Homework 2 - Due at Lecture on Wed, Feb 8th

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Instructions: You must do this homework **in groups of two**. Please hand in ONE copy of the homework that lists the **section number**, full **names** (as appear in Learn@UW) and **UW ID** numbers of all students. You must **staple** all pages of your homework together to receive full credit

Question 1. (3 points)

a. How many distinct decimal values (base-10) can be represented using 9 decimal digits?
   We have 9 positions and in each position 10 different digits (0, 1, …, 9) can sit, so the answer is 10^9
b. How many distinct hexadecimal values (base-16) can be represented using 8 hexadecimal digits?
   We have 8 positions and in each position 16 different digits (0,1, …, F) can sit, so the answer is 16^8
c. How many distinct binary values (base-2) can be represented using 32 binary digits (bits)?
   We have 32 positions and in each position 2 different digits (0 or 1) can sit, so the answer is 2^{32}

Question 2. (2 points)

Find the 2's complement of the following binary numbers:

a. 0100 1010   1011 0110
b. 1001 0000   0111 0000

Question 3. (3 points)

a. Assume that there are about 200 students in your class. If every student is to be assigned a unique bit pattern, what is the minimum number of bits required to do this?
   $128\ (2^7) < 200 < 256\ (2^8)$
   With 7 bits can represent only 128 different students, so we need at least 8 bits to assign a unique bit pattern to each student.
b. How many more students can be admitted to the class without requiring additional bits to represent each student's unique bit pattern?
   256 - 200 = 56
c. How many students need to drop the course if we have only 7 bits to represent each student uniquely?
   200 - 128 = 72

Question 4. (2 points)

Compute the following:

a. NOT(1101) AND NOT(1000)
   NOT(1101) = 0010
   NOT(1000) = 0111
   0010 AND 0111 = 0010

b. NOT(1001 OR (1010 AND 1101))
   (1010 AND 1101) = 1000
   (1001 OR 1000) = 1001
   NOT(1001) = 0110

Question 5. (4 points)

The binary number 1110 1000 is a string of 0s and 1s that can be interpreted differently depending on its data type. Please find the decimal value of the above number for the following data types:

a. An unsigned integer
   1110 1000 = $2^7 + 2^6 + 2^5 + 2^3 = 128 + 64 + 32 + 8 = 232$
b. A signed-magnitude integer
   1110 1000 = - (0110 1000) = $-(2^6 + 2^5 + 2^3) = -104$
c. A 1's complement integer
   1110 1000 = - (0001 0111) = $-(2^4 + 2^2 + 2^1 + 2^0) = -23$
d. A 2's complement integer
   1110 1000 = - (0001 1000) = $-(2^4 + 2^3) = -24$

Question 6. (6 points)

The value "-64" can be represented by strings of 0s and 1s in many different ways depending on its data type. Please show its **hexadecimal** representation for the following data types.

a. An 8-bit unsigned integer
   NA
b. An 8-bit signed-magnitude integer
   1100 0000 = xC0
c. An 8-bit 1's complement integer
   1011 1111 = xBF
d. An 8-bit 2's complement integer
   1100 0000 = xC0

e. An ASCII string (Only represent the characters between the quotation marks and assume it as a null terminated string)
- = x2D
6 = x36
4 = x34

Number: 2D3634

f. A 32-bit IEEE floating point number
$-64 = (-1)^1 * 1.00000000000000000000000*2^6 \Rightarrow$

sign: 1
fraction: 00000000000000000000000
exponent -127 = 6 => exponent = 133 = 10000101

Number: 1 10000101 00000000000000000000000

Question 7. (6 points)

a. What is the largest positive value one can represent with a 7-bit 2's complement number? Write your result in binary and decimal.
$0111111 = 2^6 - 1 = 63$

b. What is the largest positive value one can represent with an n-bit 2's complement number?
$0111...111 = 2^{n-1} - 1$

c. What is the greatest magnitude negative value one can represent with a 7-bit 2's complement number? Write your result in binary and decimal.
$1000000 = -2^6 = -64$

d. What is the greatest magnitude negative value one can represent with a n-bit 2's complement number?
$1000...000 = -2^{n-1}$

e. What is the largest positive value one can represent with a 7-bit 1's complement number? Write your result in binary and decimal.
$0111111 = 2^6 - 1 = 63$

f. What is the largest positive value one can represent with an n-bit 1's complement number?
$0111...111 = 2^{n-1} - 1$

g. What is the greatest magnitude negative value one can represent with a 7-bit 1's complement number? Write your result in binary and decimal.
$1000000 = -(0111111)_{1\text{'s complement}} = -(2^6 - 1) = -63$

h. What is the greatest magnitude negative value one can represent with an n-bit 1's complement number?
$1000...000 = -(0111...111)_{1\text{'s complement}} = -(2^{n-1} - 1)$

i. What is the maximum unsigned value one can represent with 7 quad digits? (quad number system is base-4 where only the digits 0, 1, 2 or 3 are legal)

$(3333333)_4 = 3*4^6 + 3*4^5 + 3*4^4 + 3*4^3 + 3*4^2 + 3*4^1 + 3*4^0$

$= 3*(4^6 + 4^5 + 4^4 + 4^3 + 4^2 + 4^1 + 4^0)$

$= 4^7 - 1$

j. What is the maximum unsigned value one can represent with n quad digits?

$(333...333)_4 = 3*4^{n-1} + 3*4^{n-2} +... + 3*4^1 + 3*4^0$

$= 3*(4^{n-1} + 4^{n-2} +... + 4^1 + 4^0)$

$= 4^n - 1$

Question 8. (2 points)

Give an example of an *integer* that can be represented in floating point format (32-bit IEEE format), but cannot be represented as a 32-bit two's complement integer. Show its hexadecimal representation.

The biggest 2's complement integer that can be represented with 32 bits is 2^{31} -1, so any integer number greater than 2^{31} -1, but less than 2^{128} , is the answer to this question.

so 2^{31} , 2^{31} +1, 2^{31} +2 and 2^{32} are some examples.

Question 9 (2 points)

a. What conditions indicate overflow has occurred when two 2's complement numbers are added?

   1. When 2 positive values are added and the sum's MSB is 1
   2. When 2 negative values are added and the sum's MSB is 0

b. Why does the sum of a negative 2's complement number and a positive 2's complement number never generate an overflow?

   Because the sum result would be a number more than the negative number and less than the positive number, so it is in the range of representable numbers, so there is no overflow.