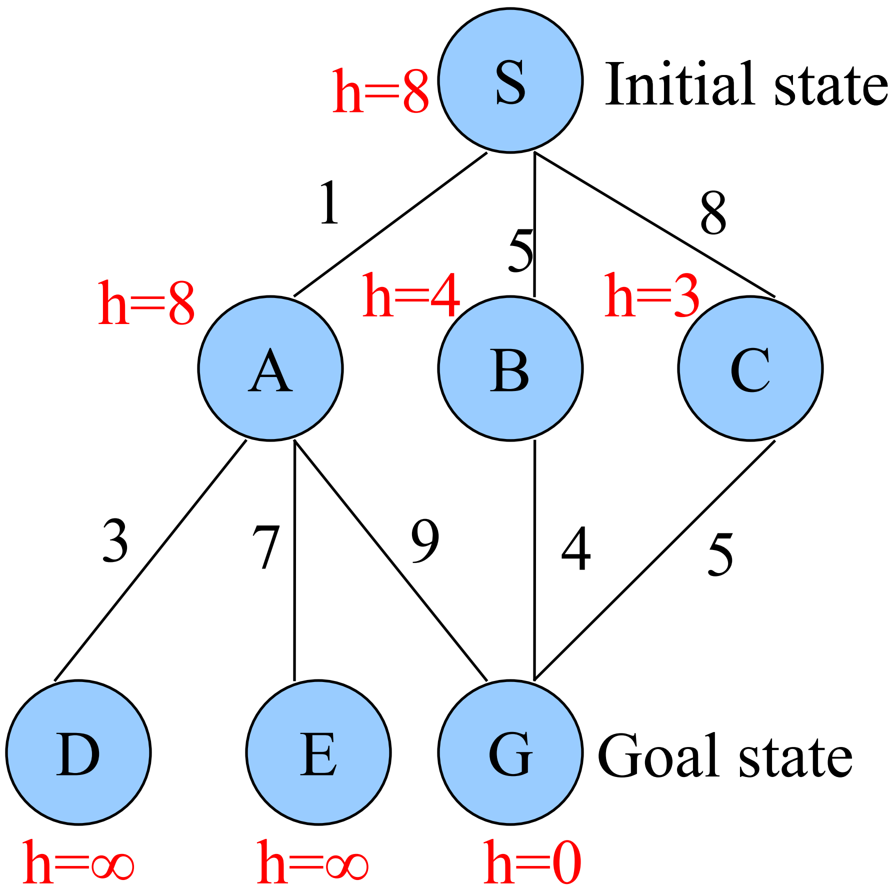
- Memory bounded search. Assume integer costs
  - Do path checking DFS, do not expand any node with $f(n)>0$. Stop if we find a goal.
  - Do path checking DFS, do not expand any node with $f(n)>1$. Stop if we find a goal.
  - Do path checking DFS, do not expand any node with $f(n)>2$. Stop if we find a goal.
  - Do path checking DFS, do not expand any node with $f(n)>3$. Stop if we find a goal.

  … repeat this, increase threshold by 1 each time until we find a goal.
- This is complete, optimal, but more costly than A* in general.

Beam search

- Very general technique, not just for A*
- The priority queue has a fixed size *k*. Only the top *k* nodes are kept. Others are discarded.
- Neither complete nor optimal, nor can maintain an 'expanded' node list, but memory efficient.
- Variation: The priority queue only keeps nodes that are at most ε worse than the best node in the queue. ε is the beam width.
- Beam search used successfully in speech recognition.

Example



(All edges are directed, pointing downwards)

Example

OPEN	CLOSED
S(0+8)	-
A(1+8) B(5+4) C(8+3)	S(0+8)
B(5+4) C(8+3) D(4+inf) E(8+inf) G(10+0)	S(0+8) A(1+8)
C(8+3) D(4+inf) E(8+inf) G(10+0) G(9+0)	S(0+8) A(1+8) B(5+4)
C(8+3) D(4+inf) E(8+inf) G(10+0)	S(0+8) A(1+8) B(5+4) G(9+0)

Backtrack: G => B => S.

What you should know

- Know why best-first greedy search is bad.
- Thoroughly understand A*
- Trace simple examples of A* execution.
- Understand admissible heuristics.

Appendix: Proof that A* is optimal

- Suppose A* finds a suboptimal path ending in goal G', where $f(G') > f^*$ = cost of optimal path
- Let's look at the first unexpanded node n on the optimal path (n exists, otherwise the optimal goal would have been found)
- $f(n) \geq f(G')$, otherwise we would have expanded n
- $f(n) = g(n)+h(n)$ by definition

  $= g^*(n)+h(n)$ because n is on the optimal path

  $\leq g^*(n)+h^*(n)$ because h is admissible

  $= f^*$ because n is on the optimal path
- $f^* \geq f(n) \geq f(G')$, contradicting the assumption at top