

CS 109 – C/C ++ Programming for Engineers w. MatLab– Summer 2012

Homework Assignment 3

Data Interpolation Using Barycentric Coordinates from Files

Due: Wednesday July 4th by 8:00 a.m., via Blackboard. Optional hard copy may be turned in during lab on Thursday.

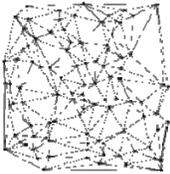
Overall Assignment

For this assignment you are to write a computer program that interpolates wind speed and temperatures based on collected data read in from files, using barycentric coordinates.

Background: Triangulation (from HW1)

Engineers and scientists are often faced with the problem of interpolating function values over an area, based on measurements taken at irregularly spaced locations. For example, weather data is measured principally at airports, which are distributed somewhat randomly about the country, and oil well test sites are also irregularly spaced. Modern computer graphics is also based on interpolating color values between the vertices of triangle meshes.

One approach to this problem is to first *triangulate* the space based on the known data points, which basically means to subdivide the space into triangles as shown in the following diagram:



Delaunay triangulation of a random set of 100 points in a plane. (Source: http://en.wikipedia.org/wiki/Delaunay_triangulation)

There are many different algorithms for triangulating a space, all of which are beyond the scope of this assignment.

Background: Barycentric Coordinates (from HW1)

Once a triangulation has been determined, and the particular triangle that contains the point of interest has been identified, then the next step is to interpolate the function values from the vertices of the triangle to the point of interest. This is where barycentric coordinates come in.

Given a triangle defined by vertices r_1 , r_2 , and r_3 , then any point r interior to the triangle can be defined by a weighted sum of the three vertices, as given by the equations:

$$r = \lambda_1 r_1 + \lambda_2 r_2 + \lambda_3 r_3$$

where λ_1 , λ_2 , and λ_3 are the barycentric coordinates, subject to the constraints:

$$\lambda_1 + \lambda_2 + \lambda_3 = 1.0$$

$$0.0 \leq \lambda_i \leq 1.0 \quad \forall i \text{ in } 1, 2, 3 \text{ for points interior to the triangle.}$$

A function f whose values are known at the vertices can be interpolated as:+

$$f(r) = \lambda_1 f(r_1) + \lambda_2 f(r_2) + \lambda_3 f(r_3)$$

Calculation of Barycentric Coordinates (from HW2)

Given the Cartesian coordinates of a point of interest, (x , y), and the Cartesian coordinates of the three vertices of a triangle, ((x_1 , y_1), (x_2 , y_2), and (x_3 , y_3)), the barycentric coordinates of the point can be calculated as¹:

$$\lambda_1 = \frac{(y_2 - y_3)(x - x_3) + (x_3 - x_2)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}$$

$$\lambda_2 = \frac{(y_3 - y_1)(x - x_3) + (x_1 - x_3)(y - y_3)}{(y_2 - y_3)(x_1 - x_3) + (x_3 - x_2)(y_1 - y_3)}$$

$$\lambda_3 = 1 - \lambda_1 - \lambda_2$$

Classification of Points Based on Barycentric Coordinates (from HW2)

Given the Barycentric coordinates of a point of interest the point may be classified as follows:

Conditions	Classification
$0.0 < \lambda_i < 1.0 \forall i \text{ in } 1, 2, 3$	Interior. (Note strict less-than operators)
One $\lambda = 0.0$, $0.0 < \lambda_i < 1.0$ for the other 2 λ s	Edge.
One $\lambda = 1.0$, The other two λ s = 0.0	Vertex
At least one $\lambda < 0.0$ or > 1.0	Exterior

Note: It may be more efficient to check for the classifications in an order other than the one presented in the table above.

Data Files

For this assignment you will be reading data in from two different data files, having the following file formats:

Weather Data File (e.g. NOAA_Data.txt) - This file has four columns of numbers, consisting of x-Coordinates (Longitudes) in the first column, y-Coordinates (Latitudes) in the second column, wind speed (mph) in the third column, and temperature (deg F) in the fourth column.

Problem Data File (e.g. problemSet1.txt or problemSet2.txt) - These files contain triangulation information and a set of coordinates for which to interpolate values:

- The first line of the file contains a single integer, indicating how many triangles are listed in the file.
- The next lines contain 3 integers per line, indicating which vertices make up particular triangles. Note that these indices are zero-based, so the number 0 refers to the first vertex (in the weather data file), the number 5 refers to the 6th vertex, etc. There should be exactly as many of these lines as the value of the first integer in the file.

¹ Source: [http://en.wikipedia.org/wiki/Barycentric_coordinates_\(mathematics\)](http://en.wikipedia.org/wiki/Barycentric_coordinates_(mathematics))

- After that are a series of lines containing two floating point numbers on each line. These are the X and Y coordinates for which you are to interpolate function values (see below.) There could be any number of these lines.

Program Details

For this assignment you are to write a computer program that interpolates wind speed and temperature data based on information read in from files and using barycentric coordinates. Your program should also classify each point as interior, exterior, edge, or vertex. Your program should first print out your name and ACCC account name (e.g. astudent), and explain to the user what the program does.

- Your program should then ask the user to enter three filenames: One for the weather data, one for the problem data, and one for the output results. You should verify that you can open each file properly and possibly ask the user to enter a new filename if the file failed to open before asking for the next filename.
- Your program should then read in the number of triangles in the problem set file, tell the user how many triangles are there, and then ask which triangle they would like to use. (For the data sets provided, the recommended answer is 6, for both problem sets.) You should verify that their answer is in range.
- Next read in all of the triangle vertex indices from the problem set data file. For this assignment you will ignore all the values except for the triangle that the user has selected. (But you still have to read them all in, to get them out of your way.)
- Now that you know which vertices you need, read in all of the vertex coordinates, wind speeds, and temperatures from the weather data file. You only need to save the data for the three vertices of the triangle you are working with, (which may not be in order.) As you read each line, count how many vertices are in the data file, and report this number.
- Finally read in the x- and y-coordinates at the end of the problem set file. For each set, calculate and report the barycentric coordinates based on the three vertices you read in from the file, report whether the point is interior, exterior, edge, or vertex, and interpolate and report the wind speed (in mph) and temperature (deg F) from the values at the vertices of the triangle.
- In the output results file, write out the x- and y-coordinates of each point calculated, along with the interpolated wind speed and temperatures. The file should end up having the same basic format as the weather data file, with four numbers per line and as many lines as you solve problems for.

Special Notes:

- **You should work out some sample problems by hand before writing any computer code.** For this problem it is probably a good idea to plot out the triangle vertices on graph paper, so you have some idea as to whether the coordinates should be interior, exterior, edges, or vertices. (The map and excel spreadsheet provided may be useful in this regard.) For each result your program calculates you should look at the function values at the vertices and your reported results, and ask yourself if the result seems reasonable. (For interior points the interpolated value should lie somewhere between the minimum and maximum vertex function values. That is not necessarily true for exterior points.)

Evolutionary Development

When dealing with large complex programs, it is generally best to work one step at a time, getting part of the program working and tested before attempting the next part. For this assignment it is probably appropriate to follow the bulleted list given above under "Program Details". First write a program that asks the user for file names and verifies that it can open the files. When that is working, modify the program to ask the user for a triangle to use and use that data, etc.

What to Hand In:

- Your code, **including a user documentation file**, should be handed in electronically using Blackboard.
- The intended audience for the documentation file is a general end user, who might want to use this program to perform some work. They do not get to see the inner workings of the code, and have not read the homework assignment. You can assume, however, that they are familiar with the problem domain (e.g. the data to be interpolated.)
- A secondary purpose of the documentation file is to make it as easy as possible for the grader to understand your program. If there is anything special the grader should know about your program, be sure to document it in the documentation file. In particular, if you do any of the optional enhancements, then you need to document what they are and anything special the TA needs to do to run your program and understand the results.
- If there are problems that you know your program cannot handle, it is best to document them as well, rather than have the TA wonder what is wrong with your program.
- Make sure that your name appears at the beginning of each of your files. Your program should also print this information when it runs.

Optional Enhancements:

It is course policy that students may go above and beyond what is called for in the base assignment if they wish. These optional enhancements will not raise any student's score above 100 for any given assignment, but they may make up for points lost due to other reasons.

- As you read the weather data, determine and report the maximum and minimum x- and y-coordinates, and wind speed and temperature.
- The data points listed at the end of the two problem sets are identical - Only the triangulation at the top is different. How do the reported results compare between the two problem sets. Is there any way to say which results are "better" for a given data point?
- For any data points that are external to the triangle chosen (e.g. triangle number 6), try to find a different triangle for which the point is internal. How do the interpolated results compare? (The "correct" triangle will probably be different for the two different problem sets. It was only coincidence that triangle 6 was a good choice for most of the given data points for both problem sets.
- Other enhancements that you think of – Check with TA for acceptability.

Future Work:

Once we learn about functions, we will be breaking the work in this assignment up into separate functions – One to calculate the barycentric coordinates and one to classify points at a minimum. Once we learn about files and arrays, we will start working with a network of triangles, with the triangle vertices and the list of which vertices make up each triangle read in

from data files. We can then use the classification function to identify which of our triangles contains the point of interest, and then interpolate function values based on the vertex values and barycentric coordinates, as in HW1.

For future assignments we will be using the concept of barycentric coordinates and triangulation to interpolate and plot weather conditions based on data from NOAA weather buoys, as shown in the following map, most likely limited to the southern half of Lake Michigan, e.g. stations SGNW3 to MKGM4 and below.

