CS 577: Introduction to Algorithms	**Homework 0**
Out: 09/07/17	**Due: Not to be turned in**

Ground Rules

- Graded problems should be done in pairs.
- You should try all problems.
- Hand in only two graded problems from one set, for each pair.
- Turn in each problem on **a separate sheet of paper**.
- Recitation sessions may discuss all problems.
- Graded problems (except for HW0) are to be turned in on the due date at the beginning of the lecture.
- Write your name(s) clearly on your submissions.

Problems (All ungraded for HW0)

1. Find a partner for doing homework.
2. Familiarize yourself with the course webpage on canvas [1]. Make sure you know all course policies.
3. Review the topics from CS 240 curriculum, especially: induction, solving recurrences, asymptotic notation, and graphs.
4. Prove by induction that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.
5. Prove that every integer (positive, negative, or zero) can be written in the form $\sum_i \pm 3^i$, where the exponents i are distinct non-negative integers. For example,

$$44 = 3^4 - 3^3 - 3^2 - 3^0$$
$$23 = 3^3 - 3^1 - 3^0$$
$$19 = 3^3 - 3^2 + 3^0$$

6. You are given a $2^n \times 2^n$ chessboard with one square missing (that we will call a hole). Prove using induction on n that regardless of the position of the hole, you can tile the chessboard with L-shaped pieces containing three squares each. That is, you can find an arrangement of the L-shaped tiles such that every square of the chessboard is covered by exactly one tile and the hole is left uncovered.
7. You are given an $n \times n$ chessboard, where $n \geq 2$. Suppose two antipodal squares at the corners are missing. For concreteness, you can think of the Northeast corner and Southwest corner squares are missing. A *domino* is a 1×2 piece (you can think of it as consisting of two adjacent squares). Prove that there is no way to tile the chessboard (with the two missing holes) by dominos. (Hint: You should separately consider n is even or n is odd.)
8. Order the following functions according to the asymptotically smallest to the asymptotically largest.

$$\begin{array}{ccc} \log n & \sqrt{n} & 5^n \\ n^{\log n} & 5^{\sqrt{\log n}} & (\log n)^n \\ 3^{n+10} & \log(5^n) & \sqrt{5^{\log n}} \end{array}$$

[1] `https://canvasinfo.wisc.edu/` , then look for CS 577

9. Consider the following sorting algorithm. We assume A is an array of $n \geq 1$ many integers. The initial call is SuperSort$(A, 1, n)$.

```
SuperSort(A,i,j){  \\sorts the subarray A[i..j]
    if (j = i) then return;
    if (j = i+1)                        \\when there are only 2 elements
      if (A[i] > A[j]) swap(A,i,j)      \\swaps A[i] and A[j]
    else {
      k = floor of ( (j-i+1)/3 );       \\integer part of the length (j-i+1)/3
      SuperSort(A,i,j-k);               \\sort first two thirds
      SuperSort(A,i+k,j);               \\sort second two thirds
      SuperSort(A,i,j-k);               \\sort first two thirds again
      }
}
```

(a) Prove using induction that this algorithm is correct, that is, it always produces a sorted array.

(b) Determine the asymptotic number of comparisons this algorithm makes.

10. (challenge problem) You are given 12 coins, and you are told that exactly 11 of which are real and one is fake. The only way to tell which is real and which is fake is by weighing them. All real coins weigh exactly the same, but the fake one weighs either heavier or lighter than the real one.

You are given a scale on which you can weigh any two disjoint subsets of the 12 coins. This is the basic operation: To weigh any chosen subset against another disjoint subset. For example, you can decide to weigh any 2 coins against another 2 coins. (There is no point to weigh 2 subsets of different cardinality. Do you see why?) Upon given the result (either the first subset weighs more or equal or less than the second one), your subsequent choice as to what to weigh can depend on this result.

(a) Devise a strategy that identifies the fake coin in no more than 3 sequential weighings. Can your method tell whether the fake coin weighs more or less than the real one?

(b) Can you devise a strategy that identifies the fake coin in no more than 3 weighings, finding out whether the fake coin weighs more or less than the real one, and the instructions on what subsets to weigh can be made parallel, i.e., which subsequent subsets to weigh do not depend on the results of previous weighings.