Incorporating User Interaction and Topological Constraints within Contour Completion via Discrete Calculus

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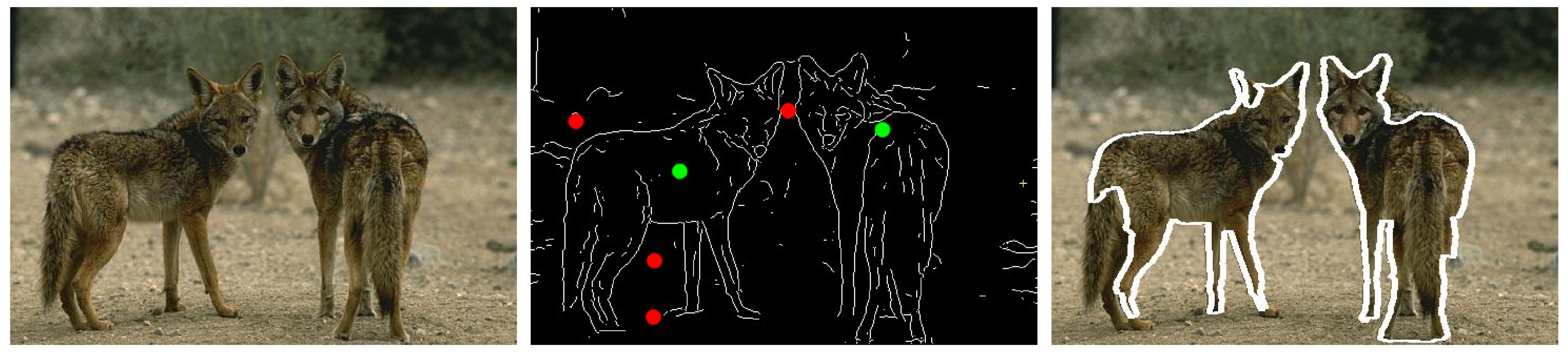
Problem Statement

Interactive multiple contour completion for edge-based segmentation



Main Intuition:
a) **#** of edgelets remains relatively **unaffected** by the image resolution
b) No implicit assumption on region homogeneity

Main Challenge

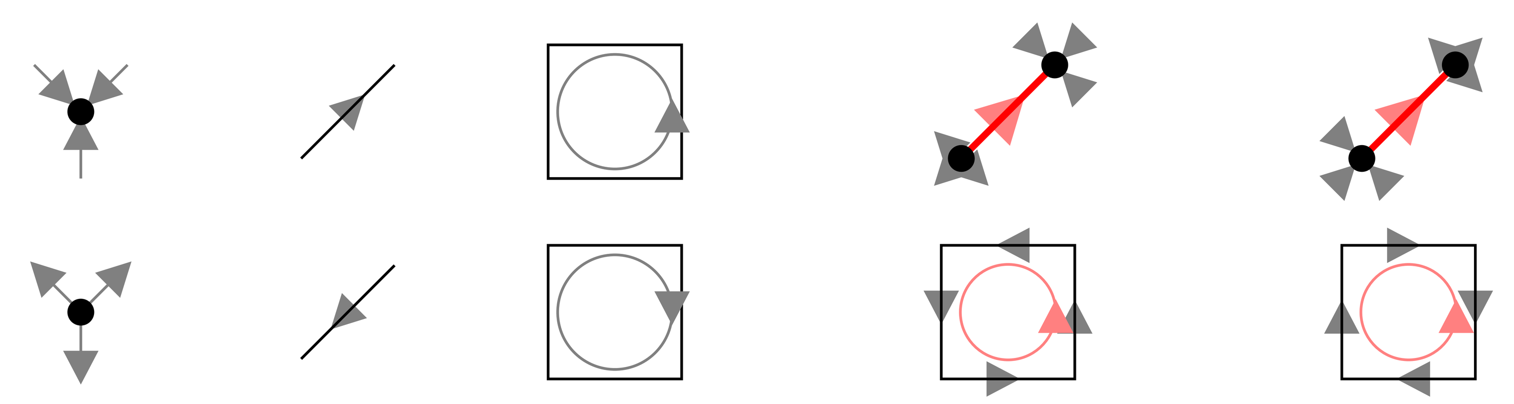
Modeling topological constraint while concurrently finding one or more minimum energy closed contours which satisfy:

- Foreground seeds must be "inside"
- Background seeds must be "outside".

Advantages of Our Method (EulerSeg)

1. Basic primitives are edgelets (Little dependence on **#** of pixels)
2. Dense strokes not needed to learn appearance model. Results do *NOT* vary with seed location (Interaction constraints are completely geometric in form)
3. Incorporating connectedness priors and specifying **#** of closures is easy (Euler characteristic)

Discrete Calculus

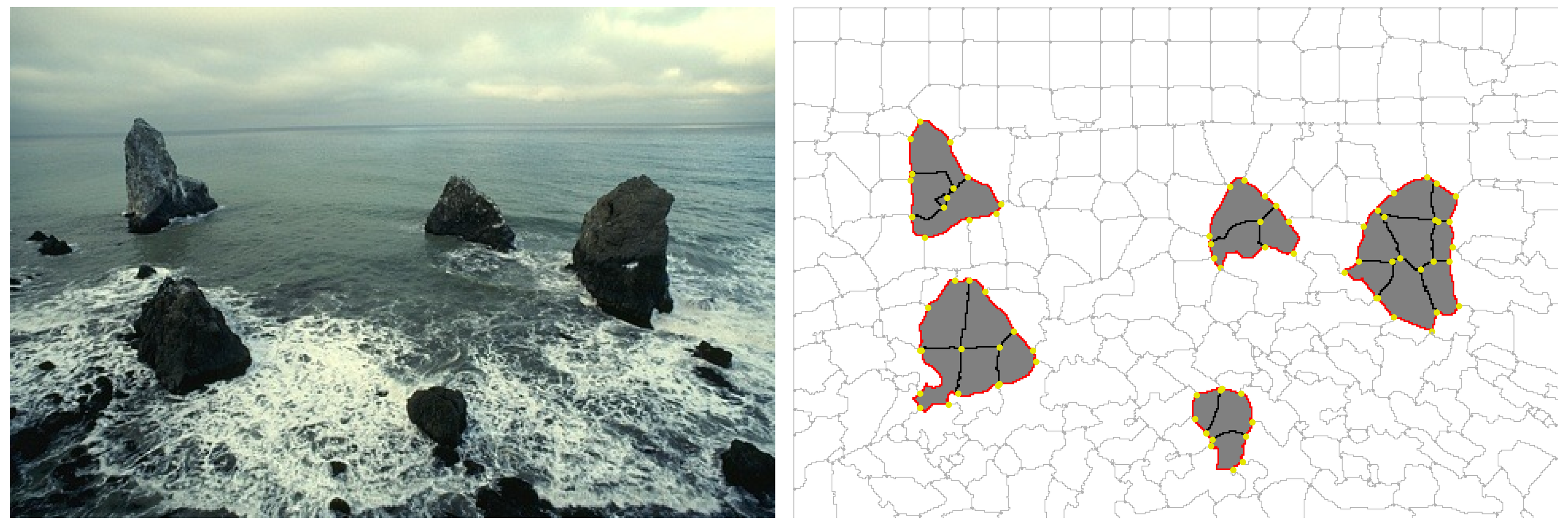


- Vertex-edge and edge-face incidence matrices:

$$\mathbf{A}_{v_k,e_{ij}} = \begin{cases} +1 & k=i, \\ -1 & k=j, \\ 0 & \text{otherwise.} \end{cases} \qquad \mathbf{C}_{e,f} = \begin{cases} +1 & e \vdash f, \text{ coherently oriented}, \\ -1 & e \vdash f, \text{ anti-coherently oriented}, \\ 0 & \text{otherwise.} \end{cases}$$

- More cell incidence matrices: $C_1 = C$, $C_2 = |C_1|$, $A_2 = |A|$, A_3: $A_{3;ij} = A_{2;ij}/d_i$ where d_i denotes the degree of node i
- Graph construction: superpixel + globalPb.

Problem Formulation



$$\min_{\mathbf{w},\mathbf{x},\mathbf{y},\mathbf{z}} \frac{\mathbf{N}^T\mathbf{w}}{\mathbf{D}^T\mathbf{w}}, \quad (\mathbf{N}\text{: edge weight vector, } \mathbf{D}\text{: edge length vector})$$

$$\text{s.t.} \quad \mathbf{w} = |C_1\mathbf{x}|, \quad 2\mathbf{y} = \mathbf{w} + C_2\mathbf{x}, \quad \text{(Cell complex constraints)}$$

$$A_3\mathbf{y} \le \mathbf{z} \le A_2\mathbf{y}, 1^T\mathbf{x} + 1^T\mathbf{z} - 1^T\mathbf{y} = n, \quad \text{(Euler Characteristic)}$$

$$\mathbf{x}_1 \le \mathbf{x} \le 1 - \mathbf{x}_0, \quad \mathbf{w},\mathbf{x},\mathbf{y},\mathbf{z} \in \{0,1\}. \quad \text{(Inside/outside constraint)}$$

Minimizing a Ratio Cost

Solved by minimizing

$$\psi(t, \mathbf{w}) = (\mathbf{N} - t\mathbf{D})^T\mathbf{w}$$

- Over admissible $\mathbf{w}$ for a sequence of chosen values of t.
- Requires $\mathbf{D} \ge 0$ and $\mathbf{D}^T\mathbf{w} \ne 0$
- With an initial finite bounding interval $[t_l, t_u]$

Pick $t_0 = \frac{t_l + t_u}{2}$, and let

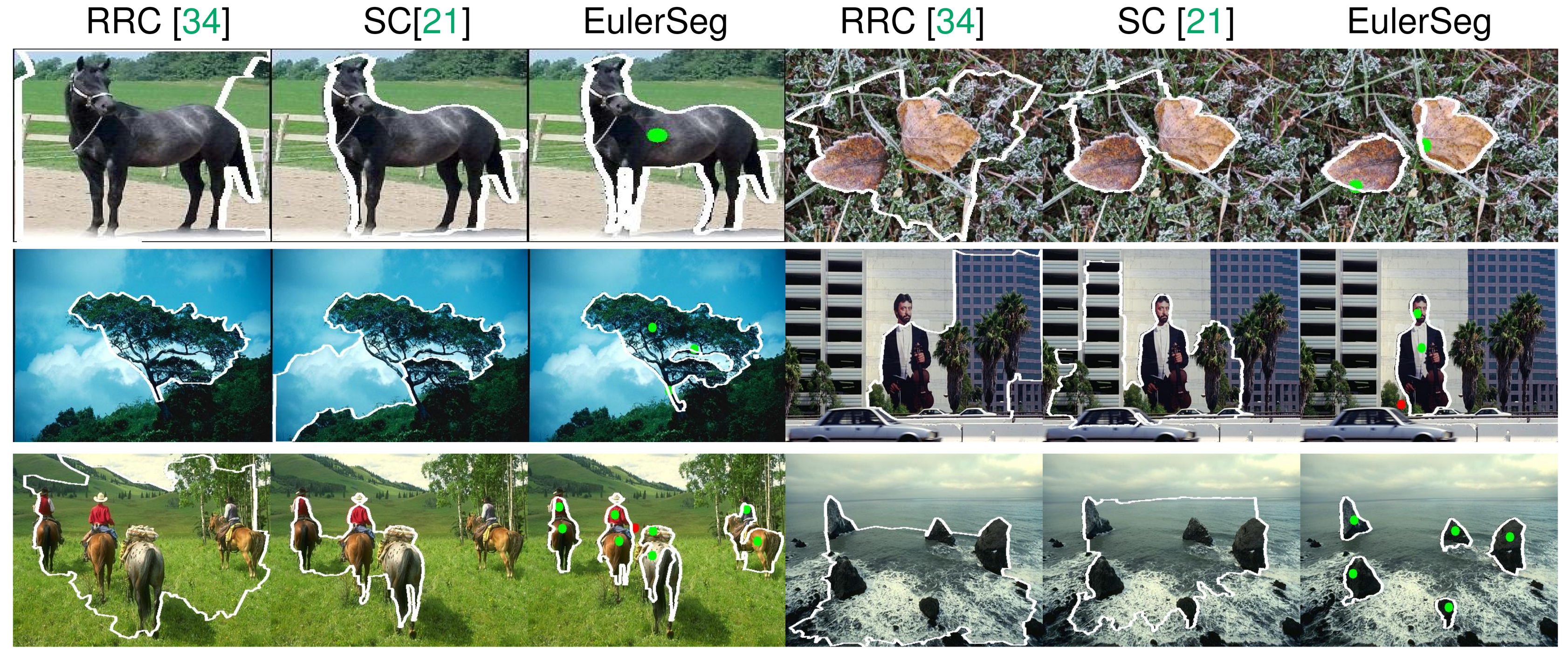
$$\bar{\mathbf{w}} = \arg\min_{\mathbf{w}} \psi(t_0, \mathbf{w})$$

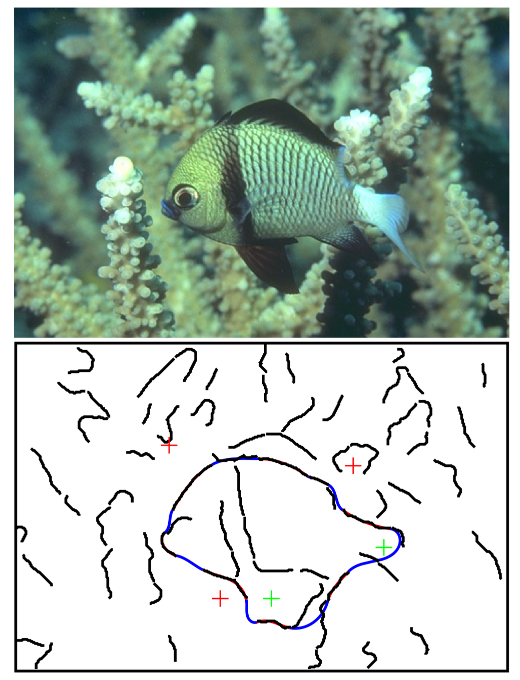
- $\psi(t_0, \bar{\mathbf{w}}) = 0$: $\mathbf{N}^T\bar{\mathbf{w}}/\mathbf{D}^T\bar{\mathbf{w}} = t_0$, terminate with solution t_0
- $\psi(t_0, \bar{\mathbf{w}}) < 0$: $\mathbf{N}^T\bar{\mathbf{w}}/\mathbf{D}^T\bar{\mathbf{w}} < t_0$, $t_u \leftarrow \mathbf{N}^T\bar{\mathbf{w}}/\mathbf{D}^T\bar{\mathbf{w}}$
- $\psi(t_0, \bar{\mathbf{w}}) > 0$: $\mathbf{N}^T\bar{\mathbf{w}}/\mathbf{D}^T\bar{\mathbf{w}} > t_0$, $t_l \leftarrow t_0$

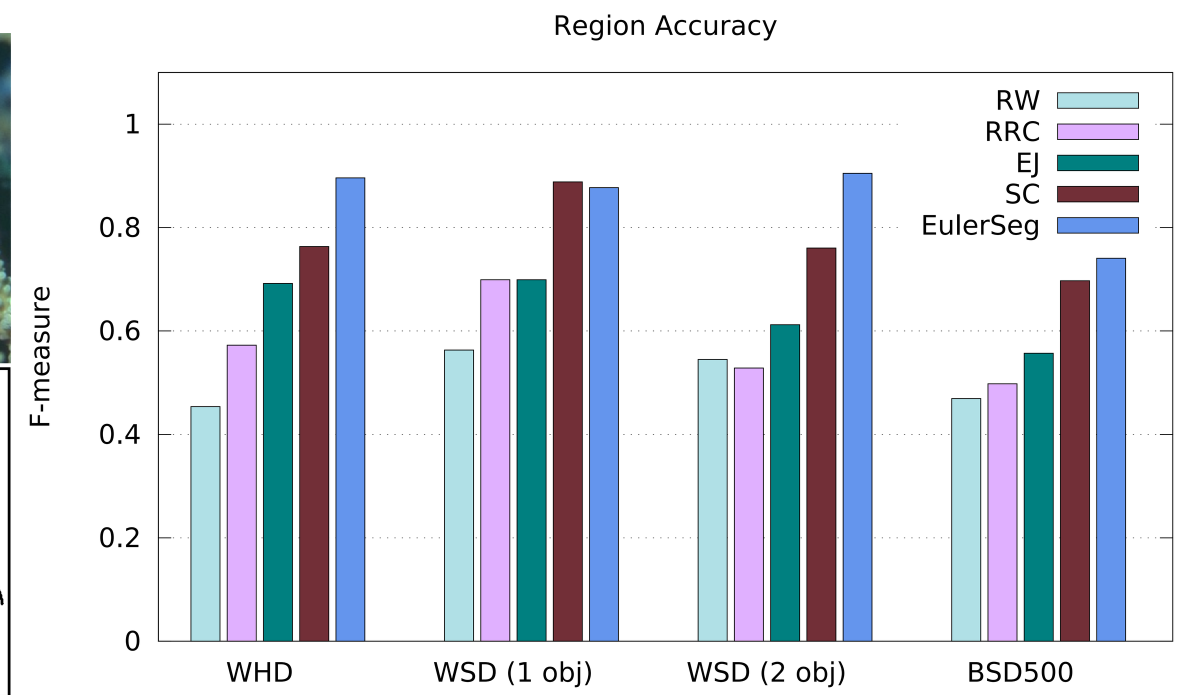
Branch and Bound Method

- **Limitations of superpixel decomposition:** occlusions or weak boundaries give cases where the set of superpixel boundary primitives do not include some valid edgelets.
- **Solution:** supplement the basic set of edgelets with additional contour pieces that bridge the "gaps" and allow a more accurate contour closure even in the presence of very weak signal variations.
- **Euler spiral for shape completion**: isotropy, symmetry, smoothness, Extensibility, and roundness.
- **Key idea:** iteratively build upon the current partial path, until we get a cycle that encloses a feasible region. Subtrees are discarded if they give rise to a self-intersecting partial path; therefore, no need to explore the entire branch and bound tree.

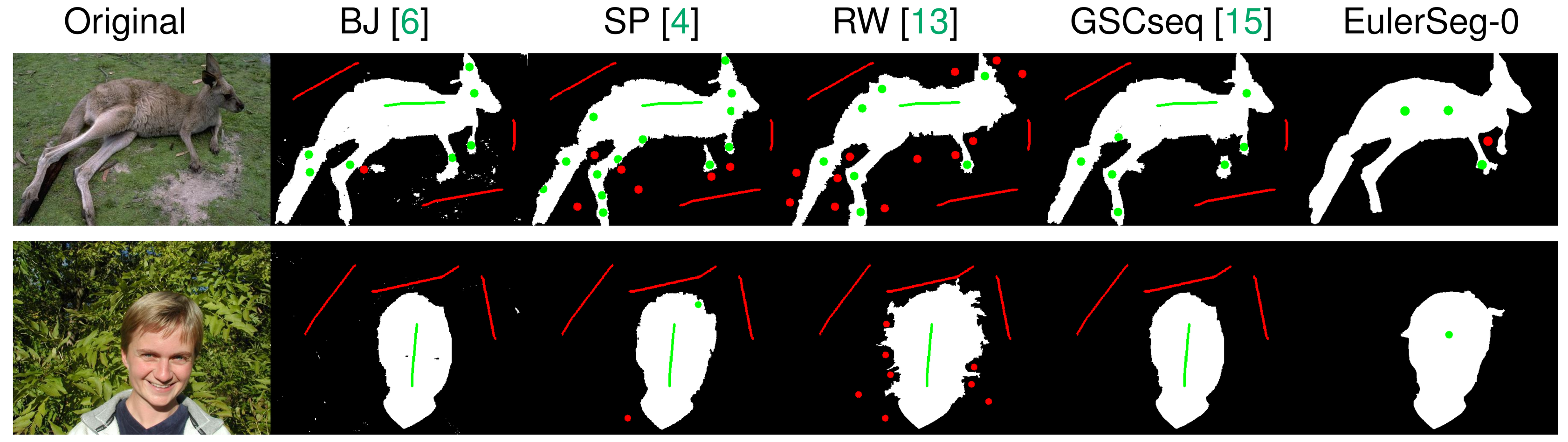
Evaluation on Contour Completion







Evaluation on Interactive Segmentation



Takeaways

- A IP model for multiple contour completion with seed constraints
- An informative seeds dataset ($\sim$ 1000 images)
- Topological constraints and interaction can be easily incorporated via discrete calculus

How much effort to reach $F = 0.95$?

Method	BJ [6]	RW [13]	SP [4]	GSCseq [15]	EulerSeg
Effort	5.51	6.48	4.54	2.30	**2.06**

Seeds can serve an important role beyond must link/cannot link

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