Introduction to Barycentric Coordinates

Due: Wednesday June 20th by 8:00 a.m., via Blackboard. Optional hard copy may be turned in during lab on Thursday.

Overall Assignment

For this assignment you are to write a simple computer program that calculates the value of a function at any point within a triangle, given the function values at the vertices of the triangle and the barycentric coordinates of the interior point at which the function value is desired.

Background: Triangulation

Engineers and scientists are often faced with the problem of interpolating function values over an area, based on measurements taken at irregularly spaced locations. For example, weather data is measured principally at airports, which are distributed somewhat randomly about the country, and oil well test sites are also irregularly spaced. Modern computer graphics is also based on interpolating color values between the vertices of triangle meshes.

One approach to this problem is to first triangulate the space based on the known data points, which basically means to subdivide the space into triangles as shown in the following diagram:

Delaunay triangulation of a random set of 100 points in a plane. (Source: http://en.wikipedia.org/wiki/Delaunay_triangulation)

There are many different algorithms for triangulating a space, all of which are beyond the scope of this assignment.

Background: Barycentric Coordinates

Once a triangulation has been determined, and the particular triangle that contains the point of interest has been identified, then the next step is to interpolate the function values from the vertices of the triangle to the point of interest. This is where barycentric coordinates come in.

Given a triangle defined by vertices \( r_1, r_2, \) and \( r_3 \), then any point \( r \) interior to the triangle can be defined by a weighted sum of the three vertices, as given by the equations:

\[
r = \lambda_1 r_1 + \lambda_2 r_2 + \lambda_3 r_3
\]

where \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are the barycentric coordinates, subject to the constraints:

\[
\lambda_1 + \lambda_2 + \lambda_3 = 1.0 \\
0.0 \leq \lambda_i \leq 1.0 \ \forall i \ \text{in} \ 1, 2, 3 \ \text{for points interior to the triangle.}
\]

A function \( f \) whose values are known at the vertices can be interpolated as:

\[
f(r) = \lambda_1 f(r_1) + \lambda_2 f(r_2) + \lambda_3 f(r_3)
\]
Program Details

For this assignment you are to write a simple computer program that calculates the value of a function at a point in the interior of a triangle, given its barycentric coordinates and the values of the functions at the three vertices of the triangle.

- Your program should first print out your name and ACCC account name (e.g. astudent), and explain to the user what the program does.

- Your program should then ask the user to enter the function values at the three vertices of the triangle and the barycentric coordinates corresponding to the first two vertices, **IN THAT ORDER**.
  - Each data item should be entered separately, with its own question and answer. I.e. with five data values to enter, your program should ask five separate questions and get a separate response for each one, rather than asking the user to enter two or more values in response to a single question.

- After performing the necessary calculations, the program should report the results, **including an echo of the user’s input**. The specific results to be reported should include:
  - The third barycentric coordinate.
  - The function value at the point of interest.

Special Notes:

- **You should work out some sample problems by hand before writing any computer code.** As an example, if the function values are 34.5, 26.8, and -13.7, and the first two barycentric coordinates are 0.25 and 0.33, then the third coordinate should be 0.42 and the function value should be 11.715. (You should verify this for yourself.)

- This problem does not involve any units. (Normally you would have to be careful about units, and possibly convert units, such as feet to meters.)

- The basic assignment should not use any loops. You may use loops in your program only if you implement one of the optional enhancements (see below), e.g. to allow the user to solve multiple problems without restarting the program.

- Since you haven’t learned how to test things yet, you can just assume that all of the user input is good, i.e. that all coordinates provided are in the range of 0.0 to 1.0, etc.

What to Hand In:

- Your code, **including a user documentation file**, should be handed in electronically using Blackboard.

- The intended audience for the documentation file is a general end user, who might want to use this program to perform some work. They do not get to see the inner workings of the code, and have not read the homework assignment. You can assume, however, that they are familiar with the problem domain (e.g. the data to be interpolated.)

- A secondary purpose of the documentation file is to make it as easy as possible for the grader to understand your program. If there is anything special the grader should know about your program, be sure to document it in the documentation file. In particular, if you
do any of the optional enhancements, then you need to document what they are and anything special the TA needs to do to run your program and understand the results.

- If there are problems that you know your program cannot handle, it is best to document them as well, rather than have the TA wonder what is wrong with your program.
- Make sure that your name appears at the beginning of each of your files. Your program should also print this information when it runs.

**Optional Enhancements:**

It is course policy that students may go above and beyond what is called for in the base assignment if they wish. These optional enhancements will not raise any student’s score above 100 for any given assignment, but they may make up for points lost due to other reasons.

- Check the data entered to verify that it is valid.
- Ask the user if they would like to solve additional problems, and if so, repeat until they indicate they are done.
- Other enhancements that you think of – Check with TA for acceptability.

**Future Work:**

For future assignments we will be using the concept of barycentric coordinates and triangulation to interpolate and plot weather conditions based on data from NOAA weather buoys, as shown in the following map, most likely limited to the southern half of Lake Michigan, e.g. stations SGNW3 to MKGM4 and below.