3 Probability

Probability vs. statistics

• *Probability* considers information from a ___________ to allow claims about properties of random samples from that population.
  
e.g. Claire’s ant farm supports 100 ants. She knows 10 of the 100 ants are poisonous, but they look like the non-poisonous ants. Probability would help her answer, “If I choose an ant at random, what is the probability that it is poisonous?” She would use information from _______________ to make a claim about _______________.
  
  She can calculate this probability *exactly*, but her claim describes what tends to happen _______________, not what happens in any single sample.

• *Statistics* considers information from a _______________ to allow claims about the properties of the population from which the sample was drawn.
  
e.g. Suppose instead that Claire doesn’t know how many of the ants are poisonous. She takes a random sample of 20 ants and finds that 2 are poisonous. Statistics would help her answer, “What proportion of the 100 ants are poisonous?” She would use information from the sample to make a claim about the population.

  Her claim will be an *estimate*; _______________ about the population will remain. The only way to get the right answer would be to examine every ant. The estimate will be useful only if paired with a description of its uncertainty: e.g. *.1±.02* is stronger than *.1±.05*. What would reduce the uncertainty in Claire’s estimate?

Probability definitions

• A *random process* is one whose outcome is _______________ due to chance. e.g.

• An *outcome* is a result of a random process. e.g.

• A *sample space* $S$ is the set of possible _______________ of a random process.

• An _______________ $E$ is a collection of outcomes (a __________ of the sample space). e.g.

• The *probability* of an outcome of a random process is the _______________ of times the outcome would occur _______________ if the process were repeated _______________.

Properties of probability

For a random process on a finite sample space,

- the probability of an event is the _______ of probabilities of outcomes in that event
- the probability of an event is between 0 and 1 \((0 \leq P(E) \leq 1)\)
- the probability ______ indicates an event that will not occur
- the probability ______ indicates an event that will always occur
- the probability that an event does not occur is one minus the probability that it does occur

e.g. Consider the 100 ants again. Here are their weights.

<table>
<thead>
<tr>
<th>Weight (mg)</th>
<th>Ant Weights</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3.0</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>3.5</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>4.0</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

• If one ant is selected at random, what is the probability its weight is less than 2.75 mg? (Assume no ant weighs exactly 2.75 mg.)

• What is the probability that a random ant weighs more than 2.75 mg?
- What is the probability that a random ant weighs between 2.75 and 3.25 mg?

\[
\text{Weight (mg)}
\]

<table>
<thead>
<tr>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>40</td>
</tr>
</tbody>
</table>

\[
\text{Ant Weights:}
\]

- 2.0
- 2.5
- 3.0
- 3.5
- 4.0

Independence

- Two events are \__________ if the occurrence of one does not change the probability of the other.

- Two events that are not independent are \__________.

Theorem: If two events are independent, then the probability that they both occur is the \__________ of their individual probabilities. (That is, if events A and B are independent, then \( P(A \text{ and } B) = \) \__________.)

e.g. Consider the sample space \( S = \{ \text{outcomes of two fair die rolls} \} \) and these events:

- \( A = \text{first die is 3} \)
- \( B = \text{second die is 1} \)
- \( C = \text{dice sum to 8} \)

Which pairs of events are independent?
Examples

1. Suppose, for cars more than 2 years old, 20% will require repairs once during a given year, 10% will require repairs twice, and 5% will require three or more repairs.
   (a) What is the probability that a randomly selected car will need some repairs?
   (b) What is the probability that a randomly selected car will need no repairs?
   (c) What is the probability that a randomly selected car will need at most one repair?

2. A die is to be rolled. Find the probabilities for these events:
   (a) \( A \): observe a 6

   (b) \( B \): observe an odd number

   (c) \( A \): observe a number greater than 4

   (d) \( A \): observe an even number and a number greater than 2

3. A mine safety chamber has a battery operated telephone and a chemical oxygen generator, each of which must work for the chamber to be helpful to miners after an accident. The phone fails 1% of the time and the generator fails 5% of the time, and these failures are independent. What is the probability that the chamber will be helpful after an accident?