Instructions.

1. Do not open the exam until I say “go.”
2. Put away everything except a pencil, a calculator, and your one-page (two sides) notes sheet.
3. Attempt all questions.
4. Show your work clearly. Correct answers without enough work may receive no credit.
5. Find the needed tables at the end of the packet. You may tear the tables sheet(s) free.
6. Don’t worry if you don’t finish the test. Just try to score as many points as you can.
7. If a question is ambiguous, resolve it in writing. We will consider grading accordingly.
8. The exam will end when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)
9. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I’ve picked up all the exams.
10. Good luck!

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Your Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Q2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Q3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Q4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Q5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Q6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Q7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Q8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Q9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Q10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
1. Suppose the length, in days, of a pregnancy from conception to birth is approximately normally distributed with mean 272 days and variance 81 days. A pregnancy is considered full-term if it lasts between 252 and 298 days. What proportion of pregnancies are full-term?

**ANSWER:**

Let $X =$ pregnancy length $\sim N(272, 9^2)$. $P(\text{full-term}) = P(252 < X < 298) = P\left(\frac{X-272}{9} < \frac{298-272}{9}\right) \approx P(-2.22 < Z < 2.89) = P(Z < 2.89) - P(Z < -2.22) = .9981 - .0132 = .9849$

2. A crate of medical gloves contains several thousand, 15% of which are defective. Eight gloves are sampled from the crate. Find the probability that exactly three of these eight are defective.

**ANSWER:**

Let $X =$ the number of 8 that are defective. Then $X \sim \text{Bin}(n = 8, \pi = .15)$. $P(X = 3) = \binom{8}{3}.15^3(1-.15)^{8-3} \approx .084$
3. Here are the numbers of blueberries on 8 randomly-chosen bushes on Flattop Mountain:

83 97 92 87 90 88 70 85

A boxplot would indicate which values, if any, as outliers?

ANSWER:
The sorted sample is 70 83 85 87 88 90 92 97. \( Q_1 = 84, Q_3 = 91 \), and \( IQR = Q_3 - Q_1 = 7 \).
Outliers are outside \((Q_1 - 1.5 \times IQR, Q_3 + 1.5 \times IQR) = (73.5, 101.5)\).

70 is an outlier.

4. The stalk-eyed fly has projections from the sides of its head with eyes at the ends. The span, in millimeters, from one eye to the other was measured in a random sample of nine flies:

8.69 8.15 9.25 9.45 8.96 8.65 8.43 8.79 8.63

Supposing that the spans have a normal distribution in the population, find a 95% confidence interval for the true mean span.

ANSWER: \( n = 9, \bar{x} = 8.778, s = .398, t_{n-1,\alpha/2} = t_{8,.025} = 2.306 \) \( \implies \) interval is \( 8.778 \pm 2.306 \times .398/\sqrt{9} = 8.778 \pm .306 \)
5. Here is a normal probability (or quantile-quantile or QQ) plot of a random sample from some population.

![Normal Q-Q Plot](image)

What does this graph tell you about whether or not the population is approximately normal?

**ANSWER:**

The points are more-or-less lined up, so it’s plausible that the population is normal. (It’s also plausible that the population is not normal, but that random variation in sampling with this small sample size nevertheless led to a plot compatible with a normal population.)

6. A battery maker claims that a battery lifetime has \( \mu = 40 \) hours and \( \sigma = 5 \) hours. Suppose a random sample of 100 batteries is selected. If the claim is true, what is \( P(\bar{X} \leq 39.8) \)?

**ANSWER:**

\( n = 100 \) is large enough (by the “\( n > 30 \)” rule of thumb) that the CLT says \( \bar{X} \sim N(40, \frac{5^2}{100}) = .5^2 \). Then \( P(\bar{X} < 39.8) = P(Z < \frac{39.8 - 40}{\frac{5}{\sqrt{100}}}) = P(Z < -.40) \approx .3446 \)
7. Suppose \( F \) is a random temperature on the Fahrenheit scale from a distribution having mean \( \mu_F = -9.67 \) and variance \( \sigma^2_F = 81 \). The corresponding temperature in kelvin is \( K = (F + 459.67) \times \frac{5}{9} \). (The kelvin is a unit of measure for temperature based on an absolute scale having minimum value zero (0 K).)

(a) Find the mean \( \mu_K \).

\[
\mu_K = E(K) = E\left((F + 459.67) \times \frac{5}{9}\right) = \frac{5}{9}E(F + 459.67) = \frac{5}{9}(E(F) + 459.67) = \frac{5}{9}(-9.67 + 459.67) = \frac{5}{9}(450) = 250
\]

(b) Find the variance \( \sigma^2_K \).

\[
\sigma^2_K = VAR(K) = VAR\left((F + 459.67) \times \frac{5}{9}\right) = \left(\frac{5}{9}\right)^2VAR(F + 459.67) = \left(\frac{5}{9}\right)^2VAR(F) = \left(\frac{5}{9}\right)^2(81) = 25
\]

8. A farmer raises chickens with weights, in grams, that are normally distributed with mean 1387 and standard deviation 161. She wants to provide a money-back guarantee that her chickens will weigh at least a certain amount. What minimum weight should she guarantee so that she’ll have to give money back only 1% of the time?

ANSWER:

Let \( X = \) weight of a chicken and \( w = \) the guaranteed minimum weight. \( P(X < w) = .01 \implies P\left(Z = \frac{X - \mu}{\sigma} < \frac{w - 1387}{161}\right) = .01 \implies \text{(from table)} \frac{w - 1387}{161} = -z_{.01} = -2.325 \implies w = -2.325 \times 161 + 1387 \approx 1012.7 \)
9. Let \( X = 1 \) if the outcome of tossing an unfair coin is heads, and 0 if it’s tails. The probability mass function of \( X \) is

\[
\begin{array}{c|cc}
x & 0 & 1 \\
p(x) & \frac{1}{3} & \frac{2}{3} \\
\end{array}
\]

(a) Find the mean of \( X \).

**ANSWER:**
\[
\mu_X = E(X) = \sum x p(x) = 0\left(\frac{1}{3}\right) + 1\left(\frac{2}{3}\right) = \frac{2}{3}
\]

(b) The variance of \( X \) is \( \sigma_X^2 = ANSWER: \)
\[
V A R(X) = \sum (x - \bar{X})^2 p(x) = (0 - \frac{2}{3})^2\left(\frac{1}{3}\right) + (1 - \frac{2}{3})^2\left(\frac{2}{3}\right) = \frac{2}{9}
\]

10. A researcher is studying the breaking strengths of a shipment of slacklines. She selects a simple random sample (SRS) of 10 lines, measures the breaking strength of each, and calculates a 98% confidence interval for the unknown shipment mean strength as 6000 ± 50 lbs. Mark each of the following statements as TRUE or FALSE.

- The probability that the true mean breaking strength is between 5950 and 6050 lbs is 98%.
- The probability that the true mean breaking strength is between 5950 and 6050 lbs is either 0% or 100%, but we don’t know which.
- If she were to select many more samples, and calculate a 98% confidence interval for each sample, she should expect that the true mean breaking strength would be within about 98% of these intervals.
- Prior to selecting her first random sample, the probability was 98% that the true mean breaking strength would be inside the soon-to-be-calculated confidence interval.

**ANSWER:**
FALSE, TRUE, TRUE, TRUE