STAT 371 Exam 2

Last name: ___________________________  First name: ___________________________

Circle discussion: 331 Tu 4:35 / 332 We 9:55 / 333 W 11:00  NetID: ___________________________

Instructions:

1. Print your name clearly.

2. Do not open the exam until I say “go.”

3. Put away everything except a pencil, a calculator, and your two pages (two sides each) of notes (formula sheets).

4. Attempt all questions.

5. Show your work clearly. Correct answers without enough work may receive no credit.

6. Find the needed tables at the end of the packet. You may tear the tables sheet(s) free.

7. Don’t worry if you don’t finish the test. Just try to score as many points as you can.

8. If a question is ambiguous, resolve the ambiguity in writing. We will consider grading accordingly.

9. The exam will end when I call time. If you continue writing after I call time, you risk a penalty. (The alternative, that you get more time than your peers, is unfair.)

10. You are welcome to turn your exam in to me before I call time. However, if you are still here in the last five minutes, please remain seated until I’ve picked up all the exams.

11. Good luck!

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Your Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>30</td>
<td></td>
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<tr>
<td>Q2</td>
<td>30</td>
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<td>Q3</td>
<td>30</td>
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<tr>
<td>Q4</td>
<td>10</td>
<td></td>
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<tr>
<td>TOTAL</td>
<td>100</td>
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</tbody>
</table>
1. Six bean plants had their carbohydrate concentrations (in percent by weight) measured in the shoot and in the root:

<table>
<thead>
<tr>
<th>Plant</th>
<th>Shoot</th>
<th>Root</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.42</td>
<td>3.66</td>
</tr>
<tr>
<td>2</td>
<td>5.91</td>
<td>5.51</td>
</tr>
<tr>
<td>3</td>
<td>4.50</td>
<td>3.91</td>
</tr>
<tr>
<td>4</td>
<td>4.77</td>
<td>4.47</td>
</tr>
<tr>
<td>5</td>
<td>5.25</td>
<td>4.69</td>
</tr>
<tr>
<td>6</td>
<td>4.75</td>
<td>3.93</td>
</tr>
</tbody>
</table>

(a) Suppose we are interested in testing whether there is a difference in population mean concentrations between the shoot and the root. To check assumptions, what graph(s) would you make and why? (Do not make graphs.)

ANSWER:
These samples are paired by plant. I would make a QQ plot of the six differences within each pair to check whether it is reasonable to assume the population of differences is normally distributed.

(b) Suppose, from your graph(s), it is reasonable to assume normality. Test whether there is a difference in population mean concentrations between the shoot and the root.

- Hypotheses:

- Assumptions:

- Test statistic:

- P-value:

- Conclusion:

ANSWER:
- Hypotheses: $H_0 : \mu_D = 0, H_1 : \mu_D \neq 0$, where $D =$ Shoot $-$ Root
- Assumptions: A column of differences calculated as $D =$ Shoot $-$ Root concentration is a simple random sample from a normal population of differences.
- Test statistic: The differences are 0.76, 0.40, 0.59, 0.30, 0.56, and 0.82, which have $n = 6, \bar{D} = .5717, s_D = .2004 \implies t = \frac{\bar{D} - \mu_0}{s_D/\sqrt{n}} \approx 6.9860.$
• P-value: $P = 2P(T_{n-1} > t) < 2(.001) = .002$
• Conclusion: Since the P-value is less than the usual threshold $\alpha = .05$, reject $H_0 \implies$
Yes, the data are strong evidence of a difference.
(c) Suppose instead, from your graph(s), it is not reasonable to assume normality. Here is output from a Wilcoxon Signed Rank test run in R:

```r
> shoot = c(4.42, 5.91, 4.50, 4.77, 5.25, 4.75)
> root = c(3.66, 5.51, 3.91, 4.47, 4.69, 3.93)
> wilcox.test(shoot, root, paired=TRUE)

Wilcoxon signed rank test

data: shoot and root
V = 21, p-value = 0.03125
alternative hypothesis: true location shift is not equal to 0
```

Suppose the assumptions of this test are met. What does the test say about whether the population root concentration is different than the population shoot concentration?

ANSWER: Since the p-value, 0.03125, is small, reject $H_0$. The data are strong evidence of different population concentration distributions at shoot and root.
2. A simple random sample of 5 Phillips Hall students weighed 132, 145, 162, 166, and 175 pounds. A simple random sample of 4 Tripp Hall students weighed 137, 147, 158, and 170.

(a) Suppose each hall’s population of weights is normally distributed. Use the Welch’s t-test to decide whether the population mean weights of Phillips and Tripp students are different. (Hint: you may use $\nu = 6$ as the degrees of freedom.)

- Hypotheses:

  $$H_0 : \mu_P - \mu_T = 0, \quad H_A : \mu_P - \mu_T \neq 0,$$
  where $\mu_P$ = the population mean Phillips weight and $\mu_T$ = the population mean Tripp weight.

- Assumptions: We have two independent SRSs from two normal populations.

- Test statistic:

  $$\bar{P} = 156, \bar{T} = 153, s_P = 17.277, s_T = 14.213, SE = \sqrt{s_P^2/n_P + s_T^2/n_T} = 10.498 \implies t = \frac{(156 - 153) - 0}{10.498} = 0.286.$$

- P-value: $P(T_6 < -0.286) + P(T_6 > 0.286) = 2P(T_6 > 0.286) > 2(0.25) = 0.50$ (R gives 0.784).

- Conclusion: Do not reject $H_0$. The data are not strong evidence that Philips students weigh more than Tripp students.

(b) Suppose each hall’s population of weights is normally distributed. Find a 98% confidence interval for $\mu_P - \mu_T$. (Hint: you may use $\nu = 6$ as the degrees of freedom. The problem says “98%,” not “95%.”)

ANSWER: $t_{\nu,\alpha/2} = t_{6.01} = 3.143 \implies$ interval = $(\bar{P} - \bar{T}) \pm t_{\nu,\alpha/2} \sqrt{s_P^2/n_P + s_T^2/n_T} \approx (156 - 153) \pm 3.143(10.498) = 3 \pm 33.00 = (-30, 36)$

(c) Suppose each hall’s population of weights is not normally distributed. Here is output from a Wilcoxon Rank Sum Test run in R:
> Phillips = c(132, 145, 162, 166, 175)
> Tripp = c(137, 147, 158, 170)
> wilcox.test(Phillips, Tripp)

Wilcoxon rank sum test
data:  Phillips and Tripp
W = 11, p-value = 0.9048
alternative hypothesis: true location shift is not equal to 0

Suppose the assumptions of this test are met (that is, we have two independent SRSs from populations having the same shape). What does the test say about whether the population of Phillips student weights is shifted relative to Tripp weights?
ANSWER: Since the p-value, 0.9048, is not small, do not reject $H_0$: “the populations are identical (with no shift of one relative to the other).” The data are not strong evidence that Phillips weights are shifted relative to Tripp weights.

3. 16 out of 200 fire trucks produced on a Chicago assembly line required extensive adjustment before they could be shipped, while the same was true for 14 of 400 trucks from an Oshkosh assembly line. (Suppose the 200 trucks may be regarded as a simple random sample from the Chicago line’s long-term work, and similarly for the 400 trucks from Oshkosh.) Are these data statistically significant evidence at level $\alpha = .05$ of a difference in the population proportions of trucks requiring extensive adjustment between Chicago and Oshkosh? Perform an appropriate test.

- Hypotheses:

- Assumptions:

- Test statistic:

- P-value:

- Conclusion:

ANSWER:

- Hypotheses: $H_0 : \pi_C - \pi_O = 0, H_A : \pi_C - \pi_O \neq 0$, where $\pi_C$ and $\pi_O$ are the population proportions of trucks needing extensive adjustment at Chicago and Oshkosh, respectively.
• Assumptions: We have independent SRSs from the two populations. The numbers of trucks requiring extensive adjustment have $\text{Bin}(n = 200, \pi_C)$ and $\text{Bin}(n = 400, \pi_O)$ distributions at Chicago and Oshkosh, respectively.

Under $H_0$, the two proportions are the same, and we estimate the common proportion as $\hat{\pi} = \frac{16 + 14}{200 + 400} = .05$. The expected numbers of successes and failures in the two samples are then $200(.05), 200(1 -.05), 400(.05)$, and $400(1 -.05)$, all of which are $> 5$, so we proceed with a test for a difference of two proportions.

• Test statistic: We found above that $\hat{\pi} = .05$, so $Z = \frac{\frac{16}{200} - \frac{14}{400}}{\sqrt{.05(1-.05)(\frac{1}{200} + \frac{1}{400})}} = 2.38$

• P-value: $P(Z < -2.38) + P(Z > 2.38) = 2P(Z < -2.38) = 2(0.0087) = 0.0174$

• Conclusion: The p-value (.0174) is less than the given significance level ($\alpha = .05$), so reject $H_0$. The data are strong evidence of a difference in the population proportions of trucks requiring extensive adjustment between Chicago and Oshkosh.
4. Suppose you are writing a contract between the producer of spliced ropes and the consumer, a parachute maker needing lines to attach a canopy to a harness.

- The producer promises that the mean breaking strength of a shipment of the lines is \( \mu = 100 \) pounds, with \( \sigma = 16 \).
- An independent lab will find \( \bar{X} \) from a SRS of \( n = 10 \) lines to test \( H_0 : \mu = (\mu_0 = 100) \) vs. \( H_1 : \mu < \mu_0 \).
- A draft contract specifies \( \bar{x}_{\text{critical}} = 97 \). If \( \bar{X} \) is below 97, \( H_0 \) is rejected and neither payment nor shipment occurs. If \( \bar{X} \) is above or equal to 97, \( H_0 \) is not rejected and both payment and shipment occur.

(a) Suppose you work for the producer (the splicing shop). If \( H_0 \) is true and the test nevertheless rejects \( H_0 \), the shipment of lines will be discarded, and you will not be paid. This happens with which probability? Mark your choice with an “X”:

- \( P(\text{type I error}) = \alpha \),
- \( P(\text{type II error}) = \beta \), or
- \( \text{power} = 1 - \beta \)

**ANSWER:**
\( P(\text{type I error}) = \alpha \)

(b) In which direction would you like to move \( \bar{x}_{\text{critical}} = 97 \) to reduce your risk of not being paid in this situation? Mark your choice with an “X”:

- I want to move \( \bar{x}_{\text{critical}} \) so that \( \bar{x}_{\text{critical}} > 97 \), or
- I want to move \( \bar{x}_{\text{critical}} \) so that \( \bar{x}_{\text{critical}} < 97 \)

**ANSWER:**
The second choice is correct (so we’ll reject \( H_0 \) when it is true less often).

(c) Suppose you work for the consumer (the parachute maker). You can’t use the lines if \( \mu = 95 \) (unless you redesign your parachute to use more of the weaker lines). If \( H_0 \) is false because \( \mu = (\mu_A = 95) \), what is the probability that the test will not reject \( H_0 \)? (In this case, you’ll use a defective shipment of lines, and then sell defective parachutes.) Mark your choice with an “X”:

- \( P(\text{type I error}) = \alpha_{(\mu_A=95)} \)
- \( P(\text{type II error}) = \beta_{(\mu_A=95)} \)
- \( \text{power} = 1 - \beta_{(\mu_A=95)} \)

**ANSWER:**
\( P(\text{type II error}) = \beta_{(\mu_A=95)} \)

(d) Which way would you like to move \( \bar{x}_{\text{critical}} \) to decrease your risk? Mark your choice with an “X”:

- I want to move \( \bar{x}_{\text{critical}} \) so that \( \bar{x}_{\text{critical}} > 97 \), or
- I want to move \( \bar{x}_{\text{critical}} \) so that \( \bar{x}_{\text{critical}} < 97 \)

**ANSWER:**
The first choice is correct (so we’ll reject \( H_0 \) when it is false more often).

(Note: Increasing the sample size may resolve the tension between producer and consumer.)