Homework #7

*Submit your homework to your support TA’s mailbox anytime prior to the due date/time. The mailboxes are to the left as you enter the Medical Science Center (1300 University Ave.) from the main University Ave. entrance.

*No late homework will be accepted for credit!

*If a problem asks you to use R, include a copy of the code and output. Please edit your code and output to include only the relevant portions.

*If a problem does not specify how to compute the answer, you may use any appropriate method.

1. A random sample of size \( n = 10 \) is taken from a large population. Let \( \mu \) be the population mean. A test is planned of \( H_0 : \mu = 12 \) vs. \( H_A : \mu \neq 12 \) using \( \alpha = 0.1 \). A QQ plot indicates that assuming normality is not unreasonable. From the sample, \( \bar{x} = 14.2 \) and \( s = 4.88 \).

   (I would suggest doing this problem with a hand calculator and statistical tables as practice for exam conditions, but you may check your answers using R if you wish.)

   (a) Since the data leave it plausible that the population is normal, and the population variance \( \sigma^2 \) is unknown, a \( t \)-test is appropriate. Compute the p-value of the test. Would you reject or not reject \( H_0 \)?

   (b) Compute the power of the test if the true population mean is \( \mu_A = 15 \).

   (c) Using \( s = 4.88 \) as our best guess of \( \sigma \), approximately what sample size would be required to achieve a power of 0.8 if the true population mean is \( \mu_A = 15 \)? Give your answer as the smallest whole number that meets the criterion.

2. A random sample of soil specimens was taken from a large geographic area. The specimens can be assumed to be independent. The amount of organic matter, as a percent, was determined for each specimen. The data are below:

   0.14, 0.32, 1.17, 1.45, 3.5, 5.02, 5.09, 5.22

   A soil scientist wants to know whether the population mean percent organic matter is different than 4%. A significance level of \( \alpha = 0.05 \) is chosen.

   (a) State hypotheses appropriate to the research question.

   (b) Graph the data as you see fit. Why did you choose the graph(s) that you did and what does it (do they) tell you?

   (c) Regardless of your conclusions from (b), use the bootstrap to perform a test of the hypotheses you stated in (a). Use \( B = 8000 \) resamplings. Compute the p-value, and make a reject or not reject conclusion. Then state the conclusion in the context of the problem. In other words, does it seem the mean organic matter level is different than 4%?

   (d) Regardless of your conclusions from part (b), use a \( t \)-test to perform a test of the hypotheses you stated in (a). Compute the p-value, and make a reject or not reject conclusion. Then state the conclusion in the context of the problem. In other words, does it seem the mean organic matter level is different than 4%? (I recommend doing this part with a hand calculator and statistical tables as practice for exam conditions, but you may check your answers using R if you wish.)
(e) Compare your answers from parts (c) and (d). Which method do you think is better?

3. A study is conducted regarding shatterproof glass used in automobiles. 26 glass panes are coated with an anti-shattering film. Then a 5-pound metal ball is fired at 70mph at each pane. 5 panes shatter. We wish to determine whether, in the population of all such panes, the probability the glass shatters under these conditions is different from .20.

(a) State the appropriate null and alternative hypotheses.
(b) Write down the test statistic, check the conditions for trusting the conclusion of the test, and calculate the observed value of the test statistic.
(c) Calculate the rejection region and draw a conclusion, given the significance level $\alpha = .05$.

4. An animal’s maintenance caloric intake is defined as the number of calories per day required to maintain its weight at a constant level. We wish to discover whether the median maintenance caloric intake, $M$, for a population of rats is less than 10g/day. We draw a SRS of 17 rats, feed each rat 10g of dry food per day for 30 days, and find that 4 of the rats lost weight, while the rest gained weight.

(a) State null and alternative hypotheses in terms of $M$.
(b) Let $B$ be the number of rats in a SRS of size 17 that exhibit daily caloric demands less than 10g/day. If $H_0$ is true, what is the distribution of $B$?
(c) What is the value of $B$ observed in the study?
(d) Calculate the p-value and draw a conclusion.