Gaze-enabled Egocentric Video Summarization via Constrained Submodular Maximization
Supplementary Material

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1. Proof of Proposition 4.1

Proof. Recall our objective $F(\mathcal{S}) = M(\mathcal{S}) + \lambda I(\mathcal{S})$ is submodular. In our implementation, We pick λ such that $F(\mathcal{S})$ is nonegative.

By the submodularity[1], we first have

$$F(\mathcal{A}) + F(\mathcal{B}) \geq F(\mathcal{A} \cup \mathcal{B}) + F(\mathcal{A} \cap \mathcal{B}), \quad \forall \mathcal{A} \subseteq \mathcal{V}, \mathcal{B} \subseteq \mathcal{V} \tag{1}$$

Following [2], we apply our Alg. 1 twice to get two local optimal solutions $\mathcal{S}_1 = \text{argmax}_{local}\{F(\mathcal{S}_1) : \mathcal{S}_1 \in \mathcal{I}, \mathcal{S}_1 \subseteq \mathcal{V}_1 = \mathcal{V}\}$, $\mathcal{S}_2 = \text{argmax}_{local}\{F(\mathcal{S}_2) : \mathcal{S}_2 \in \mathcal{I}, \mathcal{S}_2 \subseteq \mathcal{V}_2 = \mathcal{V} \backslash \mathcal{S}_1\}$. And return the maximum from these two as our final solution: $\mathcal{S} = \text{argmax}\{F(\mathcal{S}_1), F(\mathcal{S}_2)$. Given the local optimality and by Lemma 2.5 from [2] , we then have

$$2(1+\epsilon)F(\mathcal{S}_i) \geq F(\mathcal{S}_i \cup \mathcal{C}) + F(\mathcal{S}_i \cap \mathcal{C}), \quad \forall \mathcal{C} \subseteq \mathcal{I}, \quad |\mathcal{S}_i| = |\mathcal{C}|, \quad i = 1, 2 \tag{2}$$

Let $\mathcal{O}$ denote the unknown global optimal solution to the original problem $\max\{F(\mathcal{S}) : \mathcal{S} \in \mathcal{I}, \mathcal{S} \subseteq \mathcal{V}\}$. Let $\mathcal{O}_i = \mathcal{O} \cap \mathcal{V}_i, i = 1, 2$. We note $\mathcal{O}_1 = \mathcal{O} \cap \mathcal{V}_1 = \mathcal{O} \cap \mathcal{V} = \mathcal{O}$. With (2), we have

$$2(1+\epsilon)(F(\mathcal{S}_1) + F(\mathcal{S}_2)) \geq F(\mathcal{S}_1 \cup \mathcal{O}_1) + F(\mathcal{S}_1 \cap \mathcal{O}_1) + F(\mathcal{S}_2 \cup \mathcal{O}_2) + F(\mathcal{S}_2 \cap \mathcal{O}_2) \tag{3}$$

Since $F(\mathcal{S}) \geq F(\mathcal{S}_1), F(\mathcal{S}) \geq F(\mathcal{S}_2)$, we have

$$4(1+\epsilon)F(\mathcal{S}) \geq F(\mathcal{S}_1 \cup \mathcal{O}_1) + F(\mathcal{S}_1 \cap \mathcal{O}_1) + F(\mathcal{S}_2 \cup \mathcal{O}_2) + F(\mathcal{S}_2 \cap \mathcal{O}_2) \tag{4}$$

Using submodularity, we have

$$\begin{aligned} F(\mathcal{S}_1 \cup \mathcal{O}_1) + F(\mathcal{S}_2 \cup \mathcal{O}_2) + F(\mathcal{S}_1 \cap \mathcal{O}_1) &\geq F(\mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{O}_1 \cup \mathcal{O}_2) + F((\mathcal{S}_1 \cup \mathcal{O}_1) \cap (\mathcal{S}_2 \cup \mathcal{O}_2)) + F(\mathcal{S}_1 \cap \mathcal{O}_1) \\ &= F(\mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{O}) + F(\mathcal{O}_2) + F(\mathcal{S}_1 \cap \mathcal{O}_1) \\ &\geq F(\mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{O}) + F(\mathcal{O}_2 \cup (\mathcal{S}_1 \cap \mathcal{O}_1)) + F(\mathcal{O}_2 \cap \mathcal{S}_1 \cap \mathcal{O}_1) \\ &= F(\mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{O}) + F(\mathcal{O}) + F(\emptyset) \\ &= F(\mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{O}) + F(\mathcal{O}) \end{aligned} \tag{5}$$

Putting (5) back to (4), we get

$$4(1+\epsilon)F(\mathcal{S}) \geq F(\mathcal{O}) + F(\mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{O}) + F(\mathcal{S}_2 \cap \mathcal{O}_2) \geq F(\mathcal{O}) \tag{6}$$

This concludes our proof. □

References

[1] S. Fujishige. *Submodular functions and optimization*, volume 58. Elsevier, 2005. 1

[2] J. Lee, V. S. Mirrokni, V. Nagarajan, and M. Sviridenko. Maximizing nonmonotone submodular functions under matroid or knapsack constraints. *SIAM J. Discrete Math.*, 23(4):2053–2078, 2010. 1