Homework 4
Due Date: Friday, November 29, 2002

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Question 1 (20 points):
(Part a:) Express the relations shown in Figure 1 using the relations given in class and various operators (of course!). Please give a short justification for your answer.
(Part b:) Let CousinsFatherSide\((n)\), CousinsMotherSide\((n)\), and Cousins\((n)\) be the set of cousins that are separated by \(n\)-steps on the father’s side, mother’s side, and father or mother’s side. Write a recursive expression for these relations, i.e., express the relations with parameter \(n\) in terms of relations with parameter \(n - 1\) and the basic relations introduced in class.

Question 2 (20 points): Establish whether the following equations be-

| FatherInLaw : People ↔ Males |
| SisterInLaw : People ↔ Females |
| FirstCousinsFatherSide : People ↔ People |
| FirstCousinsMotherSide : People ↔ People |
| FirstCousins : People ↔ People |
| SecondCousinsFatherSide : People ↔ People |
| SecondCousinsMotherSide : People ↔ People |
| SecondCousins : People ↔ People |

Figure 1: Some relations
tween the relations are true or not. Please justify your answer. You should enter these equations in Alloy and play around with them. Assume that the relations have appropriate types.

\[(P; Q)^\top = Q^\top; P^\top\]
\[(P \subseteq Q) \Rightarrow (P; P \subseteq Q)\]
\[(Q \neq \emptyset) \Rightarrow (\emptyn; Q; \emptyn = \emptyn)\]

**Note:** The symbol \(\emptyset\) denotes the empty relation.

**Question 3 (20 points):** Translate the phone-switch example we covered in class into a model accepted by Alloy. Use Alloy to check the assertions and explain the counterexamples you obtain. Now propose a fix and incorporate it in the model.

**Question 4 (40 points):** In this question, you will write a specification for the problem described. Our aim is to model a library. There are two types `PERSONS` and `BOOKS`. There is a function `issued : BOOKS \rightarrow PERSONS` and a set `Library : Set BOOKS`. If book \(b\) is issued to a person \(p\), then `issued(b) = p`. `Library` represents the set of books in the library. There is also a set of books on reserve given by the set `Reserve : Set BOOKS`. The operations are:

- **Issue a book**
  This operation issues a book to a person. A book on reserve or already issued cannot be issued.

- **Return a book**
  This operation models the act of a person returning a book.

- **Adding a book to the library**
  This operation models a new book being added to the library.

- **Putting a book on reserve**
  This operation models a book being put on reserve. A book which is currently issued cannot be put on reserve.

- **Taking a book off reserve**
  This operation models a book being taken off reserve.
The claims (or assertions in Alloy) are:

- Issuing a book and then returning it results in the same state.

- If a book is on reserve, it should never be issued. This claim should be true before and after every operation.

**Part a:** Write a mathematical description for all the operations and claims and explain your answer.

**Part b:** Now express your design in *Alloy.*