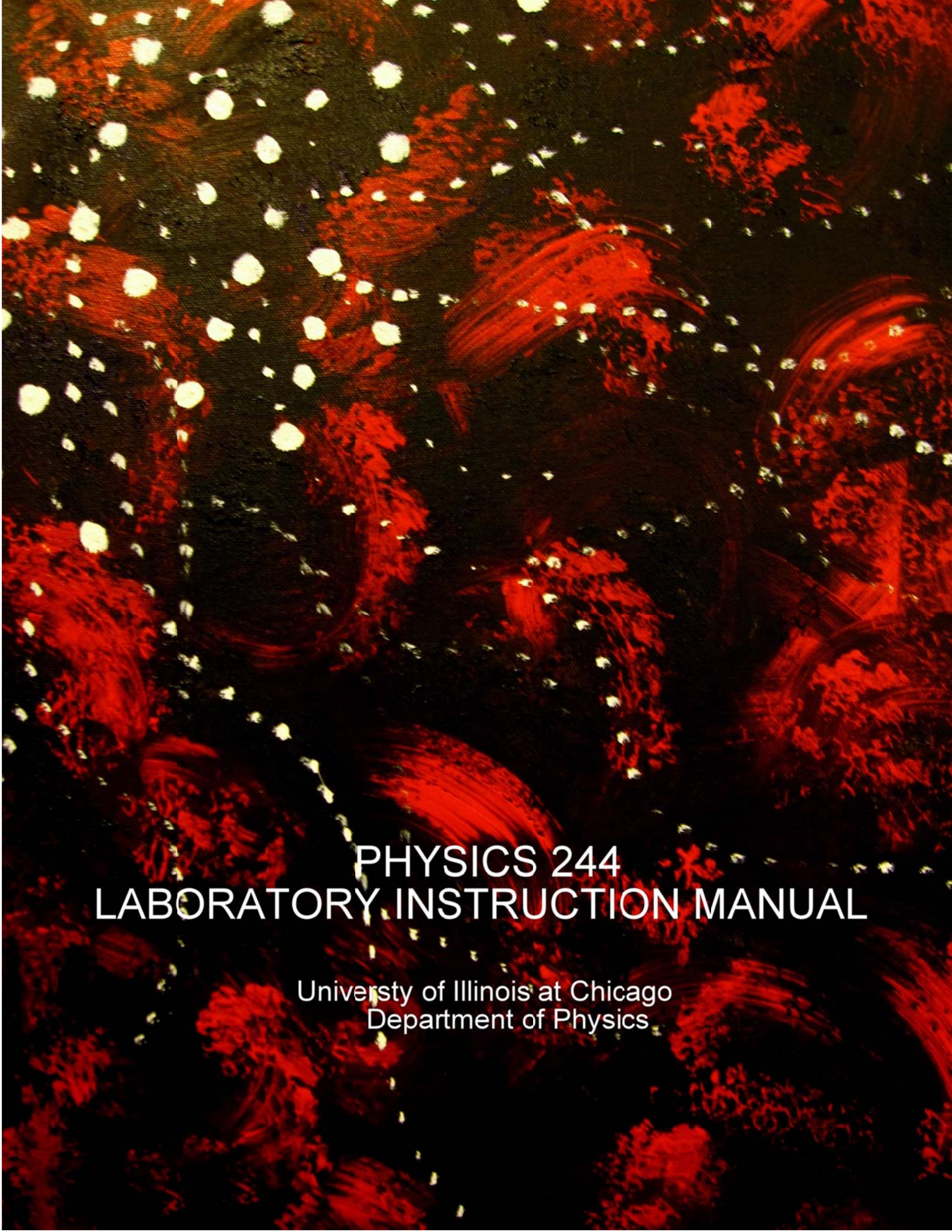


PHYSICS 244
LABORATORY INSTRUCTION MANUAL

Universty of Illinois at Chicago
Department of Physics

Revision 2010

S. Patel
Typed by: A. Laheri

Front Cover Art

I. Bucinskaite

TABLE OF CONTENTS

Introduction

Our understanding of nature around us crucially depends on measurements done in the laboratory, using "sound" laboratory techniques. The number emerging out of carefully planned experiments have the power of making a theory or breaking it! The truth for science in general and physics in particular is what we observe in the laboratory. The range of our so-called "common sense" is, unfortunately, very limited to our day to day experiences. In the real world, we have to deal with objects and phenomena, which are not at all a part of this narrow range. For example, we do not physically see particles like electrons, atoms, nuclei, etc. However we are in the middle of these particles all the time, experiencing and often using their properties with advantage in our daily lives.

Physics 244 course is an introduction to Modern Physics. Here is your opportunity to get involved in the laboratory work which brings you face to face with the microscopic world of electrons, atoms, molecules, nuclei and so on. This is indeed a fascinating world that is not part of our every day experiences. For example, in the very first experiment you would be doing in the laboratory, the study of the photoelectric effect, you will come across an exciting fact; light has a dual character. It behaves both like "particles" and "waves"! You cannot understand the measurements in the experiment unless it is assumed that light propagates as a stream of particles called photons. This is in sharp contrast with your earlier experience of studying phenomena of interference and diffraction of light. This study had convinced you that light propagates as "waves"; it is a wave phenomenon!

All the experiments described in this manual deal with making you familiar with this exciting atomic and sub-atomic world. Doing these experiments will enable you to have fun and a firm grip on the modern and often abstract concepts of basic importance. We hope that you enjoy performing these experiments and feel the excitement of learning "Modern Physics"!

The laboratory associated with this course is designed to be an integral part of the instruction. Its primary purpose is to help in comprehending more fully the concepts learned in the course. In addition, it serves to develop sound laboratory techniques and effective communication skills. To achieve these goals efficiently it is important to follow certain requirements and procedures.

GENERAL REQUIREMENTS

- **Attendance:** All labs must be completed for the student to receive credit for the course. Labs which are missed because of a legitimate excuse can be made up by arrangement with the lab instructor. The data sheet for each student must be signed by the instructor of the section in which the experiment is performed.
- **Individual work:** Each student is required to analyze his own data, plot his own graphs, make his own computations, etc.
- **Cleanliness:** If equipment is missing or found to be defective, report this to the laboratory instructor; never borrow from another station without informing the instructor. Before leaving the laboratory, make sure your station has been restored to good order.
- **Safety:** The student is expected to handle the laboratory experiment in such a manner as to avoid creating any additional hazards to himself, to other persons, or to the equipment. If a student notices any hazardous conditions in the laboratory, he should report immediately to the laboratory instructor. Electrical equipment should not be connected to the power source until the wiring has been approved by the laboratory instructor.

LABORATORY PREPARATION

A scientific experiment requires extensive study and deliberate planning. In order for the student to succeed in this lab course, he or she must prepare in two specific ways. First, the student must understand the basic physical concepts which the lab experiment is designed to investigate. Second, the student must have a grasp of the procedures which will be performed in the investigation. Laboratory preparation will account for a significant portion of the lab grade.

- **Lab materials:** The student is expected to bring a calculator, pencil and pen, and paper suitable for making data tables, doing preliminary sketches and carrying out initial calculations.
- **Prelab Questions:** All students are expected to study the experiment and the pertinent parts of the text before coming to the laboratory. Questions have been included in each lab which must be answered prior to the lab. These questions are designed to make the student think about the basic objectives of each lab, and the reason for the procedures used to obtain those objectives.
- **Data Sheets:** Students should also make advance plans for recording data, (preparing appropriate tabular forms when feasible), for plotting graphs, and for performing other tasks indicated in the instruction sheets for the experiment. The student should have a clear idea of what is going to be measured, and how.
- **Lab Quiz:** The lab TA will give a short 5-10 minute quiz at the beginning of each lab session to test the student's preparation for the lab.

DATA ANALYSIS: AN OVERVIEW

There are two kinds of data which the student will be required to obtain in this laboratory. The first kind is qualitative data. These are word or pictures descriptions of what the student saw in the course of carrying out the experiment. For instance, the student might be asked to compare the intensity of two spectral lines emitted from a hydrogen lamp. The student must simply record a verbal description of the intensities in his lab report. This kind of data is important in characterizing the behavior of the physical system one is studying. It is often used as preliminary to quantities study, since one can quickly identify the trends without being overly concerned with accuracy or precision.

Often, quantitative data are obtained for the sake of deducing a functional relationship between an important physical parameter such as time, distance, angle etc., and the physical behavior of interest, such as intensity, force, number of events etc. The data to be obtained will be numbers with the appropriate units, such as 3 mm, 10 N, 25 sec, etc. They should be recorded in tabular form, with the physical parameter being varied recorded first, and then the values of the quantities being measured. In experimental work, the functional relationship is deduced from the data by first plotting the data points, with the independents variable (the physical parameter) associated with the scale of the horizontal or x-axis. Then a smooth curve is drawn interpolating the values in-between the data points. The smooth curve represents the experimentally determined functional relationship between the physical parameter and the physical phenomenon being studied.

When the curve can be approximately described by a straight line, then the functional relationship is linear, and can by written y = mx + b, where x is the independent variable, y is the dependent variable, b is the value of y when x = 0 and m is the slope given by $\mathrm{m} = \frac{y_1 - y_0}{x_1 - x_0}$. In physics, the student must become accustomed to associating the independent variable with the "x-axis", even if the variable is called "y" in that experiment. The relationship between the two variables may not be linear. Often, however, one can still obtain a line graph by plotting functions of the (in)dependent variables, such as x^2 or $\mathrm{Log}_{10}(y)$. The lab manual will indicate when such an approach is necessary.

RECORDING DATA

In order to prepare to do the experiment, the student is expected to draw up tables in which he or she will record the data. This requires that the student anticipate what kind of behavior will be observed, i.e. will the measured quantity increase or decrease with respect to the independent variable, and at what rate. In addition, the student must consider what will be the most important aspect of the observed behavior with regard to the stated objective of the lab. In various labs the student may need to look for maxima in the measured quantity, or for the value of the physical parameter at which the measured quantity becomes zero, etc. When this is the objective, then more data points need to be taken near the maxima, or at the cut-off, and the data sheet and table should be adjusted accordingly. All tables and qualitative observations must be inspected and signed by the lab TA before the student leaves the laboratory. All original work must be done in ink. The procedure for making up a table is the following:

1. Make a title for the table describing the measurement.
2. List the conditions or the relevant quantities which remain constant during the data taking.
3. In a row across the page list first the independent variables, then the measured quantities, and later the derived quantities. After each quantity or on the line below give the dimension of the quantity.
4. The precision (error) of each measurement must be recorded. A detailed discussion of precision is given in appendix A. The precision is recorded in the form

   $$X + \Delta.$$

   This means that the quantity X is known to within Δ of the recorded value. In some, but not all, measurements the precision is the same for all measurements of that quantity. In that case the precision is entered above the dimensions in the table heading. The following is an example of a good data table.

Table of Position, Displacement and Average Velocity Versus Time			
Time t +0.01 (sec)	Position X +0.2 (cm)	Displacement $\Delta x = x(i) - x(i-1)$ + 0.4 (cm)	Average Velocity $V = \Delta x/\Delta t$ +0.4 (cm/sec)
0.00	0.00		
0.1	1.5	1.5	15.0
0.2	3.1	1.6	16.0
0.3	4.5		
0.4	6.0		
0.5			

PLOTTING DATA

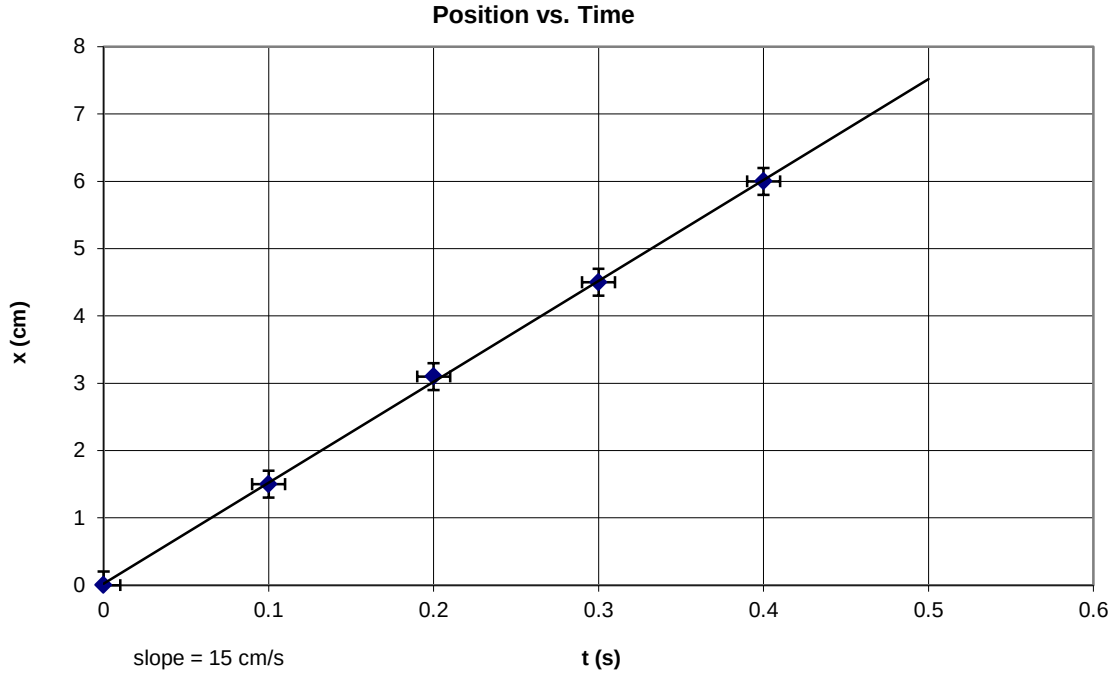
A graph is arguably the most important tool for the visual representation of data and for data analysis. The student is expected to take care in producing a graph that performs both functions well.

A graph must convey all the information necessary for a reader to understand the data at a glance. In this sense, the graph is not for the experimenter's benefit, but for those who would understand what the experimenter has done, such as your TA, or an employer. Graph paper must be used. The graph must be given a title. Both axes must be clearly labeled, and the scale of each axis chosen so that the plotted points extend across the whole graph. The data points must be clearly marked, and the precision of the data indicated with error bars. The curve used to interpolate between data points must be smooth. Do not connect points with lines since this does not add information. If the curve is a line, the calculation of the slope must be shown on the paper. In practice, plotted data are often used for the sake of comparison with the theoretical curve. In that case, the data points should not be connected by a curve. However, in our experiments, this situation will not arise.

A graph is also a visual representation of the functional relationship between the independent and dependent variables. It is therefore an analytical tool that helps the experimenter to understand the experimental results. It should be possible to use a graph in order to predict the value of the dependent variable at values of the physical parameter for which the experiment was not explicitly carried out. It is often possible to use the curve in order to determine a symbolic expression for the function. Therefore, the graph must be plotted and drawn carefully, so that the information obtained from it is as accurate as possible.

It is recommended that the student become familiar with and use one of the many graphing programs for computers now available. However, the student, not the graphing program, is responsible for ensuring that the graph meets all the above criteria.

The following is a plot of x vs. t, using the data from the table above.

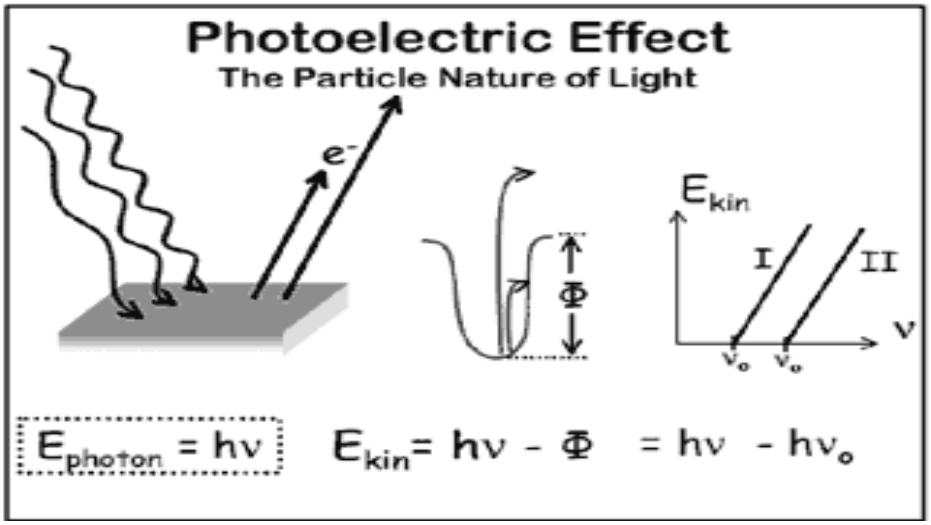


REPORT FORMAT

The student is expected to prepare a report of the experiment to be turned in one week after the experiment is performed. The laboratory report should be about 3 or 4 pages in length, excluding the data and graphs. The completed report should include all original data, signed by the TA. The following format is required:

1. **Cover Page**: This page contains the title of the experiment, the student's name, the laboratory partner's name, the laboratory station number, and the data.
2. **Title of Experiment and Objective**: The first page of the report should have the experiment title written across the top. The first section, objective, should describe in the student's own words the principal purposes that the experiment is designed to achieve.
3. **Introduction**: First, the student should explain why the experimental objective is important in understanding modern physics. Second, the student should explain how the experimental procedures allow the objective to be reached. Clearly labeled diagrams are often useful to supplement the written descriptions. Relevant material, including theory and equations when applicable, may be obtained from the laboratory experiment description, the text, or other references.
4. **Results**: This section contains the following subsections:
   - Data: containing the table of values obtained in the experiment
   - Calculations: containing at least one sample calculation of each of the different types of computation required, including the calculation of uncertainties (See Appendix A)
   - Results: Containing a tabulation of computed quantities and graphs. A few words of explanation should be included with each subsection.
5. **Conclusion**: This final section summarizes the results, including an evaluation of whether the objectives have been realized within the uncertainties of the data. If they have not, a further discussion of probable sources of systematic error needs to be included. This section should include the answers to any questions posed in the laboratory sheets.

THE PHOTOELECTRIC EFFECT



"It seems to me that the observations associated with … the production of cathode rays by ultraviolet light…are more readily understood if one assumes that the energy of light is discontinuously distributed in space."
~A. Einstein (1905)

You have earlier studied phenomena of Interference of light e.g. Young's double slit experiment. To understand the formation of "dark and bright bands" in such an experiment, as you know, one has to assume wave propagation of light. However, a study of photoelectric effect reveals that wave theory of light is of little help here and one has to consider the particle nature of light. Based on the experimental observations made of the photoelectric effect, Einstein argued that light energy must be quantized in small packets of energy proportional to the frequency of the light. The constant of proportionality is h, Planck's constant. These packets of light energy, or light particles, came to be known as photons.

In the photoelectric effect, light energy incident on a target material is acquired by individual electrons near the material's surface, permitting some of them to escape. The small size of the target electrons means that they usually interact with only a single photon at a time and the quantization of light energy can be observed by examining the amount of kinetic energy the escaping electrons possess. The experiment you will perform involves measuring the maximum amount of kinetic energy possessed by electrons leaving a metal surface as a function of the frequency of the light shining on it. Using Einstein's theoretical equation for the photoelectric effect, you will then be able to calculate the value of the Planck's constant, h, the "size" of an individual packet of light energy.

The experiment will consist of several parts. First, you will observe the line spectrum produced by a mercury vapor lamp. Second, you will, by means of colored filters, choose an approximately a wavelength which will be shining on the phototube constructed from cesium antimonide, CsSb. You will then observe the photoelectric effect (current) as function of frequency by means of an electric circuit described in that section. Based on your results from this part, you will calculate h, as well as the work function ϕ and the threshold frequency ν_0 for CsSb.

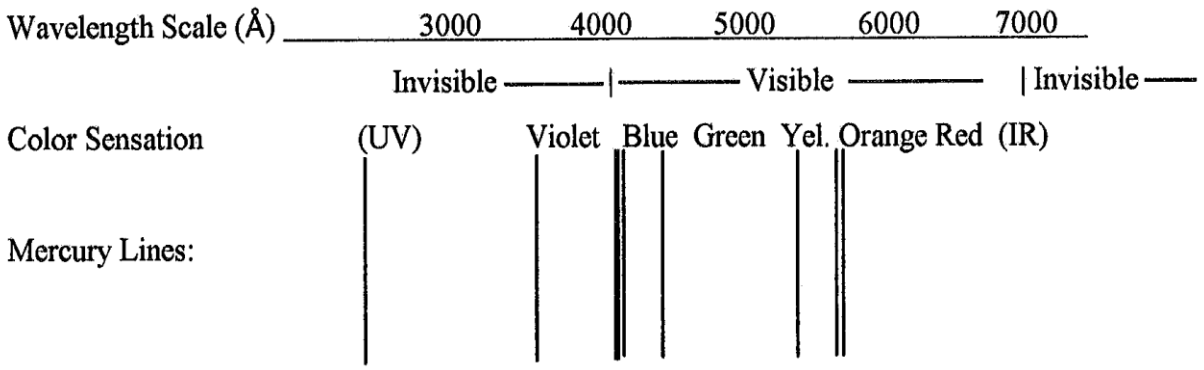
I. THE MERCURY SPECTRUM

Each station has been provided with a mercury lamp, a slotted metal mask, three filters – orange, yellow and blue – and a diffraction grating. In principle, the device acts similarly to the double slit (studied in Physics 142), but allows the transmitted light to be of far greater intensity.

Before turning on the mercury lamp, have the slotted mask (slit vertical) in place in the filter holder of the lamp. NEVER LOOK DIRECTLY INTO THE LAMP. It produced UV light which can be damaging to the eye. Now turn on the lamp and examine the light through the slide with the grating. The best method is to place the grating in the filter holder of the lamp, and then hold a piece of paper six to twelve inches away from the lamp, until the diffraction pattern from the light passing through the grating becomes distinct. Because of the room illumination and the other lamps, there will be some spurious images, but if the grating is correctly oriented (grooves vertical) you should be able to see, in addition to the zeroth-order composite mercury light at center, the first and perhaps higher orders of some of the lines in the visible region of the mercury spectrum on both sides of center. The sensitivity of the eye is greatest at about 5500 Å (green) and falls off to zero at about 4000 Å (far violet) and 7000 Å (far red). This sensitivity curve is shown on Page 5. You will see at least the following three strong lines:

Yellow	5770-5790 Å (a doublet)
Green	5461 Å
Blue	4359 Å

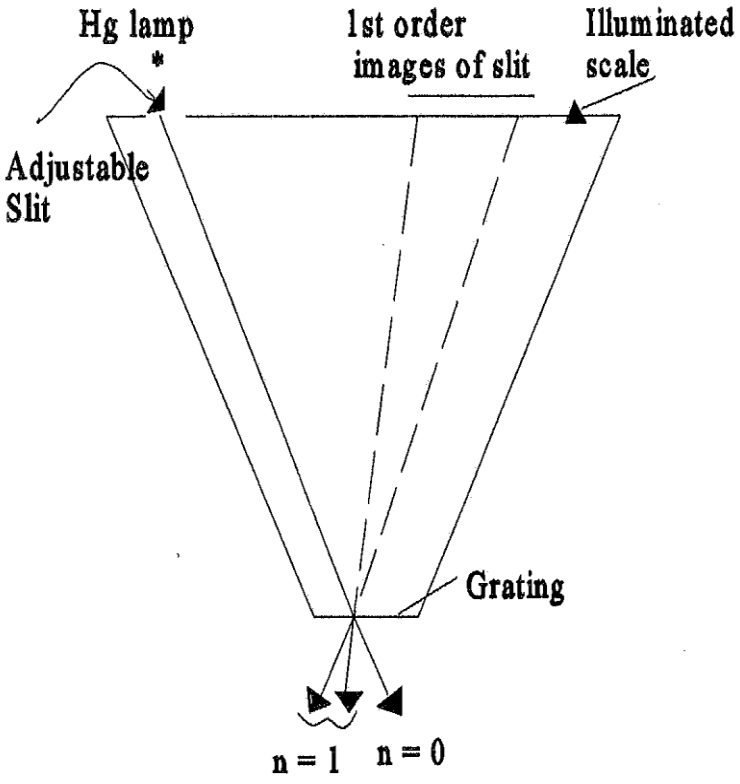
The spectrum of the sufficiently intense lines of Hg in the UV and visible regions is given in the chart below. There are many other lines in the spectrum, but they are too weak to be observed except by photographic methods. Study this chart very carefully, and make sure you understand every aspect of it. Because of the fact that the envelope of the lamp is of quartz, a material which is transparent to wavelengths as short as 1800 Å, the UV wavelengths of mercury are present in the beam from the lamp.



Wavelength (Å)	2536	3650	4047/78	4359	5461	5770/90
Frequency (10^{14} Hz	11.8	8.21	7.41	6.88	5.49	5.19
Energy (eV) per photon	4.89	3.40	3.06	2.84	2.27	2.14
Relative Intensity	Very Strong	Strong	Strong	Strong	Strong	Moderate

Note that the line intensity depends not only on the energy per photon but also on the relative number of photons undergoing the particular electronic transition in the atom as compared with the numbers in other transitions.

At some time during the period, go to the instrument; look at the composite Hg light in the figure and the 1st - order spectral lines at the right, verify roughly the wavelengths in the chart of those you are able to see and their color, and record your results in a table such as that given below. The mercury source must be positioned directly behind the slit in order to assure the strongest slit images. Record the relative apparent intensities (strong, moderate, absent, etc.), then taking into account the relative eye sensitivities and relative actual intensities given in the table, decide whether the results for apparent intensities are reasonable. It is not necessary to have the preceding experimental procedure completed in order to proceed with the other parts of the experiment which follow.

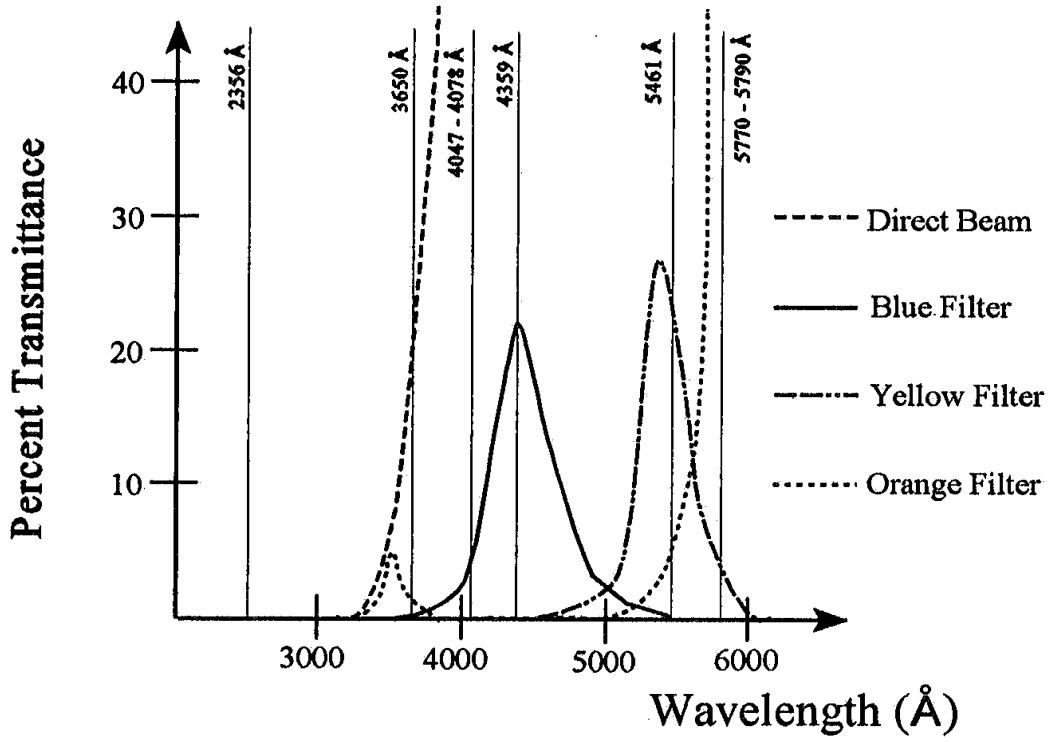


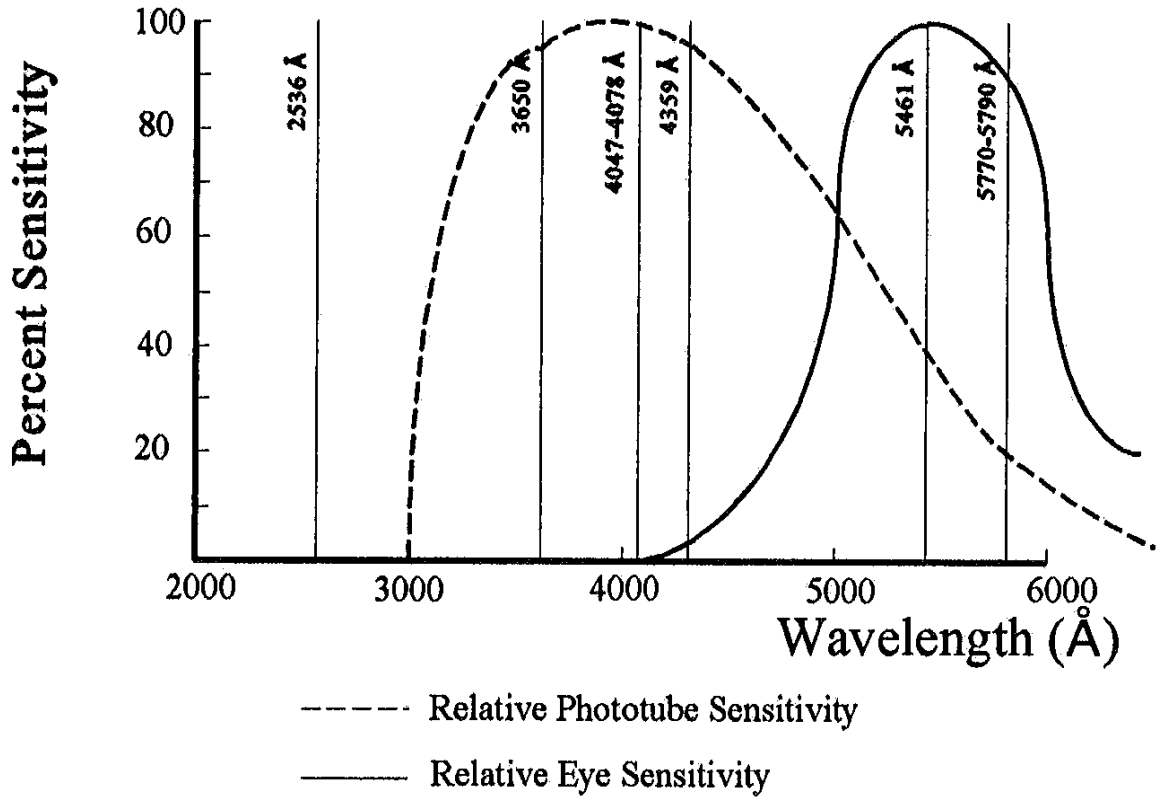
Intensity of spectral lines from Mercury vapor lamp				
λ (Å)	Color	Relative Apparent Intensity	Relativity Eye Sensitivity	Relative Actual Intensity
5770-5790			0.9	Moderate
5461			1.0	Strong
4359			0.03	Strong
4047-4078			~ 0	Strong
3650			0	Strong
2536			0	Very Strong

II. THE EFFECT OF FILTERING

The color of the filter, which is simply the composite color of the range of wavelengths remaining in the transmitted beam when white light (all visible wavelengths) is incident on the filter, gives you an idea of what region of the spectrum is transmitted by the filter (and what absorbed by it). You therefore have a rough idea of which Hg spectral lines might be passed by a given filter. The guesswork is reduced by observing the filtered beam through a diffraction grating. Note that filters usually pass more than one line. For example, note that the blue filter passes the 4359 Å blue line best but may also pass the 4047-4078 Å doublet to a certain extent. While observing the spectrum, hold each filter in turn in the beam from the lamp, and note the effect on the lines. Record your results in the table. You cannot observe the 3650 Å UV line with the naked eye.

Effect of Filters				
Hg Line		(-) Line Eliminated (√) Transmitted Poorly (√√) Transmitted Well		
λ (Å)	Color	Orange Filter	Green Filter	Blue Filter
5770-5790				
5461				
4359				
4047-4078				
3650				


Percent Transmittance
40
30
20
10
2356 Å
3650 Å
4047 - 4078 Å
4359 Å
5461 Å
5770 - 5790 Å
Direct Beam
Blue Filter
Yellow Filter
Orange Filter
3000
4000
5000
6000
Wavelength (Å)


Percent Sensitivity
100
80
60
40
20
2536 Å
3650 Å
4047-4078 Å
4359 Å
5461 Å
5770-5790 Å
2000
3000
4000
5000
6000
Wavelength (Å)
Relative Phototube Sensitivity
Relative Eye Sensitivity

III. DETERMINATION OF THE PHOTOELECTRIC WORK FUNCTION OF CsSb AND PLANCK'S CONSTANT

It should be apparent from your results in Part II that it is possible by means of the filters to isolate reasonably well some of the mercury lines. We shall use these wavelengths in conjunction with a photoelectric cell, or phototube, to determine Planck's constant , the work function and threshold frequency of the material used in the cathode of the phototube.

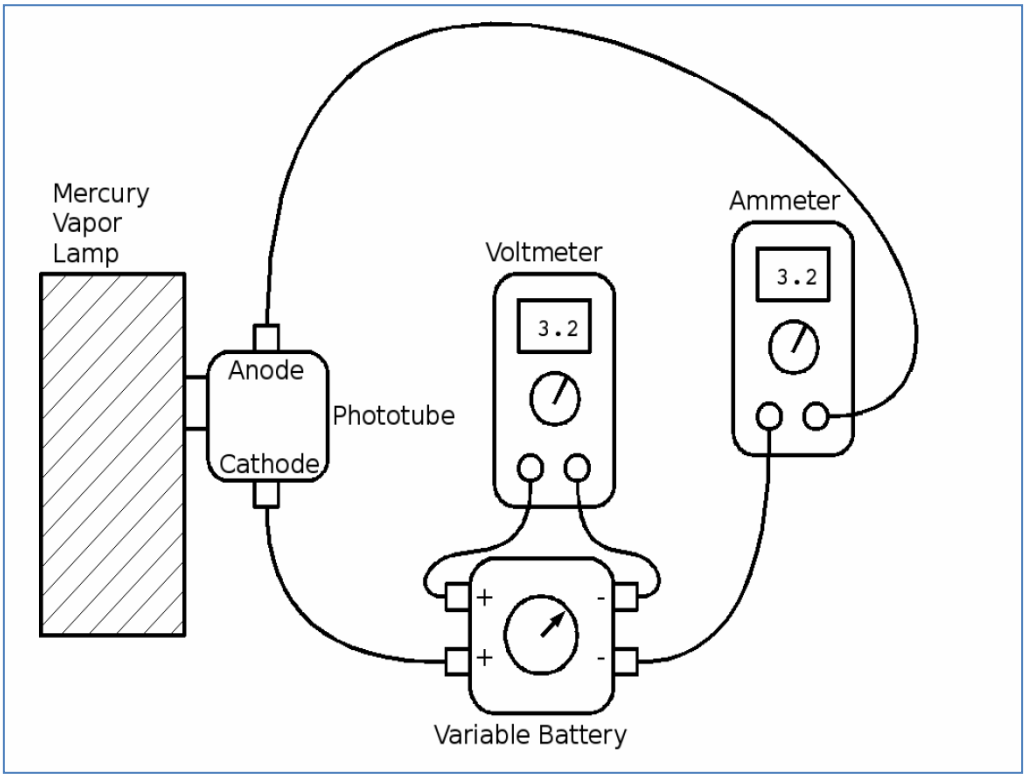


Diagram of Set up

The phototube has in it a semi-cylindrical photocathode of cesium antimonide, CsSb, a material with a low work function, plus a central wire on the cylindrical axis as an anode. Just as the eye varies in its response to different wavelengths, the phototube does also. The glass envelope passes radiation down to 3000 Å. This means that the strong 2536 Å UV line from the mercury lamp is blocked but that the 3650 Å UV line and above can produce effects. The complete sensitivity curve is found on page 5.

The idea behind the experiment is straightforward. You will shine the filtered light from the mercury vapor lamp on the cathode of the phototube. According to Einstein's theory, the electrons near the surface of the cathode absorb single photons from the light, and gain all the energy carried by the photons. If the energy absorbed is sufficient, the electron escapes the material, overcoming the barrier, called the work function ϕ of the material. The emitted "photo-electrons" escape with a kinetic energy, E_k. Therefore, when light of sufficiently high frequency shines on the cathode, a current is generated as the electrons ejected from the cathode flow to the anode, completing the circuit. Our circuit is shown in the photograph given at the beginning and schematically shown here.

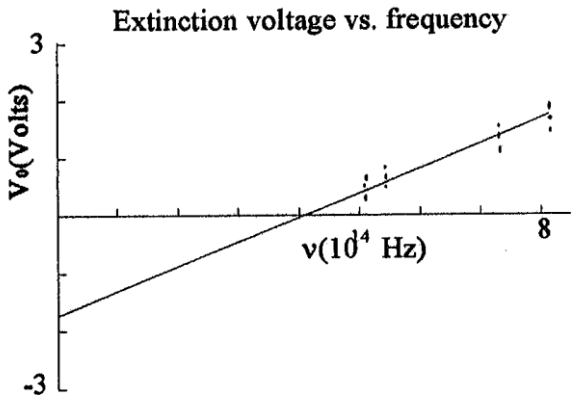
On one side of the metal shield covering the phototube is a double aperture through which the light may enter. The strip between apertures is intended to block that part of the beam which would otherwise fall directly on the anode, sometimes complicating matters by causing photoemission from anode as well as cathode.

By controlling the voltage difference between the cathode and the anode, we are able to control the potential energy barrier, eV, faced by each electron as it crosses over to the anode. For a given frequency of light, there will be a certain voltage, called the stopping potential V_0 at which the energy barrier is equal to the maximum kinetic energy of the ejected electrons. The electrons are stopped and the current in the circuit goes to zero at V_0. By recording the extinction voltage required for a given frequency of light, we can calculate the amount of energy carried by a photon of that frequency and the work function of the material. Remember that the photo-electric current will be zero, not only when V_0 is reached but also when the voltage is made more negative than V_0. Therefore you are recommended to play with the voltage near V_0, so that the current oscillates between 0 and 0.01 A.

We can now go ahead with finding the stopping voltage associated with each of the filters (as well as for the direct beam). However, we must remember that the filters do not isolate perfectly the spectral lines from the lamp. You must use your data from parts I and II, as well as the Einstein theory of the photoelectric effect, in order to judge correctly which of the spectral lines passed by the filter is responsible for the photoemission from the cathode being stopped by the stopping voltage. Remove the slit from the lamp. Place the orange filter in the holder, turn on the lamp and place it snugly against the extender, perhaps covering with the rubbing cloth (for Part D) to help in shielding from the room lights. Start with zero source voltage and advance the voltage just to the point where the current is exactly zero. (If a further increase shows a negative current, this is probably an indication that there is some anode emission, despite our precautions). The best we can do is to make a note of this and consider it a source of error in our measurements. Practice a few times before recording voltages. Now bring the current to zero and read the stopping voltage V_0. Record V_0 in the given table. Repeat twice, and record all three readings. Repeat the procedure, in succession, for the orange, yellow, blue filters and the direct beam. If that order is followed, you should find that the extinction voltages obtained are in the order of lowest to highest.

Filter	Voltage V_0			Line Responsible	
	Trial 1	Trial 2	Trial 3	λ (Å)	ν(Hz)

Plot your experimental points on coordinates V_0 vs. ν, and draw the straight curve which best represents the points. Let the V_0 scale range from +3 to -3 volts. Plot all three trials for each filter. Extend the curve with a broken line through the negative V_0 axis at ν=0. Using a large slope triangle, measure and record on the curve its slope, in units of volts/hertz. Also record the V_0 intercept in volts, and the ν-intercept, in Hz.



Use these values in connection with the Einstein photoelectric equation

$$h\nu = \phi + E_{K\,max}$$

and the stopping-voltage relationship

$$eV_0 = E_{K\,max}$$

to determine both Planck's constant h and the cesium-antimonide work function ϕ and threshold frequency ν_0. <u>You must be careful to use the correct units.</u> Show all calculations (in algebraic form), as should always be done, and put your results in a table as follows:

	Experimental Value	Published Value
Planck's constant h(j*sec)		
Work function ϕ of CsSb (eV)		1.8 eV
Threshold Frequency ν_0 of CsSb (Hz)		
Threshold wavelength λ_0 ($=c/\nu_0$) of CsSb (A)		6800 A

PRELAB QUESTIONS:

Q1: What is a photon? How much energy does an electron receive when it absorbs a photon of frequency ν?

Q2: If the electron from Q.1 must use an energy equal to ϕ, the work function of the phototube cathode, in order to escape the cathode's surface, how much kinetic energy does it have once it escapes?

Q3: What is the energy required for an electron of charge –e to travel across a potential difference of –V?

Q4: If there are two frequencies of light, with $\nu_1 > \nu_2 > 0$, in a light beam shining on a phototube whose work function is ϕ_0, at what level of applied voltage across the phototube will all the electrons which received a photon of frequency ν_2 and ν_1 be stopped? Which of these voltages is greater?

Q5: Use the charts given on page 5. For the orange filter, which of the two spectral lines should determine the stopping voltage for the phototube? For the blue filter?

BRAGG SCATTERING

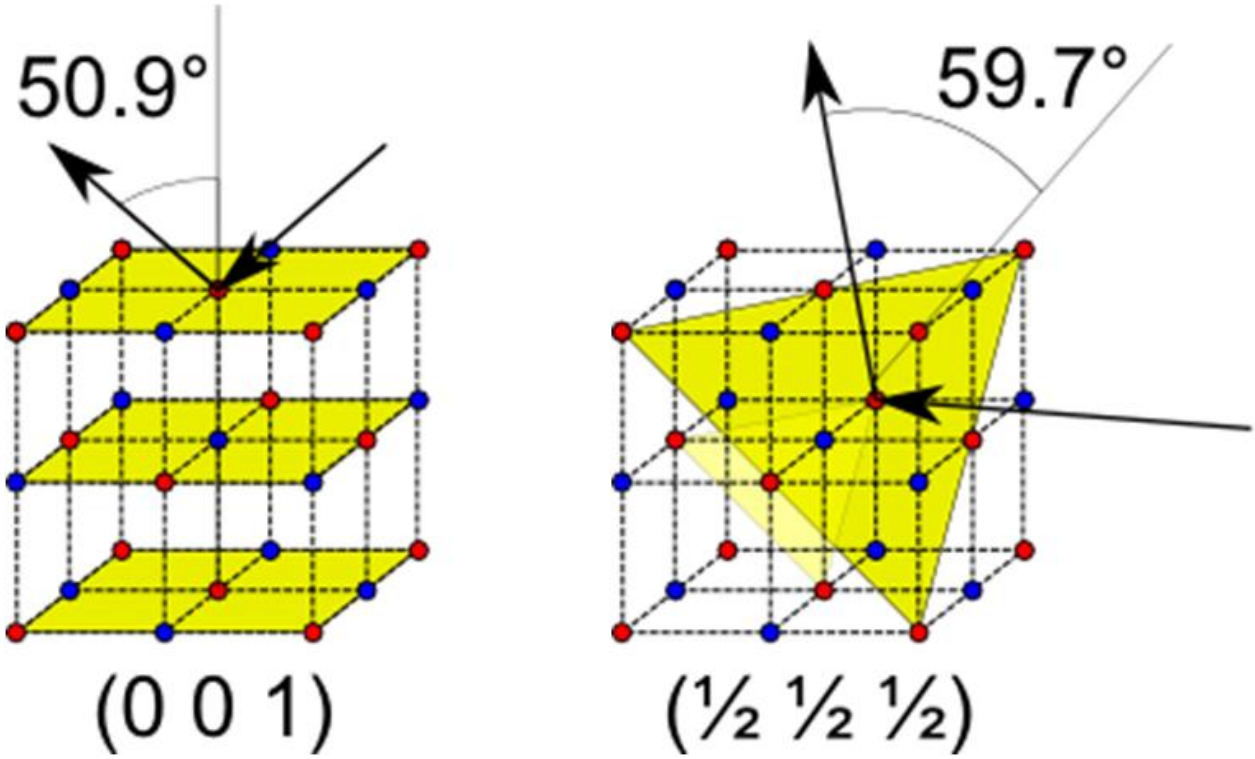


Diagram showing Crystal Planes

I. Introduction

When Wilhelm Röntgen discovered X-rays in 1895, he was not sure about their nature and hence, he called them "X-rays". The wave nature of some radiation can be determined by using a diffraction grating with the appropriate grating spacing. If the radiation has a wave nature, then interference will occur, and a diffraction pattern will result behind the grating. This was observed for the light from the mercury vapor lamp passing through the grating in Experiment #1 The Photoelectric Effect. If X-rays were made of waves, as was thought, however, they had to be waves of very short wavelength, because no diffraction pattern could be produced using manufactured gratings.

Max Von Laue had the idea to use the known, regular structure of atomic crystals as diffraction gratings for X-rays, since the spacing in such crystals was known to be very small, of the order of a few Angstroms. If one could shine X rays on a crystal whose structure was already known by some other means and produce a diffraction pattern, then one could demonstrate the wave nature of the radiation and calculate its wavelength. Von Laue successfully demonstrated that this idea was correct, and later W.L. Bragg and W.H. Bragg developed the use of crystals as diffraction gratings for X-rays. They designed an X-ray spectrometer from a crystal of known

structure in order to analyze the spectral content of X-rays and to use monochromatic X-rays for probing the structure of unknown crystals.

Unlike diffraction through diffraction gratings, X-ray diffraction in crystals involves interference between waves reflected off the planes of atoms in the crystal. Reflection off a single plane of atoms, shown in Figure 1.a, is similar to reflection of visible light off a smooth surface, like a mirror. Although each atom scatters the incoming radiation in a spherical pattern, when the scattered light off all the atoms in the plane is combined, cancellation due to interference constrains the reflected wave to leave the surface at the same angle at which the incident wave struck the surface.

In a regular atomic crystal, planes of atoms are layered one on top of the other, separated by a distance, d. Not all of the radiation is reflected by the upper plane, but much of it passes through to the next plane, where some percentage of that radiation is reflected back. The reflected waves from the adjacent planes will have a path difference, Δ, determined by the distance of separation between the planes d and the angle of incidence θ_i. This can be seen in Figure 1b.

From figure 1b, it can be seen that the path difference between the waves scattered from adjacent planes is given by the formula,

$$\Delta = 2d\sin\theta_i$$

Here θ is the *grazing angle* of incidence as shown in the figure.

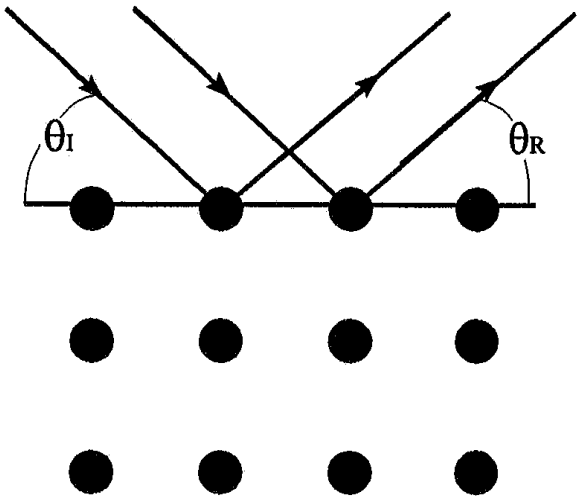


Figure 1a. Bragg scattering from centers in a single plane.

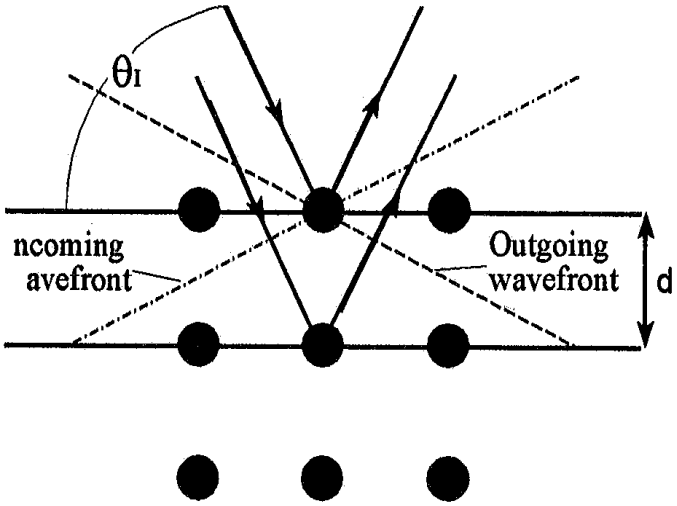


Figure 1b. Bragg scattering from centers in adjacent planes.

If the path difference is equal to an integral number of wavelengths, n = ±1, ±2, ±3, …, then the reflected radiation will add constructively, and a peak in the intensity of the reflected radiation will be observed. Thus, the Bragg formula for the maxima in the intensity of the light reflected off an atomic crystal is given by

$$\Delta = n\lambda = 2d\sin\theta_i$$

Radiation of a known wavelength can be used to investigate the crystalline structure of unknown materials. The purpose of this experiment is to introduce and familiarize the student with this important technique, and understand the principles behind it. In practice, however, the control of the X-ray source in order to produce monochromatic radiation (radiation having a single wavelength) and the preparation of the crystalline sample is not easy. Therefore, we will investigate Bragg scattering using a scale model of a crystal constructed out of metal spheres arranged in a cubic pattern in a Styrofoam block. Instead of X-rays, we will use microwaves of wavelength 3 cm. The physical mechanism, however, is the same as for the X-ray scattering in atoms.

The large scale enables us to measure the crystalline structure directly and to use electromagnetic radiation from a more controllable source. All the same, this is not an easy experiment to perform, especially because the microwave amplitude is very sensitive to the laboratory setup. However, by making careful measurements, you should be able to observe maxima in the scattered radiation at the angles predicted by the Bragg scattering formula for scattering off several planes in the "crystal".

The steps in the experiment are as follows: You will first measure the spacing between spheres in our model of the "crystal". Then you will calculate the spacing between planes for three different orientations of the "crystal". Then you will take measurements of the amplitude of the scattered signal for various angles of the incoming radiation relative to each of the scattering planes, and compare the location of each of the experimentally determined maxima with the theoretical values. Finally, you will use your experimentally determined values of θ_{max} in order to calculate the wavelength of the microwave radiation, and compare it with the given value of 3 cm.

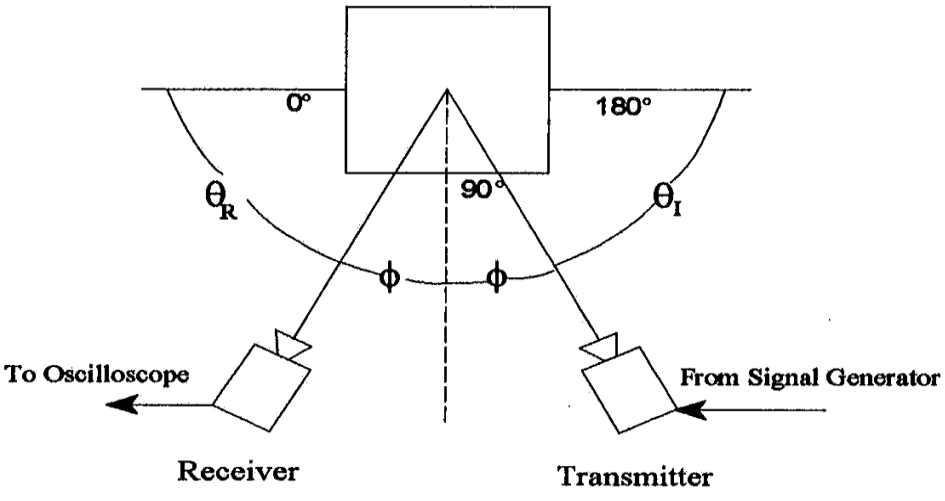


Figure 2. Experimental apparatus layout

Step by Step Instructions

Testing the function generator, oscilloscope, microwave transmitter, and microwave receiver

Step 1 – Test the function generator and oscilloscope

- Set function generator to the following settings:

o Function button should be set to sine wave (as opposed to square and triangle)
o Frequency range button set to 100 Hz (or 500 Hz if you have a blue and white BK Function Generator), Frequency dial set to around 600 Hz (on display)
o Dial "Duty Cycle" to zero.

- Set oscilloscope, either channel one or two, to:

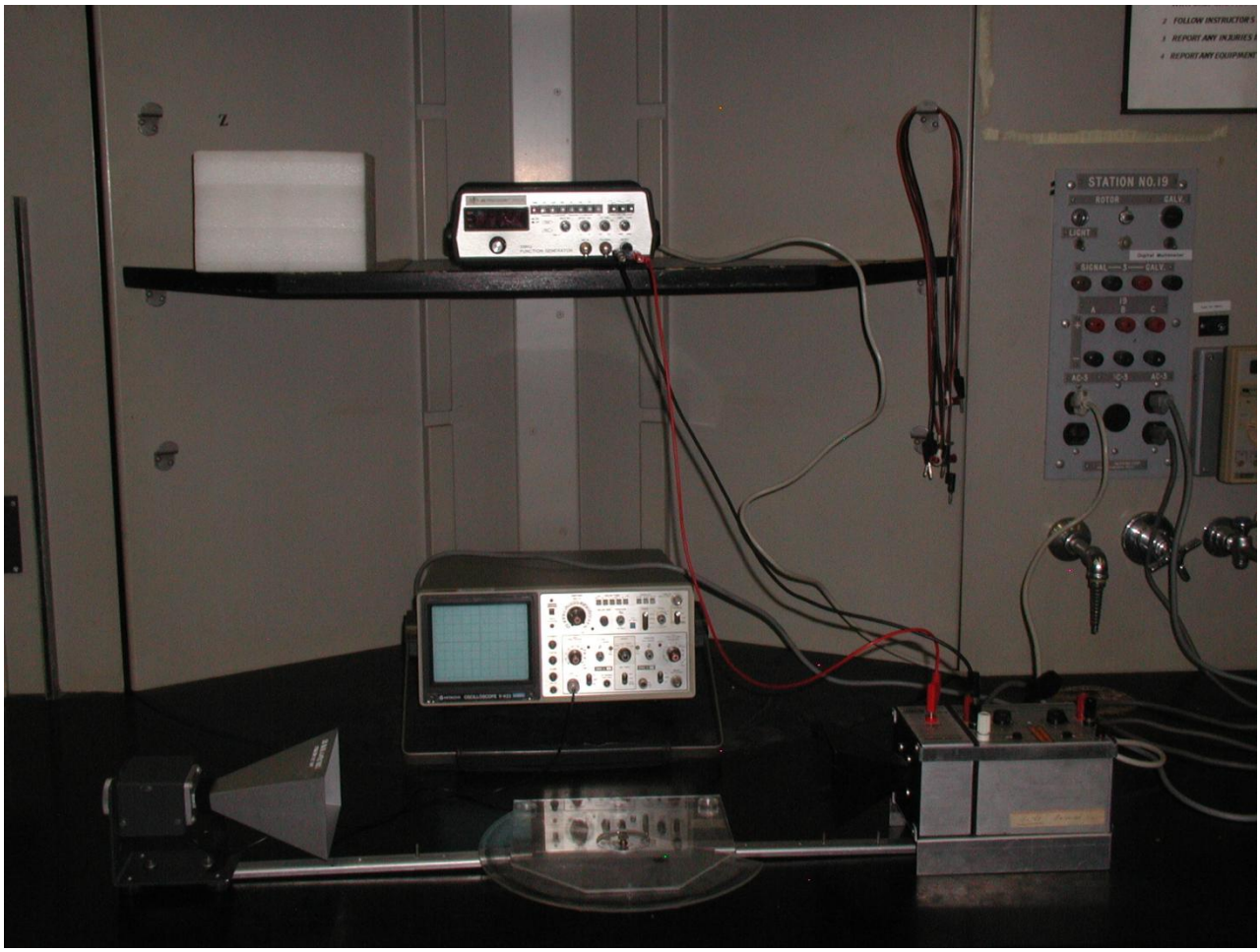
o 2 or 5 Volts/div
o Sweep time of 0.5 ms

- Connect red and black wires from the output side of the function generator to the oscilloscope input, using the banana/BNC adaptors (right).
- Verify that you get a stable signal (should look like a stationary sine wave).
- Vary the function generator amplitude and frequency knobs and observe the effect.
- Vary the oscilloscope gain (Volts/div) and sweep (ms) knobs and observe the effect.

Step 2 – Test the microwave generator and receiver

- Connect the function generator to the microwave generator (red wire goes to the "modulation input," black to ground).
- With the generator directly facing the receiver horn, connect the horn to the oscilloscope.
- Adjust gain knob on oscilloscope until signal is clear. If the signal looks strange (top shaved off), reduce the amplitude on the function generator.

Congratulations! You are ready to go. Once you have suitable settings for the experiment, make sure you do not make any further adjustments to the function generator or the oscilloscope.



II. APPARATUS

The microwave spectrometer, shown in Figure 2., consists of a microwave source, receiver, frame to enable angular measurements and a 100 element model of a crystal of metal spheres mounted in a polyethylene cube. The amplitude of the received microwaves is observed on an oscilloscope. The wavelength of the generated microwaves is 3 cm.

Measure the distance between the centers of two of the ball bearings to the nearest half millimeter. Be sure to include some estimate of your error in gauging the centers of the balls by eye. Record the measurement as a ± Δa.

Make eight more measurements of the spacing between different spheres and record the results in a table. Find the average value of a, ä, and the standard deviation, σ, of your data and compare the standard deviation with your previous error estimate. Assuming that the spacing is actually the same between all the spheres, σ should be about 1/√9 (or ⅓) Δa.

III. BRAGG PEAKS

Mount the source and receiver at 0^0 and 180^0 respectively, i.e. opposite to each other; verify that without the cube a signal is observed. Note: the frequency of the signal seen on the oscilloscope is not the frequency of the microwave signal itself, but of the signal imposed by the signal generator. This signal is introduced only for the sake of being able to observe the radiation with the oscilloscope. We will use the oscilloscope readings only for the sake of determining the relative amplitude of the received signal.

Adjust the amplitude of the signal by turning the amplitude knob on the signal generator and see if the pattern on the oscilloscope responds accordingly. Then set the amplitude to the highest setting that retains a good sinusoidal shape and DO NOT ADJUST THE AMPLITUDE FOR THE REST OF THE EXPERIMENT. Now mount the cube so that one face of the cube is perpendicular to the 90^0 line on the protractor, as shown in Fig. 3. The center of the cube should be over the pivot point of the protractor.

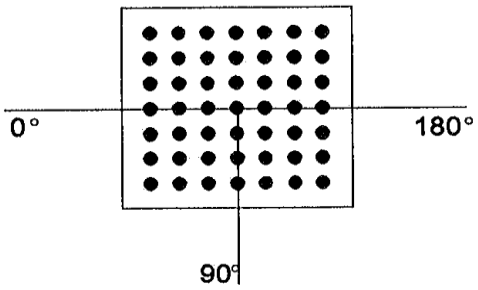


Figure 3. Orientation of the cube for normal incidence.

Using Eqn. (2) and your measured value of a, predict the values of θ_i for the peaks in amplitude for this orientation of the cube when n = 1, 2, 3, ..

When taking measurements, BE SURE TO KEEP $\theta_i = \theta_r$. Starting with $\theta_i = \theta_r = 10^0$ (See Fig. 2), record the observed amplitude of the scattered signal as a function of θ in increments of 5^0 up to 75^0. MAKE SURE THAT THE ORIENTATION OF THE CUBE DOES NOT CHANGE AS YOU MOVE THE SOURCE AND RECEIVER. Make a plot of the relative amplitude in arbitrary units as a function of θ. You should notice peaks near the values you calculated for Q.3. Take additional readings for smaller increments of the angle on both sides of the peaks, in order to determine more precisely the experimental value of each maximum. Make sure that you record errors in the values of θ as well as in the amplitude. Use your plot to determine your experimental values of θ_{max} including an estimate of $\Delta\theta_{max}$ for each peak. Check whether your experimental values agree within experimental error to those calculated.

IV. CRYSTAL STRUCTURE

Rotate the cube by 45^0 above the vertical axis as shown in Fig. 4.

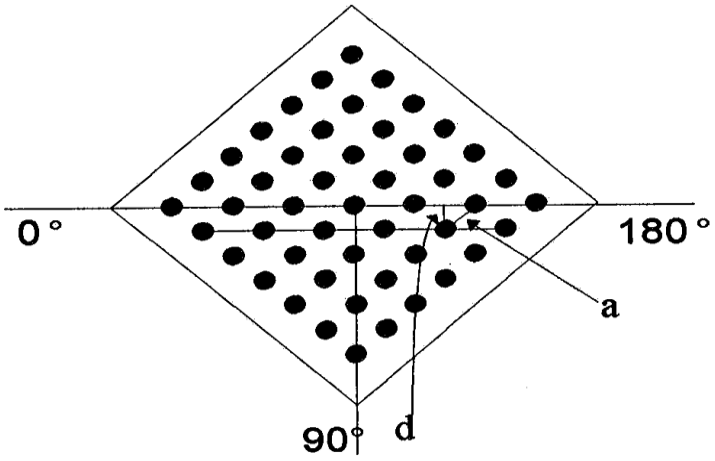


Figure 4. Orientation of the cube for part IV

Calculate the spacing, d, between the scattering planes for this orientation of the cube.
Also calculate the angle(s) where a peak should be found for this orientation. Make sure to check for n > 1.

As in part III, record the amplitude of the received signal as a function of the incident and reflected angle. Vary the angle from 20^0 to 50^0 in increments of 5^0, plot your points, and then take more data points at smaller increments of θ near the peak. Use your plot to determine the experimental value of θ_{max} and include your estimate of the error. Does your experimental value agree within experimental error to the value you calculated?

V. MORE CRYSTAL STRUCTURE

Orient the cube as in Fig. 5.

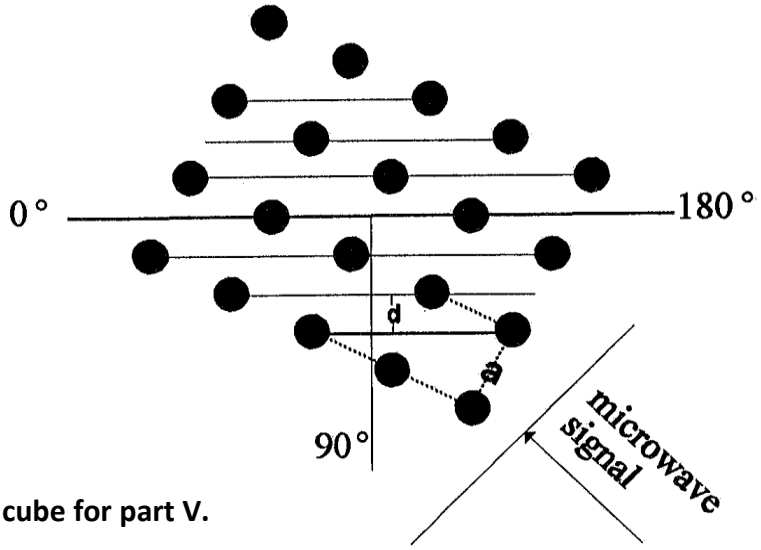


Figure 5. Orientation of the cube for part V.

Calculate the distance between scattering planes for this new orientation of the cube.
Predict the angles where a peaks in scattered radiation should be observed for this orientation of the cube.

As in the two previous sections, record the amplitude of the scattered radiation as a function of the incident angle. Vary the angle from 40^0 to 70^0 using increments of 5^0. Plot the points, locate the probable position of the peak and then take more data points near the peak. Use your plot to determine the experimental value of θ_{max} and include your estimate of the error. Does your experimental value agree within experimental error to the value you calculated?

VI. MEASUREMENT OF λ

As was mentioned in the introduction, Bragg scattering can also be used to determine the wavelength of the probing radiation. Using your experimentally determined values for d and θ_{max} from part III, calculate the wavelength of the microwave radiation. Make sure that you perform an error analysis. Does your result agree, within experimental error, with the accepted value of 3 cm?

PRELAB QUESTIONS

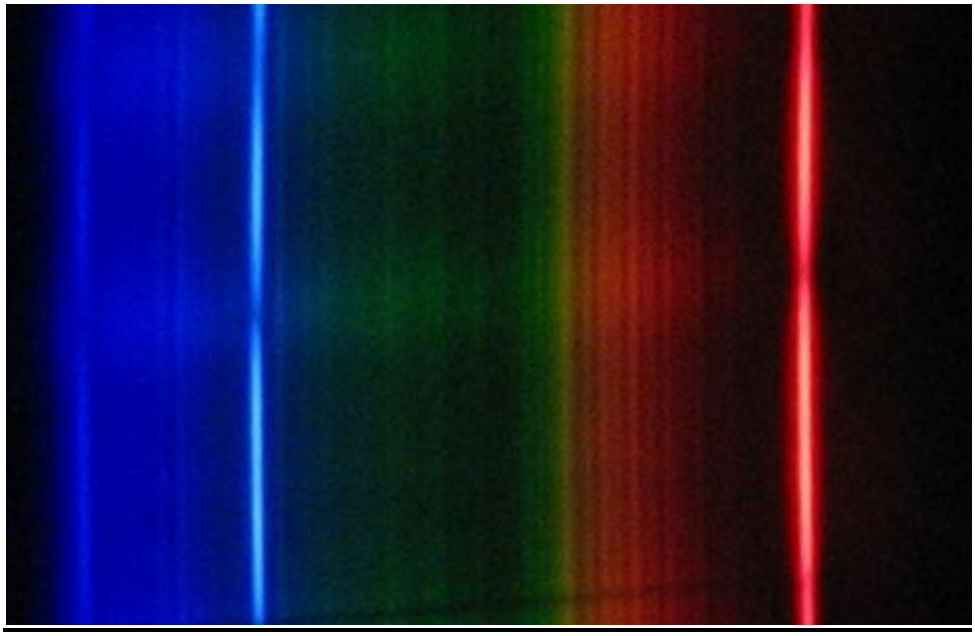
Q.1. If d = 4.5 cm and λ = 1.5 cm, how many maxima will be observed and at what angles?

Q.2. If the spacing between spheres in a simple cubic lattice is a = 4.5 cm, what is the spacing between scattering planes, d, if the cube is rotated by 45^0, as in Fig. 4?

Q.3. What is the angle of rotation for the orientation of the cube in Fig. 5 (That is, by how much was the cube rotated from the position in Fig. 3 to obtain the orientation in Fig. 5)?

Q.4. Describe briefly how diffraction through a crystal lattice could be used to isolate a single wavelength from X-rays of many wavelengths. (This is the principle behind and X-ray spectrometer.)

THE EMISSION SPECTRUM OF HYDROGEN



Photograph of Hydrogen Spectrum

I. INTRODUCTION

One of the most interesting phenomena in nature is the fact that elemental substances such as sodium, mercury or hydrogen absorb and emit light only at a number of well-defined frequencies, characteristic of the element. These atomic absorption and emission spectra, as they are called, became one of the most important tools used to investigate the quantum mechanical nature of matter at the atomic scale. They were the motivation behind the development of the Bohr model of the Hydrogen atom.

For hydrogen, the spectral lines of a given element can be grouped into series of lines, labeled n_s = 1, 2, 3, etc, and the lines in each series can be numbered from n = n_s + 1 to n = ∞. An experimentally derived expression for the wavelength of the spectral line as a function of the number of the line in the series was obtained by Balmer for the visible series in the hydrogen spectrum

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

While Rydberg later generalized the Balmer results for all the series for the hydrogen atom n_s

$$\frac{1}{\lambda} = R\left(\frac{1}{n_s^2} - \frac{1}{n^2}\right)$$

Where R = .0110 nm^{-1} is the Rydberg constant.

In this experiment, we will use a diffraction grating and a spectrometer to resolve the hydrogen light into its visible spectrum, and determine the wavelength of the spectral lines as a function of the initial orbital energy level, n_u, where n_u is the quantum number of the "upper" orbital, from where the energy transition starts, ending in to the orbital with quantum number n_s. The visible spectral lines are part of the Balmer series, for which we know that $n_s = 2$. We will plot $1/\lambda$ vs. $1/n_u^2$ to verify that this is a linear relationship, and also predict the value of R, the Rydberg constant. In doing so, we will have demonstrated the quantum mechanical nature of the atomic behavior in hydrogen, viz., a discrete, characteristic line spectrum of energies. We will also use one of the spectral liens from the mercury vapor lamp of known wavelength (See Exp. #1) to verify the grating spacing.

II. APPARATUS AND PROCEDURE

The use of the spectrometer is described in Appendix C at the back of this lab manual. Special care must be taken to reduce as much as possible any extraneous light so that all four visible lines in the hydrogen spectrum can be seen. It is a common mistake to identify the strong blue-violet line as the much weaker violet line. You should increase the slit width in order to observe the violet line. Remember, however, that the wider the slit, the greater the error in determining the diffraction angle. In general, you should use the minimum slit width at which the spectral line can be observed in order to obtain precise and accurate results.

Make sure that the diffraction grating is placed on the center stand so that the light from the telescope is incident normally upon the grating. For normal incidence, the positions of the spectral lines are given by

$$m\lambda = d\sin\theta,$$

Where m is the spectral order, λ is the wavelength of the spectral line, d is the distance between rulings on the grating, i.e., the "grating constant" and θ is the diffraction angle.

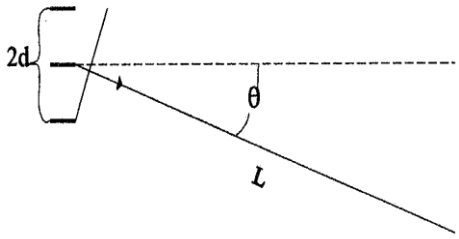


Figure 1. Sketch for deriving the relationship between, the slit spacing and angle of diffraction.

Q.1. Derive the expression mλ = dsinθ for light passing through two slits separated by a distance d using Fig. 1. Assume that d << L, the distance from the diffraction grating to the point of measurement. As in the Bragg scattering formula, you should calculate the path difference for light passing through the two slits and then impose the condition for when the light will interfere constructively. Note, however, that this formula is not the same as for Bragg scattering. Explain the difference.

The angle θ is measured from the m = 0 spectral line. However, in order to decrease the percentage error of our measurement of the angle, we will measure the angle for the m = 1 spectral line of the same wavelength λ both to the right and to the left of the m = 0 line and take the difference between the two recorded angles. Dividing by 2, we then obtain θ, but with an error ½ as large as if we had measured θ from the m = 0 line.

Now measure 2θ for m = 1 for each of the four visible lines in the spectrum. The short wavelength lines will be closest to the m = 0 line. Calculate the wavelengths of the lines you can see, expressing these to no more significant figures than are justified by the data, and plot 1/λ vs. $1/n_u^2$ (using your experimentally obtained values of θ). Which shape curve is expected? Use a table as follows:

Hydrogen line-color	Red	Blue-Green	Blue-Violet	Violet
Grating constant d (Å)				
Reading (deg.)—right				
Reading (deg)—left				
Difference (=2θ) deg.				
θ deg				
Sin θ				
λ (Å)—Experimental				
For graphing: Quantum Number N_u				
$1/n_u^2$				
$1/\lambda_{exptl}$				

Use the Rydberg equation

$$\frac{1}{\lambda} = R\left(\frac{1}{n_s^2} - \frac{1}{n_u^2}\right)$$

To show that the y-intercept is R/4. Find R from your graph, then use it (a) to find the short wavelength limit of the Balmer series, and (b) in conjunction with the Rydberg equation and the Einstein relationship

$$E_{Photon} = hc/\lambda$$

to determine E_I, the ionization energy of hydrogen. In a table, compare the two results with the published values.

III. DETERMINATION OF THE GRATING CONSTANT

Now, using a line of known wavelength, let's test the information given by the manufacturer about the grating. As a standard for calibrating, use one of the strong lines of mercury (See Exp. # 1). Take first – and second-order readings <u>on both sides of zero-order</u>, and from these determine the grating constant d and the number of grooves per unit width. As usual, express your result to no more significant figures than are justified by your data. Compare your result with that obtained from the manufacturer's value given on the grating. Use a table as follows:

Line-color from mercury spectrum		
λ (Å)		
Spectral order m	1	2
Reading (deg) – right		
Reading (deg) – left		
Difference (=2θ) deg.		
θ deg.		
Sin θ		
Grating Constant d (Å)		
Grating lines per mm (expt)		
Grating lines per mm (manufacturer)		

Make sure that you record the error in your measurements and calculate the error in your experimentally determined value of d.

With due consideration to the uncertainty in your result, does your result agree with that of the manufacturer? If not, what is a likely reason for the difference?

PRELAB QUESTIONS

Q.1. Q1 in the text above, on Page 3 of 5.

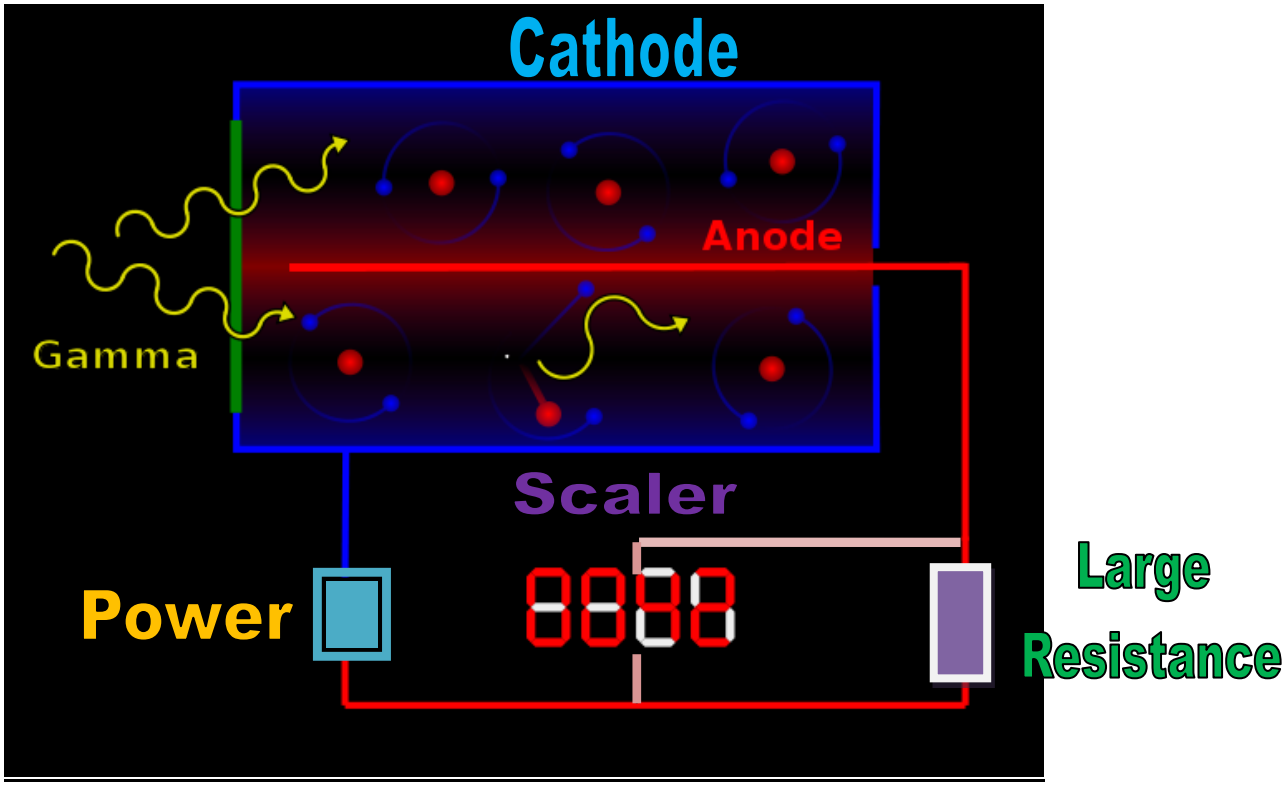
Q.2. Using your textbook to help you, derive the expression for R using the Bohr model. Explain briefly how the quantum mechanical or “wave” nature of the orbital motion affects the energy of the nth electronic state in hydrogen and creates the discrete energies of transition observed in the emission spectrum of hydrogen.

Q.3. Why can’t we see the transitions to the n = 1 ground state?

Q.4. What is the mechanism in the hydrogen lamp which causes the hydrogen molecules to dissociate into atoms and the atomic electrons to be in higher energy orbital states?

Q.5. Explain how will you make the grating, perfectly normal to the incident light beam from the collimator of the spectrometer.

THE GEIGER-MüLLER COUNTER, AND THE STATISTICS OF COUNTING RADIOACTIVE DECAYS



The remainder of the labs in this course concerns the decay of atomic nuclei and the byproducts of their decay. While atomic spectra were a window for investigating the quantum mechanical nature of the electronic motion about the atomic nucleus, radioactive decays are one of the most significant tools for investigating the quantum mechanical nature of atomic nuclei. They tell us about the constituent particles that make up the nucleus, as well as the forces that hold them together.

The purpose of this experiment is twofold. The first part deals with the operation of a Geiger-Müller counter, an instrument used frequently to detect and measure radiation from nuclear decays, particularly alpha and beta radiation. We will make and experimental survey of the operation and use of the counter, which will be our main tool in investigating radioactive decays. In particular, we must learn what are the Geiger counter's optimal operating conditions. The second part of this experiment is concerned with the statistical methods required to interpret the results of radiation counting measurements.

PART 1. THE GEIGER-MüLLER COUNTER

DISCUSSION

The Geiger-Müller counter, which is generally called the Geiger counter, belongs to a class of radiation detectors whose operation is based on the phenomenon of ionization. Invented about 1908 by Hans Geiger, the Geiger counter is still widely used as a radiation detector, particularly in medical applications.

The Geiger counter (see Fig. 1) is a hollow metal conducting cylinder generally filled with argon, an inert gas. A thin wire which runs along the axis of the cylinder is kept at a high positive potential (about 900 V) with respect to the metal cylinder which is grounded. This results in an electric field which is directed radially outward from the central wire to the surrounding cylinder.

When charged particles enter through a thin-walled "window" section, interactions with the argon gas occur. These interactions may result in ionization, the removal of an electron from an atom of the gas. The electron and the positive argon ion are accelerated in opposite direction in the radial electric field. The electrons, because of their much lower masses, move much faster than the argon ions and so make additional collisions with other argon atoms. If the potential difference between the central wire and the hollow cylinder is sufficiently large, the electrons will acquire enough energy between collisions to knock loose other electrons in these collisions. This process results in an avalanche of electrons which produces a momentary current flow, or pulse, in the central wire.

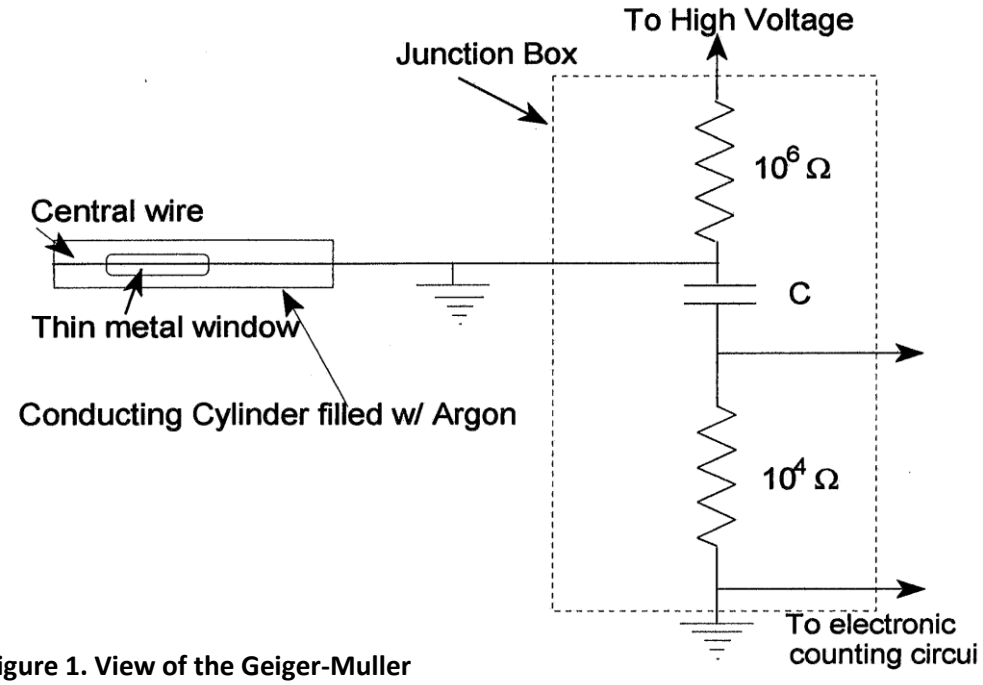


Figure 1. View of the Geiger-Muller counter

GEIGER-MüLLER PULSES

The electrons produced in the avalanche are collected by the central wire and flow into the junction box shown schematically in Fig. 1. This produces a time-dependent potential difference between the ends of the lower resistor. The resulting pulse, shown in Fig. 2, may be observed with an oscilloscope. Note that the time scales discussed here are associated with the response of the electronic circuit. The interaction between the radioactive particle and an argon atom is over a much shorter time span. In normal operation, these pulse signals are connected to electronic counting circuits, as described later.

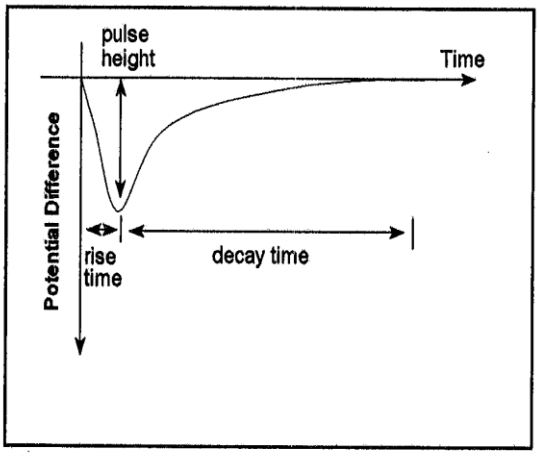


Figure 2. The pulse from a Geiger-Müller tube viewed on an oscilloscope.

The positive argon ions produced in the avalanche process reduce the electric field inside the Geiger tube and prevent an unlimited buildup of the avalanche. In addition, as long as a large number of ions surround the central wire, other charged particles which enter the tube cannot initiate avalanches: the Geiger tube is "dead" until the ions are neutralized. Although the ions eventually drift to the outer metal cylinder where they are neutralized by acquiring electrons, this process alone is too slow to make the device, as described, a useful instrument for counting radiation. To obviate this problem, a small amount of ethyl alcohol is added to the argon gas in the tube. The alcohol serves to neutralize the ions by giving up electrons to argon ions in collisions. This effect may be observed by looking at Geiger pulses on an oscilloscope with charged particles entering the Geiger tube at a high time rate. After a single pulse (produced by one of these particles) is seen, the tube is dead and no other pulses appear for a definite time interval. After this interval small pulses are observed, eventually reaching full height. This behavior is illustrated in Fig. 3.

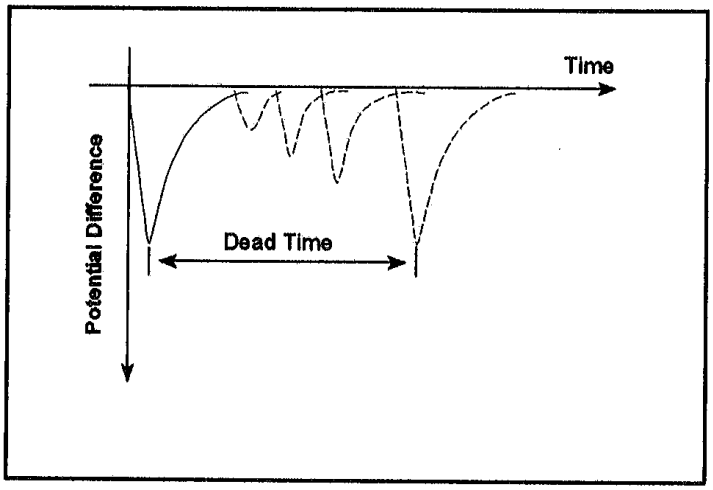
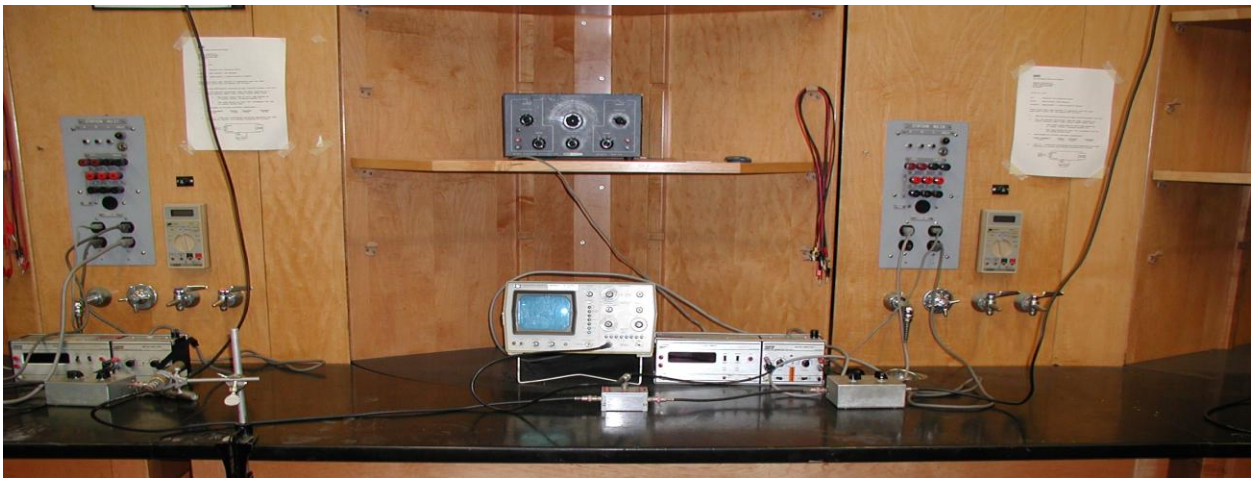


Figure 3. Operational dead time for the Geiger-Müller tube.



GEIGER COUNTER OPERATION

As mentioned above, the electrical pulse signals produced by a Geiger tube in response to ionizing radiation are connected to electronic circuits. A block diagram of a typical Geiger counter is shown in Fig. 4. First, the pulses are amplified to a convenient size. The amplified pulses are then sent to a discriminator.

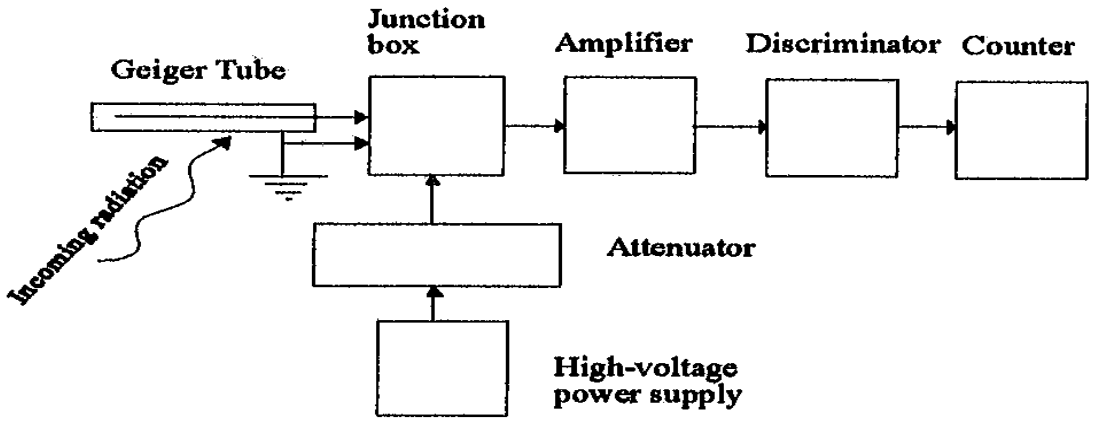


Figure 4. Block diagram of the Geiger counter electronics

This device produces an output pulse only when the input pulse is greater than a predetermined (usually variable) threshold level, thereby discriminating against spurious pulses. Finally, the discriminated pulses are individually counted in a device known as a scaler.

Since the height of the pulses produced by the Geiger tube increases with the voltage applied between the tube and the central wire, it is necessary to adjust this voltage so that each count registered by the counter corresponds to a single charged particle passing through the tube. If

the high voltage is set too low, pulses produced by charged particles will be too small to activate the discriminator. On the other hand, if the high voltage is too large, some of the ions produced in avalanches can gain sufficient energy in drifting to the outer cylinder to eject electrons from the metal surface. These secondary electrons can initiate new avalanches, thereby producing spurious or multiple pulses. Thus, the high voltage must be set sufficiently high to register a count for each charged particle passing through the tube, but no so high that spurious counts are recorded. Usually, there is a range of high voltage that satisfies these requirements.

This range is called a "plateau", corresponding to the fact that the number of counts per unit time from the tube remains nearly constant over the interval. A typical graph of counting rate as a function of high voltage is shown in Fig. 5. The voltage difference between the central wire and the tube must be within the plateau region in order for the statistics obtained to be a true measure of the physical system.

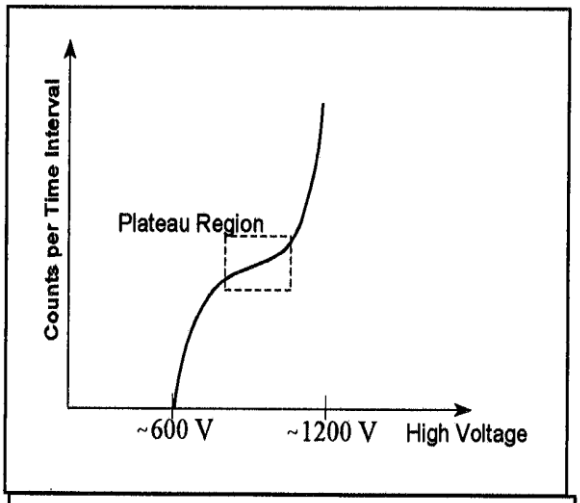


Figure 5. The Geiger-Müller plateau.

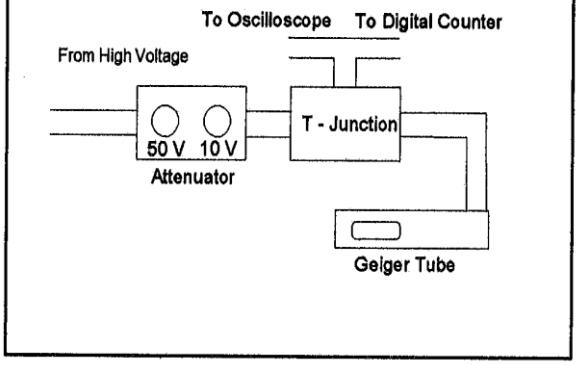


Figure 6. Connections to the Geiger tube.

SETTING THE VOLTAGE ACROSS THE GEIGER-MüLLER COUNTER

The apparatus employed in this experiment is indicated in Fig. 6. The high voltage required to operate the Geiger tube is produced by a power supply set to -1200V, and is distributed to each experimental station. To permit adjustments of the high voltage, an attenuator box is connected between the Geiger tube and the station high voltage, as shown in the diagram. The "50V" switch leaves the high voltage unchanged when set to the "O" position, reduces the high voltage by 50V when set to the "1" position. it reduces the high voltage by 2 x 50V = 100V when set to the "2" position, and so forth. The "10V" switch functions in a similar manner, reducing the high voltage in 10V steps. Thus, the attenuator box serves to reduce the high voltage applied to the Geiger tube in a combination of 10V and 50V steps determined by the switch settings.

PART I. EXPERIMENT

INVESTIGATION OF GEIGER PULSES

In this investigation, the electrical pulses by a Geiger counter are studied by connecting the oscilloscope to the pulse output of the junction box (see Figs. 1 and 6) using one branch of a "T" connector. The oscilloscope push-button controls should be set as follows:

Control	Setting
Trigger	AUTO
Slope	- (Minus)
Mode	AC
Source	INT

To start, the vertical sensitivity might be set to 20 mV/div, and the sweep speed to 10 μsec/div. The trigger-level control determines the signal level at which the sweep will begin.

In today's experiment, β particles emitted by a radioactive $^{204}_{81}Tl$ source will be detected by the Geiger counter. (In standard notation Tl denotes the element thallium, 204 indicates the total number of particles – protons and neutrons – in the nucleus, and 81 indicates the number of protons in the nucleus, i.e., the atomic number of thallium.)

Check out a $^{204}_{81}Tl$ source, being sure to sign the checkout sheet and to record the source number. Place the source so that the disk stands on edge with the aluminum foil facing toward the Geiger tube, as shown in Fig. 7. The Geiger tube should be located about 5 – 10 cm from the source. Adjust the attenuator to provide a high voltage of -900V on the Geiger tube.

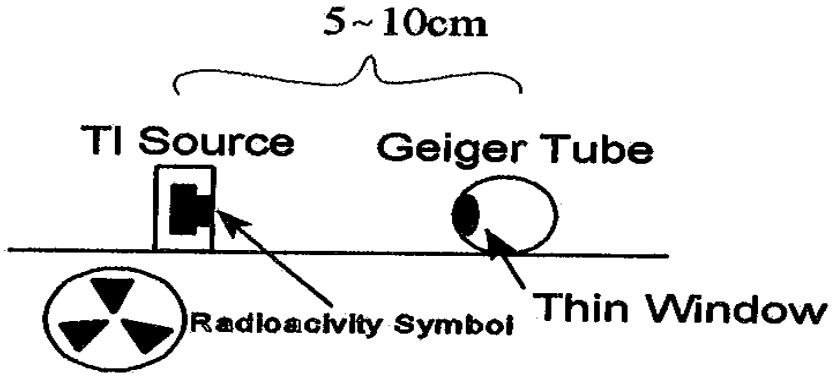


Figure 7. Orientation of the Tl source with respect to the Geiger tube.

Observe the Geiger pulses on the oscilloscope screen. Set the oscilloscope sensitivity and sweep speed controls to appropriate values for viewing the pulses. Be sure that the "calibrate" controls are properly set. Normally, the time scale for the rise time and the fall time are very

different, and it will be necessary to change the scale on the oscilloscope to observe both phenomena well. In addition, it may be quite difficult to observe the rise time because the oscilloscope does not trigger properly. Usually, by adjusting knob # 12 as shown in Appendix D toward the negative side and increasing the signal intensity, it is possible to get a good picture of the rise time of the signal.

Q.1. Make a careful sketch of a single Geiger pulse, including voltage and time scales. Be sure that the rise of the pulse is visible.

Q.2. Estimate the rise and decay time of a Geiger pulse.

The phenomenon of dead time may be observed by moving the source close to the Geiger tube so that the β^- particles emitted by the Tl source enter the tube at a high rate. Under these circumstances, the pulses seen on the oscilloscope should appear as shown in Fig. 3. A slower sweep-rate setting on the oscilloscope may be required.

Q.3. Estimate the dead time of the Geiger tube.

DETERMINATION OF THE GEIGER-MüLLER PLATEAU

In this investigation the pulse output from the junction box is connected to an electronic counter. This is in effected by connecting a cable from the input of the counting circuit to the used branch of the "T" connector on the junction box (see Fig. 6). Leave the oscilloscope connected to the other branch.

The controls on the counter should be set as follows:

Control	Setting
Count Input Switch	APS-OUT
Hold Mode	Pulse*
DC-AC	AC

On "Gate", the start-stop switch is bypassed, and the counter is "on" continuously. This is a useful position when testing the equipment. The counter may now be operated using only four controls. First, the "Hold" button starts and stops counting process. If the circuit is counting, pressing the button stops the counting; if the counter is not counting, pressing the button starts the counting. The "Reset" button is used to clear the counter before starting a measurement. The "Gain" and "DC Offset" controls are related to the amplification and discrimination function of the circuit. The gain needed depends on the size of the input pulses, which in turn depends on the voltage applied to the Geiger tube. Do not use excessively high gain. **Set the D.C. offset near the center of the range in which counting occurs for the gain used.**

Position the Tl source and Geiger counter as shown in Fig. 7. Adjust the high voltage to -650V by setting the "10V" switch to "0" and the "50V" switch to "10" on the attenuator box. Start the counter and slowly increase the high voltage in 50V steps until counting begins. At this point Geiger pulses should begin to appear on the oscilloscope. Remove the source to check that the pulses are indeed due to radiation emitted by the source and not to spurious effects. Record the number of counts registered in 20 sec.

Now increase the high voltage in 50V steps and record the corresponding number of counts collected in 20 sec in a table. Continue this procedure until full – 1200V of power supply is applied to the Geiger tube. (You may need to use 20v steps in the plateau region and where the counting rate changes abruptly.)

> Q.4. Plot the number of counts per 20 sec interval as a function of the applied high voltage. Draw a smooth curve through the data points, and compare your result with Fig. 5.

Set the high voltage to a value near the center of the plateau. This is the proper operating voltage for the Geiger tube and should be left constant for the remainder of the experiment.

> Q.5. Measure and record the number of counts collected in four successive 20 sec intervals. Are all of these values equal?

PART II. STATISTICS OF COUNTING RADIOACTIVE DECAYS

DISCUSSION

The answer to Q.5 indicates an apparent anomaly: The results of several measurements performed under identical conditions are not all identical. This feature is not really anomalous but is inherent in measuring phenomena like radioactive decay. In classical physics, a measurement of a deterministic process is expected to yield a "correct" answer. If different trials yield slightly different answers, that is a result of a random errors in the measurement process. An example of this is the student who measures the length of a piece of paper, and obtains lengths which are randomly distributed about a mean value. The assumption is that, by obtaining a better measuring device, one could theoretically reduce the measurement error to zero.

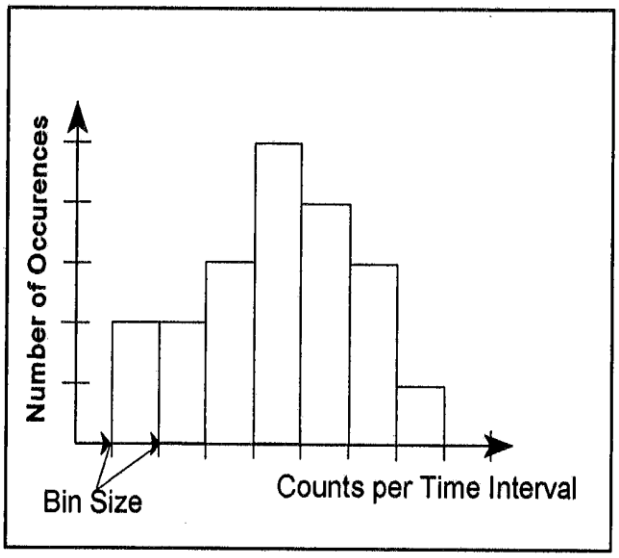


Figure 8. A distribution histogram.

In our experiment, the measurement consisted of counting the number of β^- particles emitted in radioactive decays during a definite time interval. There are errors in the measurements process, such as in the time interval, in the efficiency of the Geiger tube and counter, etc… In addition to these errors, however, there is an additional source of randomness in the data that has nothing to do with the measurements process. Instead, although the decay mechanism is the same for each member of a group of identical radioactive nuclei, there is no way to predict exactly when a particular nucleus will decay. On the average an equal number of nuclei will decay in equal time intervals, but the actual number decaying in any specific interval may vary significantly from the average value. Thus, there is no "correct" answer to which we can compare our experimental findings. Instead the phenomenon we are measuring is inherently random, so that the best we can hope to do is determine the mean value and characterize how the data is distributed about the mean. It is a result of the fact that on the scale of the atomic nucleus, particles obey quantum or "wave" mechanics, and therefore only the probability of an event is determined by the physical conditions. We note, however, that random errors in the measurement and randomness in the phenomenon being measured generally can be treated using the same statistical tools.

MEASUREMENT STATISTICS

A set of n identically performed measurements of a single, well-defined x, will yield the slightly different values x_1, x_2, x_3, …, x_n because of errors in the measurement. The measurements are conveniently displayed in a distribution histogram, having some appropriate bin size Δx, as shown in Fig. 8.

The results of such a set of measurements are generally characterized by two parameters. These are the average or mean value, defined by

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad (1)$$

And the standard deviation, given for a reasonably large value of n by

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{(n-1)}} \qquad (2)$$

The value of σ_x represents the uncertainty expected in a single measurement. That is, the result of a single measurement, x_i, would be presented as $x_i \pm \sigma_x$. For the case of a very large number of measurements having purely random measurement errors, the distribution curve approaches the familiar bell-shaped "normal" distribution. In this case 68% of all the measurements are included within the band of width $\pm \sigma_x$ about $\bar{x}$. Thus, there is a 68% chance

measurements are included within the band of width ± σ_x about $\bar{x}$. Thus, there is a 68% chance that another single measurement would result in a value lying within this band.

The uncertainty of the mean value $\bar{x}$ is, of course, much smaller, since more information – the entire set of n measurements – is available. The standard deviation of the mean value is given by $\sigma_{\bar{x}} = \sigma_x/\sqrt{n}$, and the results of the entire set of measurements may be presented as $\bar{x} \pm \sigma_{\bar{x}}$.

RADIATION STATISTICS

When the measurement consists of counting particles produced in radioactive decays, the random nature of the decay process results in a special kind of data distribution. As an example, consider the experiment of counting the number of pulses obtained in a definite time interval Δt using a Geiger counter apparatus. If this measurement is repeated, the result will be a set of number N_1, N_2, N_3, …, N_n corresponding to the number of counts recorded in each trial. Selecting an appropriate bin size, this date may be displayed in a distribution histogram, such as the one shown in Fig. 8. A mean value, Ň, can be calculated for the number of counts in the time interval, Δt. However, this mean value has a different interpretation than in the previous section. It is not an approximate value of the "true" number of counts in a time interval Δt. There is no "true" number of counts in a given time interval! Instead, Ň is an approximate value of the average number of counts that would be obtained if the counting experiment were repeated for an infinite number of times. Equivalently, it is an approximation of the average value per time interval Δt that would be obtained if the experiment were performed over an infinite time interval.

In addition, a standard deviation can be calculated to characterize the data distribution for our counting experiment. The distribution of the data has a shape such that the standard deviation, defined generally by Eq. (2), is also equal to the square root of the mean value,

$$\sigma_N = \sqrt{N} \qquad (3)$$

This standard deviation is independent of the measurement process and is solely a result of the random nature of the decay events. Therefore, an extensive set of counting measurements is not longer required to obtain an estimate of the uncertainty of a counting measurement. Since the data distribution is approximately independent of the measurement process, if N_1 counts are recorded in a single measurement, then the uncertainty of this value is known to be approximately $\pm\sqrt{N_1}$, and the single measurement may be presented as $N_1 \pm \sqrt{N_1}$. The relative uncertainty is given by

$$\frac{\pm\sigma}{N} = \frac{\pm\sqrt{N}}{N} = \pm\frac{1}{\sqrt{N}} \qquad (4)$$

From this result it is seen that the relative uncertainty of a single counting measurement is inversely proportional to the square root of the number of counts recorded. This consideration is very important in experiments involving radiation counting. For example, consider an experiment in which 121 counts are registered in 20 sec. The uncertainty in this value is ± $(121)^{1/2}$ = ± 11. Of course, if the apparatus is not functioning properly, the total error may be considerably larger. Thus, the result may be presented as 121 ± 11 counts. The relative uncertainty is ± (11/121) x 100% = ± 9%. Now, had the measurement run for 80 sec, a value of about 484 counts would have been obtained. This result is presented as 484 ± $(484)^{1/2}$ 484 ± 22 counts. In this case, the relative uncertainty is ± (22/484) x 100% = ± 4.5%. Counting four times as long and recording four times as many counts reduces the relative uncertainty by $1/\sqrt{4}$, or one-half. Obviously, a longer counting interval would result in a still smaller relative uncertainty. Thus, precision in a radiation counting experiment can be improved only at the expense of significantly increasing the time duration of the experiment.

The standard deviations defined in Eqns. 2 and 3 are strictly equivalent only when all errors due to the lack of precision in the measurement process have been eliminated. Part of the purpose of this experiment is to show whether these two measures of the data distribution are equivalent in practice. In future experiments, we will assume that they are equivalent and use the simpler method given in Eqn. (3). We emphasize again that this error is the result of the randomness of the process being observed and cannot be removed by improving the precision of the measurement. This "inherent randomness" in quantum mechanical processes is a characteristic of quantum mechanics.

PART II EXPERIMENT

Using the orientation shown in Fig. 7, position the Tl source so that about 200 counts are recorded in 20 sec by the Geiger counter apparatus. Then, make a series of thirty measurements of the number of counts registered in a 20 sec interval. Tabulate your data. In making these measurements, take care in starting and stopping the counter so that all time intervals are equal to within 1/5 sec.

Q.6. Using a bin size of 10 counts, display your data in the form of a counting distribution histogram.

Q.7. What is the most probable bin? What is the probability that a single 20 sec measurement will yield a number of counts falling in this bin?

Q.8. What is the central value of the most probable bin?

Q.9. Compute the mean number of counts per 20 sec interval. How does this value compare with the central value of the most probable bin?

Q.10. Compute the standard deviation of your distribution using the general definition given in Eqn. (2). The procedure is outlined in appendix A.

Q.11. Recompute the standard deviation using the special relation given in Eqn. (3), valid for the radioactive decays.

Q.12. From your histogram, estimate the standard deviation by including a sufficient number of bins on each side of the mean value to include 68% of the 30 measurements. (Simply calculate the % of the counts in the most probable bin and in those adjacent; then decide which bins, or fractions thereof, should be included to give about 34% on each side of the center of the most probable bin.)

Q.13. Which, if any, of the two measures of the standard deviation in Q.10 and Q.11 would you expect to be smaller? Compare and discuss the results of Q.10, 11 and 12.

Q.14. Suppose that a single measurement gave a value equal to your mean counting rate. Find the relative uncertainty of the result.

The results of a longer set of counting measurements may be simulated by considering the first two 20 sec measurements to be one 40 sec measurement, the third and fourth 20 sec measurements to be another 40 sec measurement, and so forth. In this manner, obtain counting data for fifteen 40 sec intervals by adding together the counts obtained in successive pairs of 20 sec measurements. Tabulate the data.

Q.15. Using a bin size of 10 counts, display this data as a counting distribution histogram.

Q.16. Compute the mean number of counts per 40 sec.

Q.17. Compute the standard deviation using Eq. (3).

Q.18. Compute the relative uncertainty of a single 40 sec measurement equal to your mean value.

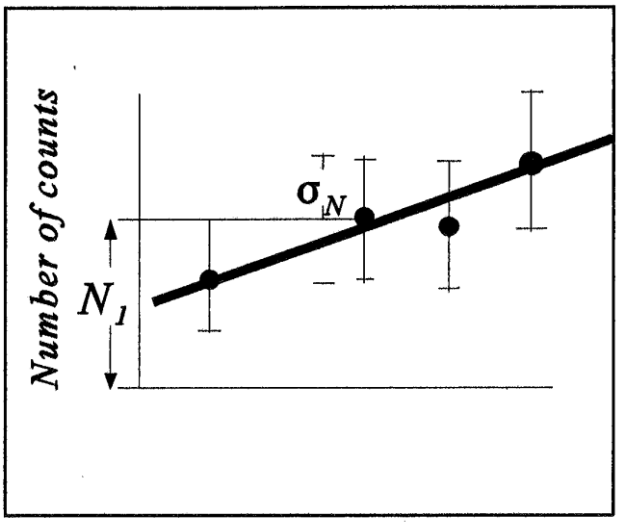


Figure 9. The use of error bars in radiation counting experiments.

Q.19. Compare and discuss the relative uncertainties obtained in 20 sec and 40 sec intervals (see Q.14, Q.18.).

UNCERTAINTIES IN SINGLE COUNTING MEASUREMENTS

As discussed earlier, a single measurement of N_i counts has an inherent uncertainty of

$\pm\sqrt{N(i)}$. In graphical displays this uncertainty is represented by the use of error bars which extend this amount above and below the point representing N_i counts. This procedure is illustrated in Fig. 9. The result of a series of measurements is then characterized by a smooth curve which, generally, passes through most of the error bars associated with the individual data points.

Q.20. Use this procedure to add error bars to your graph of the Geiger plateau data of Q.4. Did you happen to draw your curve so that it is consistent with the uncertainty?

PRELAB QUESTIONS

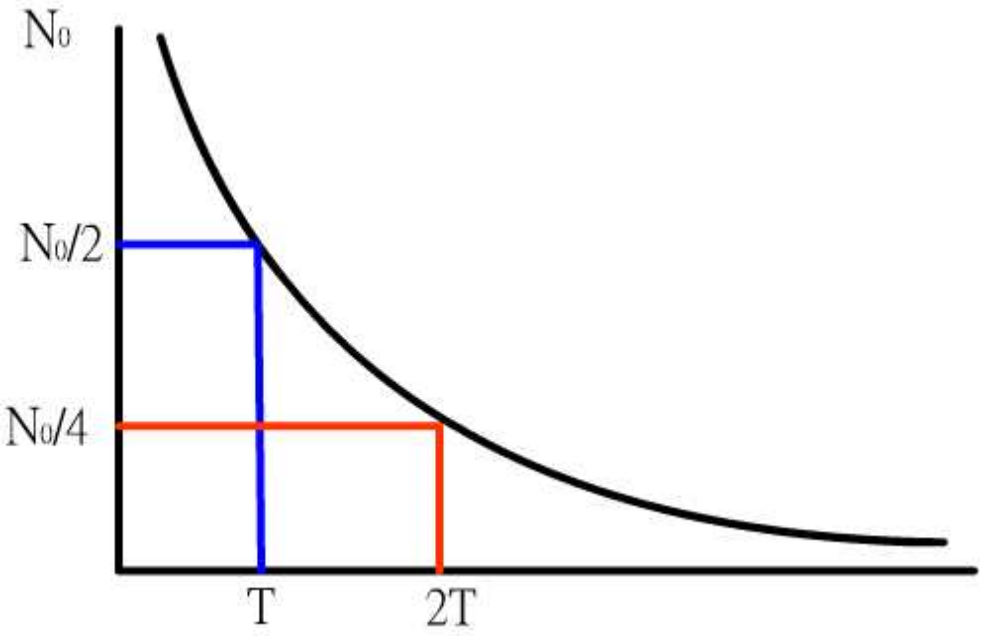
Q.1 What causes the electron avalanche in a Geiger tube which signals the presence of a charged particle, what eventually stops the avalanche, and by what process is the tube restored for the next incoming particle?

Q.2. Using simple circuit theory, explain why a pulse of electrons from the Geiger tube generates the pulse on the oscilloscope illustrated in Fig.2. See Fig. 1.

Q.3. Explain briefly why it is necessary to “plateau” the Geiger counter before running our experiment.

Q.4. Compare and contrast data distributions caused by random errors in measurements with those resulting from random physical process.

Nuclear Lifetime



In Experiment # 4 we emphasized the fact that radioactive decays are by nature a random process, and that this is a result of the quantum mechanical nature of the motion of the particles that make up the nucleus. However, while we cannot predict when a given nucleus will decay, we can measure and sometimes predict (using quantum mechanics) the probability that a certain decay process will occur within a given time period. The time interval over which the nucleus has a 50% chance of undergoing decay is called that half-life, and a measurement of the half-life is a good way of characterizing the instability of a nucleus. The purpose of this lab is to determine the half-life of a certain excited and unstable nucleus, ^{137}Ba.

I. INTRODUCTION

The decay of unstable nuclei does not occur a fixed time after formation, but is distributed over a wide range of times. Quantum mechanics allows knowledge only of the probability that a type of nuclei will decay, not the exact time of any particular nucleus to decay.

The probability that any nucleus will decay in a certain unit of time is described by the decay constant, r. The rate at which nuclei will decay at time t, R(t), is proportional to the number N(t), of radioactive nuclei present at time t. The constant of proportionality or the decay constant r, is independent of when the nucleus was formed. We can then write,

$$R(t) = rN(t) \qquad (1)$$

R(t) is the rate of decrease of N:

$$\mathrm{R(t)} = -\,dN/dt \qquad (2)$$

So,

$$dN/dt = -R_n \qquad (3)$$

The solution is an exponential since the rate of decay of N Is proportional to N:

$$\mathrm{N(t)} = N_0 e^{-rt} \qquad (4)$$

Here N_0 is the number of nuclei present at tine t=0. There are two common ways to describe the lifetime of the nucleus. The mean life τ is defined as the time in which N drops to 1/e its original value. It is inversely proportional to the decay constant, r.

$$\tau = 1/r \qquad (5)$$

The half life, $t_{1/2}$ is the time in which the number of nuclei decreases to half its original value.

$$\mathrm{N}\left(t_{1/2}\right) = N_0 e^{-rt_{1/2}} = \frac{1}{2}N_0 \qquad (6)$$

$$-\mathrm{r}t_{1/2} = ln(1/2), \text{ or} \qquad (7)$$

$$t_{1/2} = ln2/r = \tau ln2 = 0.693/r \qquad (8)$$

An important conclusion is that an absolute t=0 time is not defined. Half of the existing nuclei at time t will decay during a period t to t + $t_{1/2}$ independent of when the measurement was started.

Graphs of Eqn. (4) are shown in Fig. 1. Figure 1a. is a plot on linear graph paper. The same data is shown plotted on semi-$\log_{10}$ graph paper in Fig.1b. The points in the second graph fall on a straight line whose intercept is N_0 and whose slope is

$$-\frac{\log e}{\tau} = \frac{\log N(t2) - \log N(t1)}{(t2 - t1)} \qquad (9)$$

The equation of the line is then:

$$\log \mathrm{N(t)} = \log N_0 - \mathrm{t}/\tau \log \mathrm{e} \qquad (10)$$

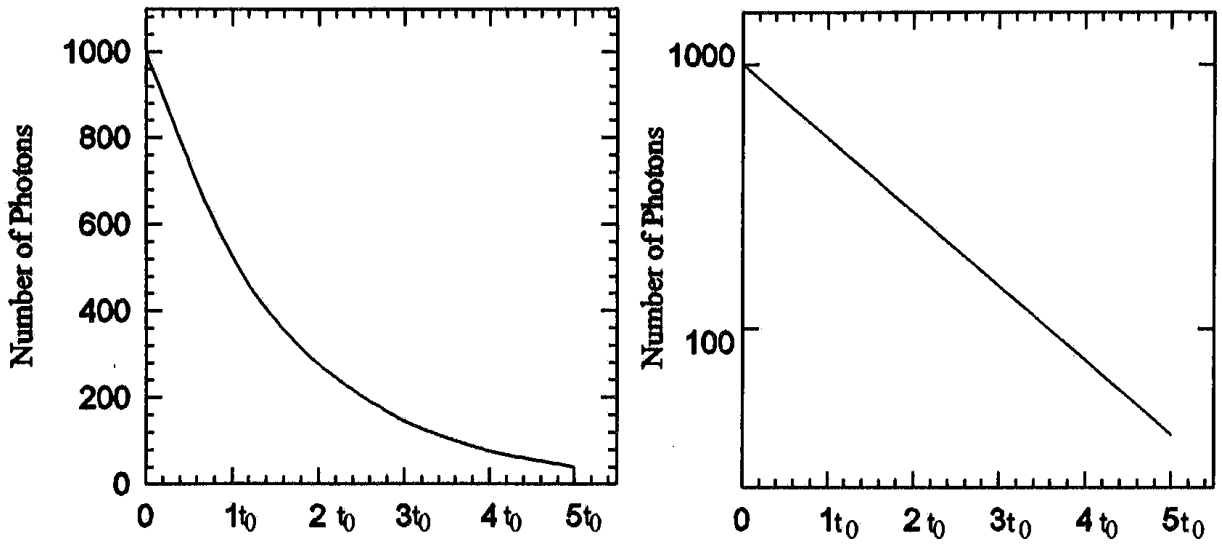


Figure 1. number of nuclear decays versus time.
(a) Plotted on linear and (b) plotted on semi-log graph paper.

In order to measure the lifetime of an excited nucleus, a significant percentage of the nuclei must decay during the time of observation: our lab session. From Eqn. (11) below, we can show that a nucleus with a long lifetime, e.g. 10 years = 8.8 x 10^5 hours, would only have 1.1 x 10^{-4}% decay in 1 hour.

$$\frac{N_0 - N_t}{N_0} = 1 - e^{-\frac{t}{\tau}} \qquad (11)$$

This would be exceedingly difficult to measure, so a nucleus with a short half-life is used in this lab. A parent nucleus, ^{137}Cs, is used to generate the daughter nucleus ^{137}Ba, which is in an excited and unstable state. The decay of the unstable ^{137}Ba nucleus releases a 0.66 MeV photon as shown in Fig. 2. The half-life of ^{137}Cs is 30 years whereas the hal-life of ^{137}Ba is 2.6 minutes.

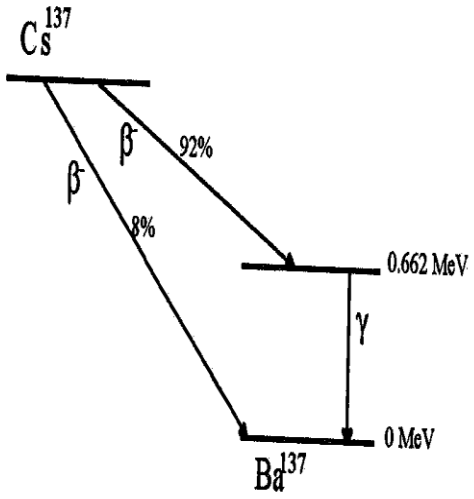


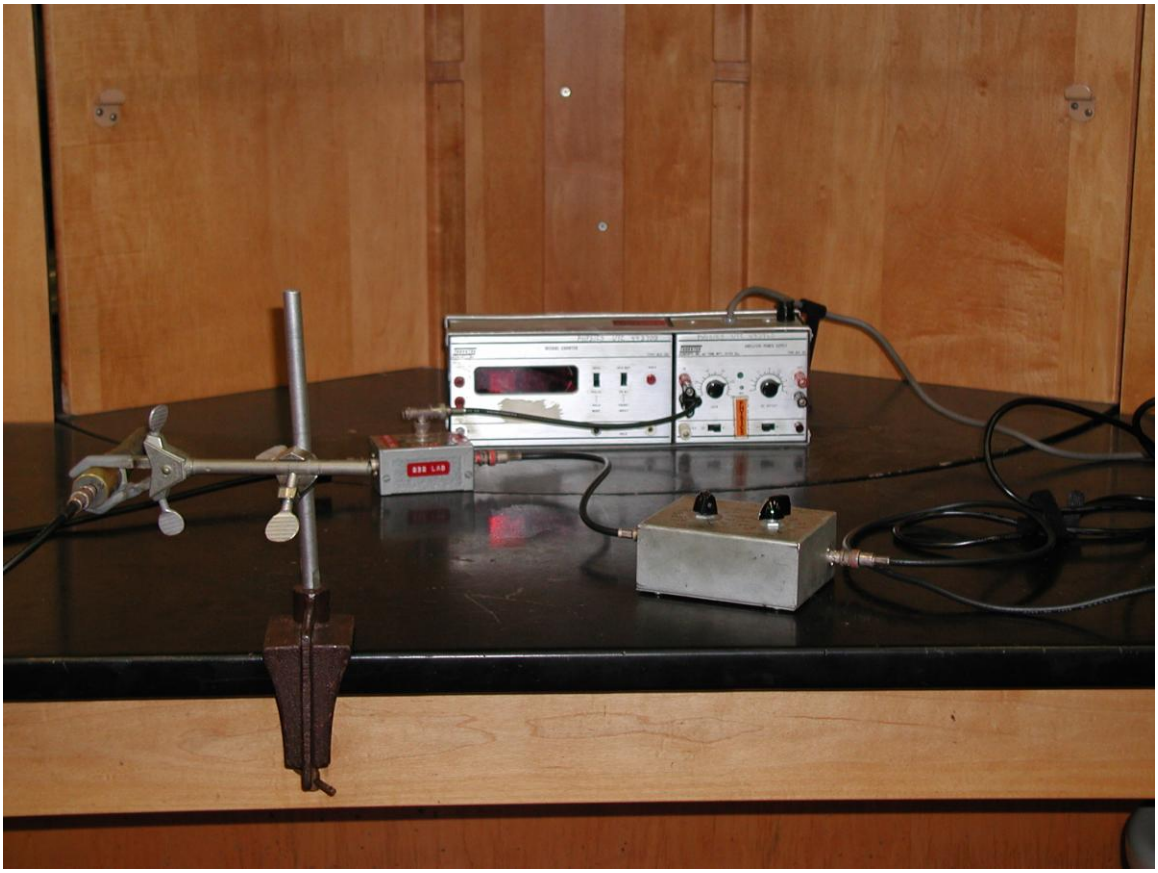
Figure 2. Decay scheme of ^{137}Cs to ^{137}Ba.

II. <u>Warning</u>

The primary radioactive source of ^{137}Cs is contained in a bottle and is not sealed as in other experiments. Cesium decays to Barium inside a source bottle. The Barium is then chemically extracted with a HCl and NaCl solution and stored in a separate container. DO NOT OPEN ANY CONTAINERS to avoid contamination. THE LAB INSTRUCTOR WILL HANDLE THE Ba

GENERATOR KIT and provide you with a sealed Ba sample. Because these are not permanently sealed samples, you must wear plastic gloves and aprons.

Unfortunately, the extraction process is not 100% efficient and your Ba sample will most likely contain some additional Cs nuclei. You will have to include the Cs decay as part of the background radiation count in your data analysis. In addition, the extraction process requires a sufficiently long period of time that it may be necessary to work with larger groups. You may have to wait some time for your sample. But you must be prepared to take data as soon as the sample is ready.



III. <u>DETERMINATION OF ^{137}Ba HALF-LIFE</u>

You will use the Geiger counter as in Experiment #4 in order to count the number of photons emitted from the Ba^{137} nucleus as it de-excites to the ground state. First, you must set up the Geiger tube and plateau the counter, using a $T\ell^{204}$ source. Next, determine the background count with no radioactive source nearby for 2 minutes, and divide by four. This is your background rate for a 30 second time interval. Be sure to determine the statistical error of your

background rate. Next, obtain a Ba^{137} sample from the lab instructor and IMMEDIATELY begin taking consecutive readings of the number of counts emitted from the sample every 30 seconds. You should take counts for at least 7 minutes, and continue recording the number of decays until the number becomes essentially constant for 4 consecutive time intervals. This is your actual background rate which may be different from the background rate which you measured with no sources present. Estimate the error in your actual background rate.

In a table, calculate the counts per time interval and the statistical error for each count, assuming a normal distribution. Then determine the corrected number of counts by subtracting the actual background rate. The error in the corrected number of counts should be calculated using the formula in Appendix A for the difference between two quantities.

Plot the number of decays per time interval on a semi-$\log_{10}$ plot. Include your calculated error bars; if you use regular graph paper, use the formula for error in logarithms given in Appendix B. Determine the mean life and half-life from your data and record them using the proper units. Estimate a range of lifetimes which could fit the data points and reports this as an estimate of the statistical error.

Q.1. Is your result for the half-life of ^{137}Ba consistent with the accepted value?

Q.2. Discuss any systematic errors which could affect your result.

PRLEAB QUESTIONS

Q.1 Show that Eqn. (4) satisfies Eqn. (1).

Q.2 How would Eqns. (9) and (10) change if we used $\log_e$ instead of $\log_{10}$?

Q.3 If 40% of an unstable substance remained after 10 hours, what is the half-life of that substance?

Q.4 What is the probability that a single nucleus which still remains in the sample from Q.3 will decay within the next 1 hour?

Q.5 There are two decay processes in a sample, a primary one with a half-life of 5 minutes and a residual one of 30 counts per 30 sec which is essentially constant over the lab period. The initial number of counts is 300. What is the apparent half life of the primary decay process if we fail to correct for the contamination?

BETA DECAY OF NUCLEI

The purpose of this experiment is to investigate the beta decay of nuclei by studying the beta particles emitted by $^{204}_{81}Tl$. The experiment consists of measuring the momentum distribution of the emitted betas using a simple magnetic spectrometer. Analysis of the data will indicate that beta decay involves three particles in the final state, and that the third particle – the electron neutrino – has essentially zero rest mass. In addition, analysis of the data will show the need for relativistic kinematics.

INTRODUCTION

I. THE NATURE OF THE BETA-DECAY PROCESS

Soon after Henri Becquerel's discovery of natural radioactivity in 1896, it was found that three different types of particles can be emitted in this process: alpha (ά) rays were found to be helium nuclei; beta (β) rays were found to be negative or positive electrons; and gamma (γ) rays were found to be electromagnetic radiation.

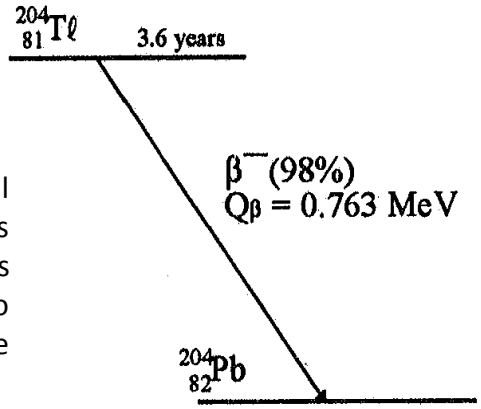


Figure 1. Energy-level diagram for the beta decay of $^{204}_{81}Tl$ to $^{204}_{82}Pb$

In beta decay, the charge of the nucleus changes by one unit, but the total number of nucleons does not change. Originally, it was though that the negative beta decay of a parent (P) nucleus to a daughter (D) nucleus consisted of the reaction

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + {}^{0}_{-1}e \qquad (1)$$

in which the upper indices denote nucleon number and the lower denote charge. A schematic representation of such a decay, the beta decay of $^{204}_{81}Tl$ to $^{204}_{82}Pb$, is shown in Fig. 1.

In a decay such as indicated in Eqn. (1), in which a single particle initially at rest decays into two particles in the final state, each of the two particles emerges with a unique value of energy and momentum. Thus, an experiment measuring the momentum of the beta (electrons) emitted in the process described by Eq. (1) should produce the results shown in Fig. 2. All betas should have the same momentum, except for a small smearing due to the finite precision of the experimental apparatus.

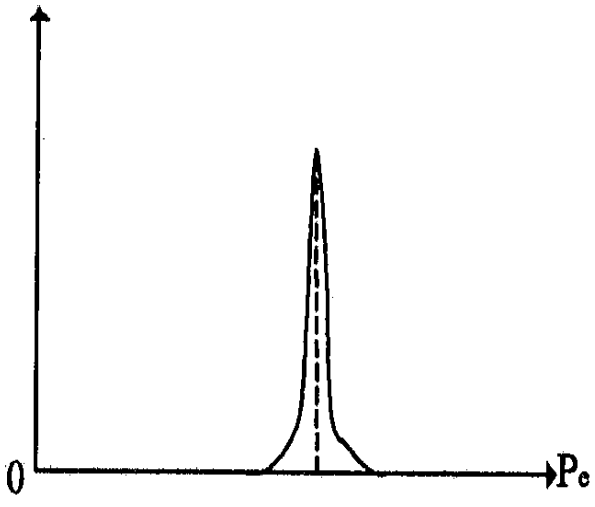


Figure 2. Beta momentum spectrum for a two-body final state.

However, when such experiments were performed, the distribution of beta momentum was found to be continuous as shown in Fig. 3. Instead of all emitted betas having the same momentum, it was discovered that the betas exhibited a continuous spectrum over a range of momentum from zero to a maximum value, $P_{e\,max}$, that was characteristic of the particular decay under investigation.

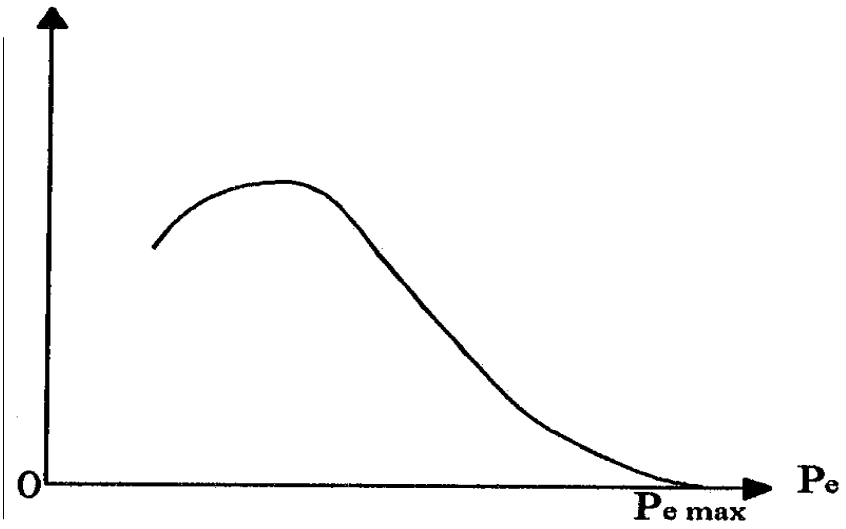


Figure 3. Experimentally determined electron momentum spectrum in beta decay.

It is now known that beta decay produces a three-body final state, and the negative beta decay consists of the reaction

$$ {}^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + {}^{0}_{-1}e + \nu_e. \qquad (2) $$

The third particle is known as neutrino. (The particle denoted by the symbol ν_e is called the electron anti-neutrino; the electron neutrino ν_e, is emitted in positive beta decay; other types of neutrinos are emitted in the weak decays of certain elementary particles.) The fact that beta decay was thought to be a two-body process resulted from the elusive nature of neutrinos: they have zero charge and zero or near zero rest mass, yet carry energy and momentum, and they interact so weakly with matter that they were not detected directly until 1953.

The continuous nature of the beta spectrum, as shown in Fig. 3, is thus understood in terms of the three-body final state. The total energy available for the decay is shared between the three particles, and all three together serve to conserve momentum. As a consequence, there is no unique solution for the momentum and energy of any of the three bodies; a range from zero up to some maximum value is possible, and the values for any particular decay depend upon the angles at which the three particles are emitted.

Three of the infinite numbers of possibilities are shown in Fig. 4.

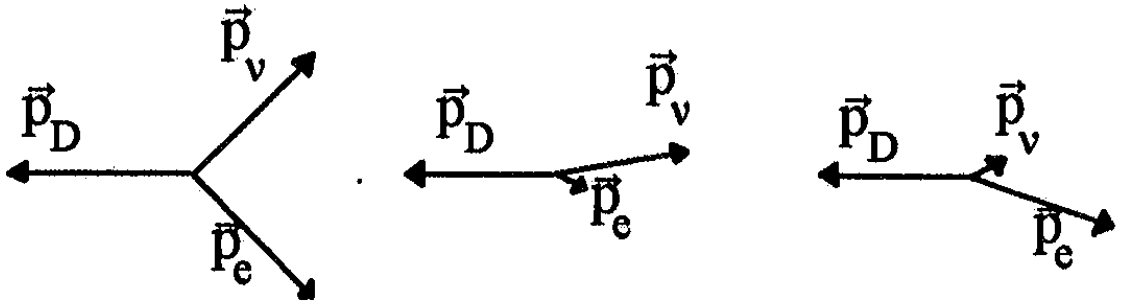


Figure 4. Three possible final states in beta decay

II. ENERGY CONSIDERATIONS: THE NEUTRINO MASS

If the neutrino is assumed to have a rest mass m_ν then application of total energy conservation to the beta-decay process of Eqn. (2) gives,

$$\{M_p c^2 - Z m_e c^2] = [M_D c^2 - (Z + 1)m_e c^2 + K_D] + [m_e c^2 + K_e] + [m_\nu c^2 + K_\nu]. \qquad (3)$$

Here, M_p and M_D are neutral atomic (not nuclear) rest masses of the parent and daughter, respectively, m_e is the electron rest mass, and K_D, K_e and K_ν are the daughter, electron and neutrino kinetic energies.

The total energy release, or Q-value, for this process is then,

$$Q = K_D + K_e + K_\nu = [M_p c^2 + m_\nu c^2]. \qquad (4)$$

Now, m_ν is known to be very small, at best, in comparison with M_p and M_D, and so Q is well-approximated by the difference, $M_p c^2 - M_D c^2$. Typically, this rest-mass energy difference is of the order of one MeV, which is shared between the three final-state particles. Since the rest-mass energy of the daughter is very much greater than 1 MeV, the daughter always moves with speeds very much smaller than c, even if the entire energy released were acquired by the daughter; thus, the daughter can be treated using non-relativistic relations. On the other hand, the rest-mass energies of the beta and the neutrino are not large compared with 1 MeV, and so they must be treated relativisitcally.

Furthermore, it is easy to show that, because of the large rest-mass energy of the daughter compared to both the electron rest-mass energy and the energy released, the energy acquired by the daughter is negligible. Consider a decay of the type shown in Fig. 4, where the decay energy is shared almost entirely between the daughter and the beta. Here, momentum conservation takes the form

$$O = P_D + P_e. \qquad (5)$$

or

$$P_D \approx P_e.$$

Combining this result with the relation

$$K_D = P_D^2/2M_D \quad (6)$$

for the daughter, and with

$$(K_e + m_e c^2)^2 = P_e^2 c^2 + (m_e c^2)^2 \quad (7)$$

for the beta, leads to the result

$$K_D \approx \frac{K_e(K_e + 2m_e c^2)}{2M_D c^2} \quad (8)$$

Since the largest possible value of K_e is Q (see Eqn. (4)), then

$$K_{d\,max} \frac{Q(Q + 2m_e c^2)}{2M_D c^2} << Q. \quad (9)$$

This result may be used to determine an upper limit for the neutrino rest mass. Since the daughter's kinetic energy is negligible, Eq. (4) takes the form

$$K_e + K_\nu \approx [M_p c^2 - (M_D c^2 + m_\nu c^2)] \quad (10)$$

Now, in decays of the type shown in Fig. 4(c), essentially all of the decay energy is carried away by the beta which has a momentum near $P_{e\,max}$ and has essentially its maximum kinetic energy, $K_{e\,max}$; the neutrino energy, K_ν, is essentially zero. Hence, Eq. (10) yields

$$m_\nu c^2 \leq (M_p c^2 - M_D c^2) - K_{e\,max} \quad (11)$$

Thus, in a given beta decay, a measurement of $K_{e\,max}$ serves to place an upper limit on the neutrino rest mass.

III. MEASURING THE BETA MOMENTUM SPECTRUM

The momentum of charged particles is usually determined by measuring their deflections in a known uniform magnetic field. The special device using this principle to determine the momentum spectrum of the electrons emitted in beta decay is called a beta-ray spectrometer, and its basic features are shown in Fig. 5. A beam of electrons from a beta

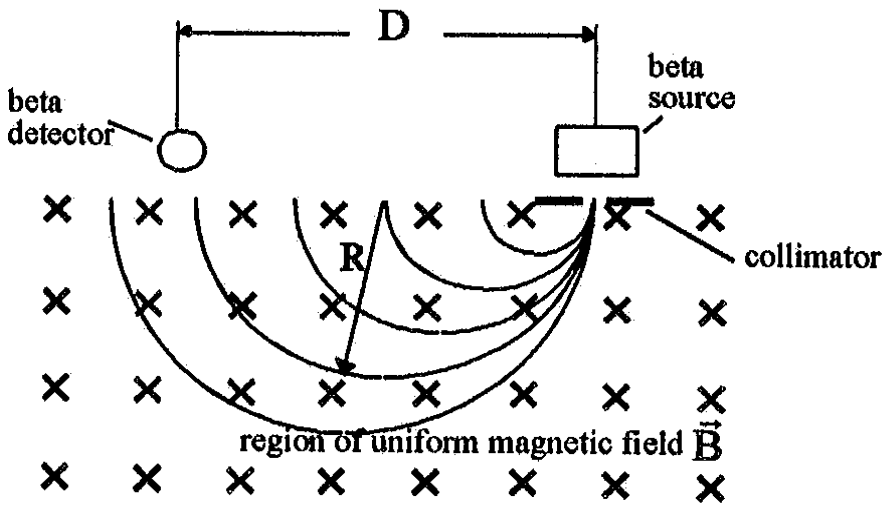


Figure 5. Beta-ray Spectrometer

source enters a region of uniform magnetic field. Electrons of different momenta follow circles of different radii, and the counting rate at various distances D = 2R is measured. This determines the momentum spectrum since, from Newton's second law and the Lorentz force law, it follows that

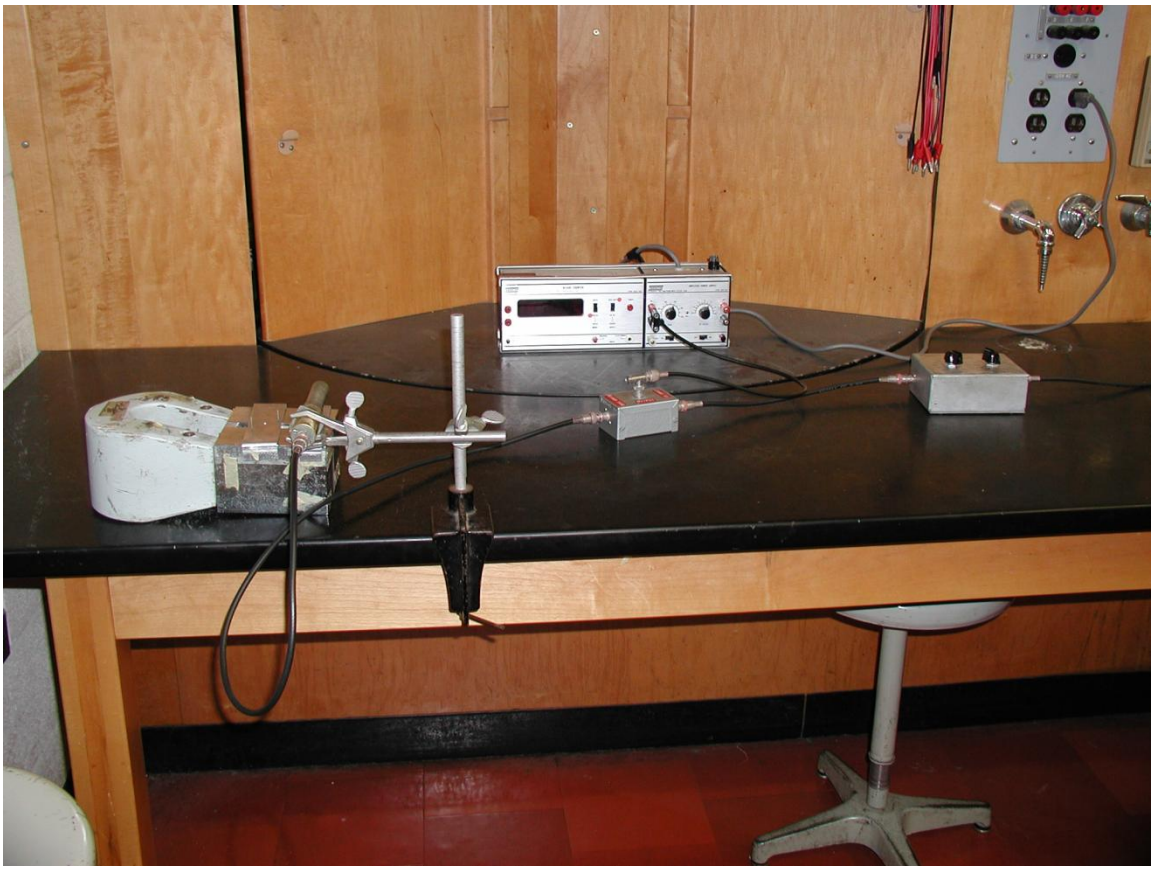
$$p_e = eBR, \qquad (12)$$

A result valid under relativistic, as well as non-relativistic, conditions. Thus, the data of counting rate as a function of distance is easily converted to the momentum spectrum, $\Delta N/\Delta p_e$ vs. p_e.

THE EXPERIMENT

I. WARNING

A very strong permanent magnet is used in this experiment. If you wear a watch, be sure to keep it far away from the magnet; otherwise, it may become magnetized and cease functioning properly. In addition, avoid disturbing the steel pole pieces – the rectangular blocks attached to the magnet, and used to provide a uniform field (see Fig. 6); if handled improperly they may snap shut violently, perhaps catching a finger between them. For similar reasons, keep ferromagnetic materials away from the magnet.



II. APPARATUS

The rudimentary beta-ray spectrometer used in this experiment is shown in Fig. 6. It consists of a strong permanent magnet, two pole pieces to provide a region of uniform field, a $^{204}_{81}Tl$ source mounted on a movable aluminum bracket containing a small hole which acts as a collimator, a Geiger-Muller-tube electron detector, and an aluminum bracket with a small slot which is located under the detector window in order to collimate the entering betas.

III. PROCEDURE

Check to see that the counting circuit is connected correctly, and that the counter controls are set properly. Check out a $^{204}_{81}Tl$ source. Place it 5 – 10cm from the Geiger-Muller tube, and measure and tabulate the counting rate as a function of high voltage. Plot the data, including error bars, and determine the proper operating voltage.

With the source removed, determine the background rate counting for 2 minutes. Calculate the background rate and its uncertainty per 30 sec.

Use a small compass (whose poles have been checked using the known North direction) to determine the polarity of your spectrometer magnet. Measure the strength of the B field between the pole pieces using a Gauss meter. Record this information. The field strength should be between 0.1 and 0.135 Tesla in order for the full momentum spectrum to be seen. Assemble the spectrometer and source as shown in Fig. 6. Determine the sign of the charge of the emitted beta particles. Record your information

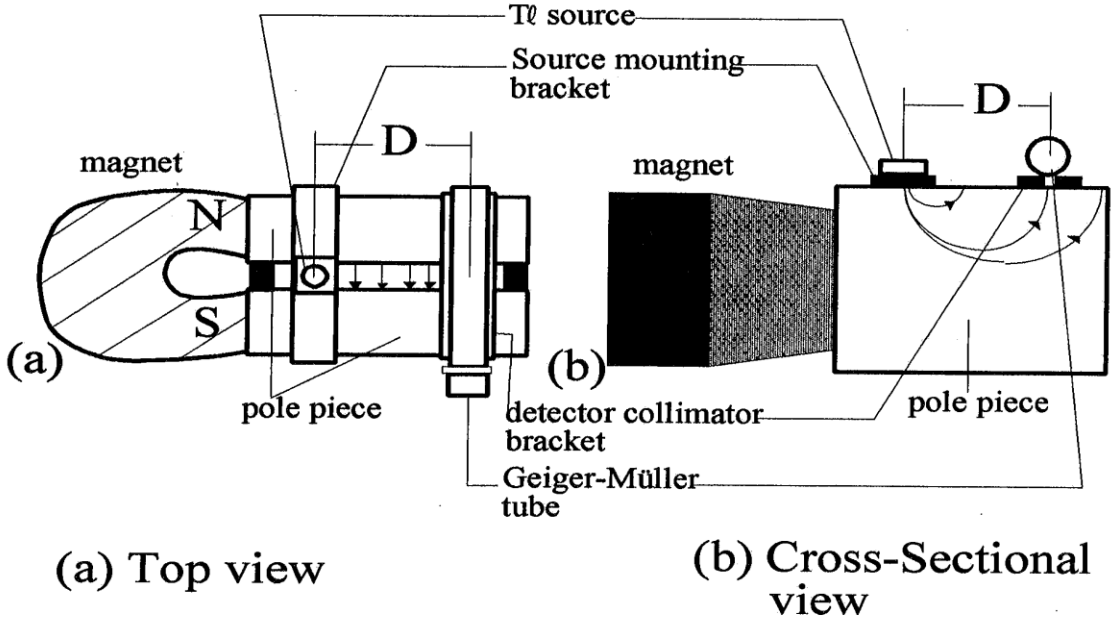


Figure 6. Experimental apparatus. Note: the orientation of the magnet may <u>not</u> be the same as in the sketch

IV. THE BETA MOMENTUM SPECTRUM

Use the spectrometer to determine the spectrum of the emitted betas. It is generally more convenient to leave the Geiger-Muller tube and its collimator fixed near the ends of the pole pieces, and to move the source bracket to vary the center-of-source to center-of-counter distance, D.

Begin counting at the smallest possible value of D. Measure and record the counts per 60 sec twice. Determine the spectrum by moving the source away in 0.5cm steps, and taking two 60-sec readings at each position. Continue data taking until the counting rate has fallen to the background level. SINCE THE END-POINT (i.e, MAXIMUM BETA MOMENTUM) OF THE SPECTRUM IS IMPORTANT, go back and take additional data at closely spaced D settings (say, 1mm or 2 mm apart) in the region just before and just after the counting rate diminished to the background value.

Record your data in a table containing the following columns: 1)D ; 2)R; 3) raw counts for each of the two trials; 4) background; 5) corrected counts for each of the two trials; 6) average corrected counts per 60 sec. Be sure to include the uncertainties of all quantities. Calculate the uncertainty in the corrected number of counts as in Experiment # 5. Don't forget to take into account the fact that you made two trials!

Plot a graph of the average corrected counting rate vs. R, include error bars. Since R is directly proportional to p_e (see Eqn. (12)), this graph has the same shape as the beta momentum spectrum, dN/dp_e vs. p_e. From your graph, determine the maximum value of the radius, R_{max}, which corresponds to the end-point of the beta spectrum. Estimate the uncertainty of this quantity.

Use the value of R_{max} to calculate the maximum beta momentum, $p_{e\,max}$ (Eqn.(12)). Include the uncertainty of this quantity. Using the value of $p_{e\,max}$, determine the maximum beta kinetic energy, $K_{e\,max}$, and its uncertainty (see Q.6)

V. THE UPPER-LIMIT OF THE NEUTRINO REST-MASS

Finally, use your data to set an upper limit on the rest-mass energy of the neutrino (see Eqn. (11)). Express your results as a multiple of the electron rest-mass energy.

This experiment involves the beta decay ${}^{204}_{81}Tl$ to ${}^{204}_{82}Pb$ via the reaction

$$ {}^{204}_{81}Tl \rightarrow {}^{204}_{82}Pb + {}^{0}_{-1}e + {}^{0}_{0}\nu_e $$

A table of rest masses and equivalent energies for particles related to this decay is included below.

Particle	Rest Mass (u)	Rest Energy (MeV)
${}^{0}_{1}e$	0.000549	0.511
${}^{1}_{0}n$	1.008665	939.550
${}^{1}_{1}p$	1.007276	938.256
${}^{204}_{81}Tl$ (atom)	203.97386	189997.166
${}^{204}_{82}Pb$ (atom)	203.973044	189996.403

PRELAB STUDY AND QUESTIONS

Q.1 Determine the energy released in the decay. Where does the energy come from?

Q.2 Derive Eqn. (4) from Eqn. (3). Where did the electron rest mass go?

Q.3 Verify the numerical calculation the statement that the ${}^{204}_{82}Pb$ daughter can be treated non-relativistically. Use the criterion that $\beta = v/c = pc/E_{tot} < .001$ is non-relativistic.

Q.4. Derive Eqn. (8), beginning from Eqn. (5) and including all required steps. What are the assumptions in Eqns. (5&6)?

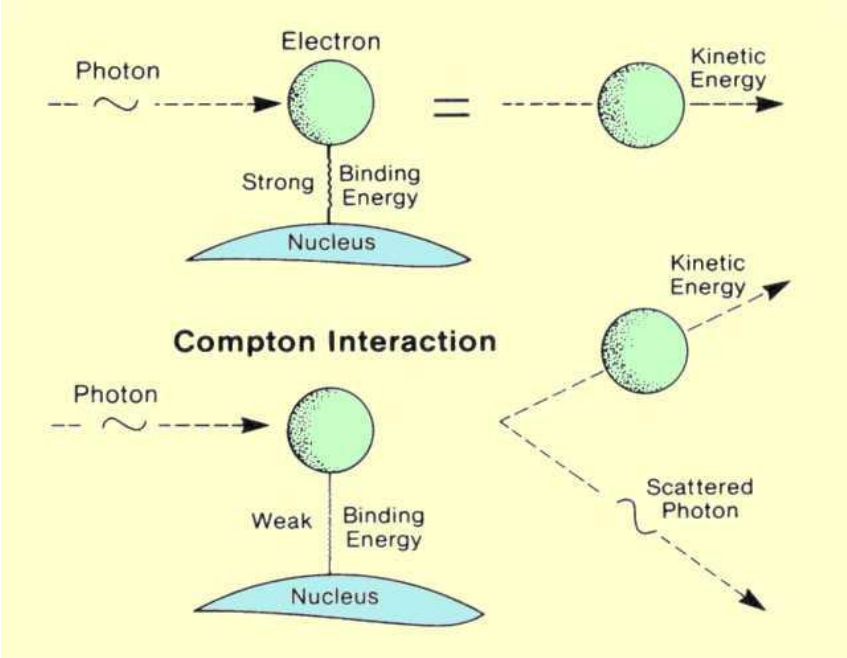
Q.5. Find the larges possible value for the kinetic energy of the ${}^{204}_{82}Pb$ nucleus, and verify that this quantity is negligible in comparison with the beta and neutrino kinetic energies.

Q.6 Determine an expression for the maximum beta kinetic energy, $k_{e\,max}$, in terms of the maximum momentum, $p_{e\,max}$, and the rest-mass energy, $m_e c^2$. Did you use the relativistic energy equation in your calculation?

Q.7. Starting from Newton's second law, the criterion for uniform circular motion and the Lorentz force law, derive Eqn. (12).

Q.8. Why will the device shown in Fig. 6 work for beta rays, but not for gamma rays (photons)?

INTERACTION OF GAMMA RADIATION WITH MATTER



The purpose of this experiment is to measure the attenuation of gamma rays in matter. The technique is to measure the number of gamma rays per unit time passing through an absorber as a function of the absorber thickness. The attenuation will be shown to decrease exponentially as a function of the thickness of the material. The rate of attenuation will be characterized by an absorption coefficient.

DISCUSSION

I. SOURCE OF GAMMA RAYS AND ISOLATION FROM OTHER RADIOACTIVITY

Nuclei which are unstable (or radioactive) have decay products which may include electrons and positrons, also called betas, gammas, and alpha particles. We have studied beta decay in experiment # 6. Alphas are actually Helium (He) nuclei. Gamma rays originate in a nucleus which is in an excited state. The excited nucleus will eventually decay to a lower level with the emission of a single photon which carries off exactly the energy difference between the two levels. One such example is shown in Fig. 1. Cs^{137} decays with the emission of an electron (β^-) to an excited state of Ba^{137}, 92% of the time with a half life of 27 years. The excited state of Ba^{137} decays down to the ground state with the emission of a gamma ray of energy 0.662 MeV with a half-life of 2.6 minutes (See experiment # 5). 8% of the time Cs^{137} decays directly to the ground state of Ba^{137} with the emission of a beta having K_E = 1.18 MeV.

In this experiment, we wish to study the gamma radiation resulting from the above decay and its ability to penetrate matter. However, there is a problem because the Geiger counter is not

able to distinguish between the beta and gamma rays which are emitted when Cs^{137} decays. Because photons have no charge, we cannot use magnetic or electric fields to deflect them, as with the beta ray spectrometer. However, as we will see in Experiment #8, the range of charged particles in matter is very small compared to that of gamma rays. If we insert a 2 mm thick iron plate in the path of the radiation, we can stop even the 1.18 MeV electrons completely, without affecting the number of gamma ray photons significantly. Therefore, after passing the radiation from the Cs^{137} through the 2 mm absorber we have almost a pure photon beam of energy 0.662 MeV.

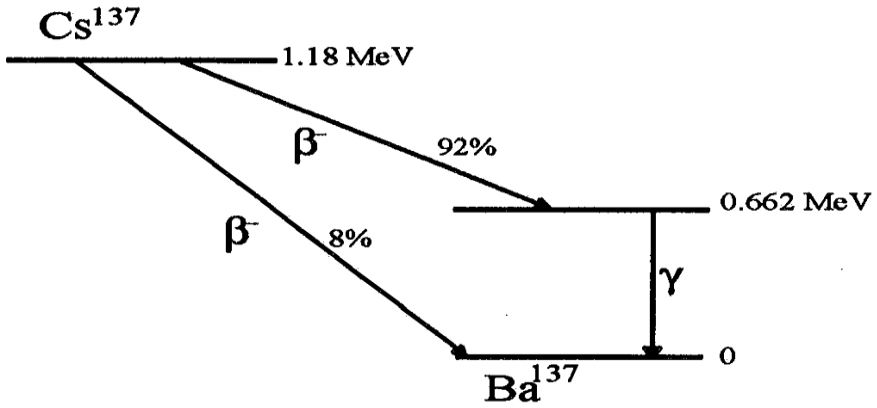


Figure 1. Energy levels and decay scheme of Cs^{137} to Ba^{137}

II. INTERACTION OF GAMMA RADIATION WITH MATTER

The main difference between the passage of charged particles through matter and the passage of uncharged particles through matter is that a charged particle is subject to a Coulomb force whenever it comes near another charged particle in the material. Gamma rays, on the other hand, interact only when they come in contact with something and during that interaction are either absorbed or scattered so that they are taken out of the incident beam. A gamma ray, therefore, either passes through the material intact or is lost, while a charged particles loses some energy with each encounter and comes through with a lower energy than it started. Indeed if the material is thick enough, the charged particle may energy would be degraded until it stops. The distance required to stop a charged particle is called the "range". Gamma rays do not exhibit such a range.

The interactions of gamma rays with matter are of three kinds:

1. Photoelectric Effect, where an electron absorbs the photon and, if the photon frequency is high enough, is ejected from the material with a kinetic energy $E_k = h\nu - \phi$, where ϕ is the work function of the material.

2. Compton Scattering, where a photon collides with another charged particle, usually an electron, and is scattered through an angle ϕ coming out with a lower energy and a new, larger wavelength; and

3. Pair Production, where a photon gives rise to (near a large atom), a pair of particles; an electron and a positron, appear.

The photoelectron effect predominates at low energies, 10 – 1000eV, Compton scattering is predominat from 1000 eV to 2 MeV, and pair production predominates above that energy. Regardless of the mechanism, the discussion given below is appropriate for predicting the behavior of a beam of photons as it passes through matter.If N_0 photons are sent into a material of thickness x then the decrease in the number of photons in any section of the material of thickness dx is proportional to the number of atoms it might encounter (which is directly proportional to the thickness) and to the number of photons in the beam, N. (Fig 1.) .

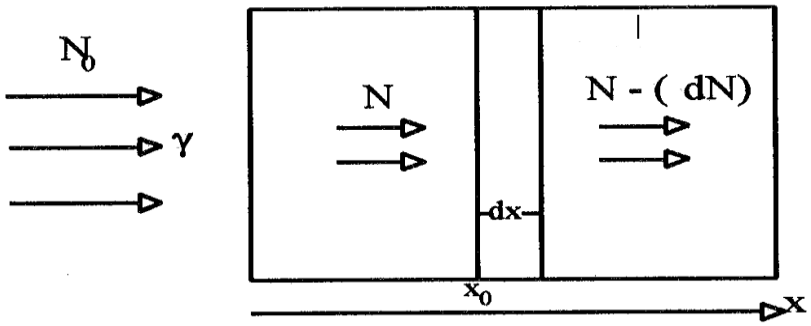


Figure 2. N_0 photons enter a material of thickness x. N photons in the material reach the distance x_0, and N – (dN) pass through to x_0 +dx.

$$dN = -\mu N\, dx \qquad (1)$$

where μ is the proportionality constant and N is the number entering dx, at x. The minus sign shows that there is a decrease in N.

If we now collect terms in Eqn. (1), it becomes

$$dN/N = -\mu dx \qquad (2)$$

Integrating Eqn. (2), we get ($N=N_0$ at $x=0$)

$$N = N_0 e^{-\mu x} \qquad (3)$$

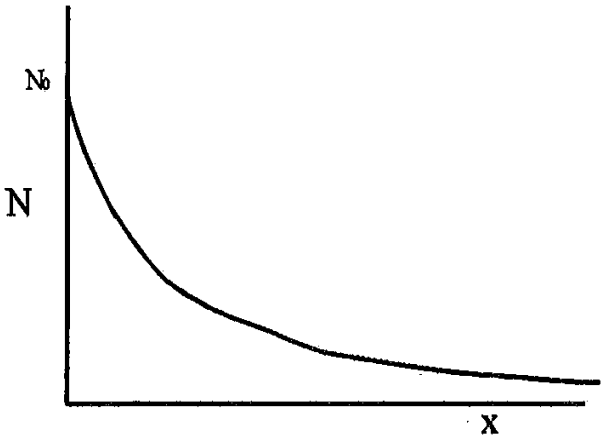


Figure 2a. Plot of N versus x for $N = N_0 e^{-\mu x}$

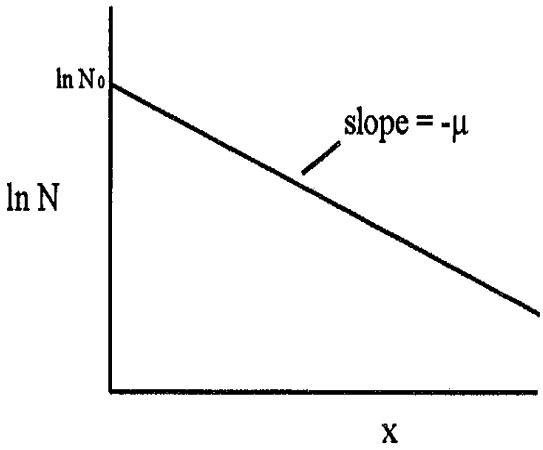
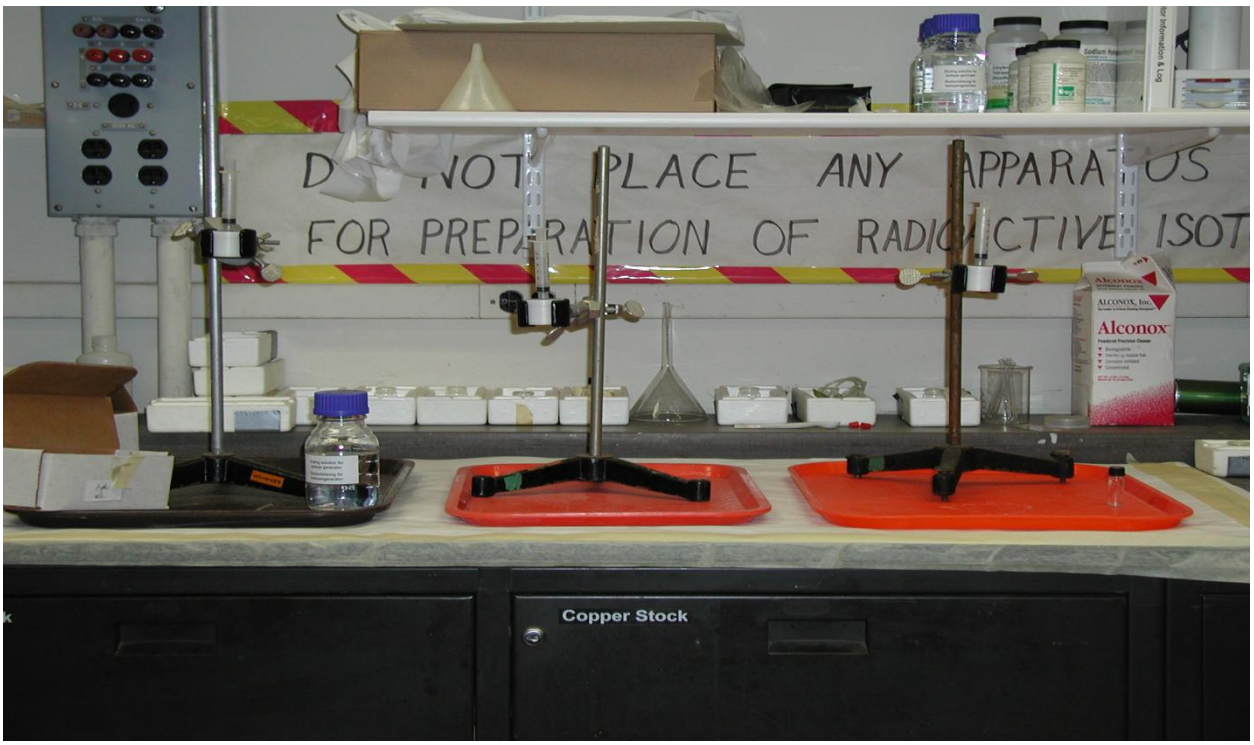


Figure 2b. Plot of N versus x

Fig 2a is a plot of N versus x as given in Eqn. (3). Fig. 2b is a plot of the ln N versus x. In Fig. 2b the curve is a straight line having slope –μ. Later we will plot the data as in Fig. 2b so that we may extract the coefficient μ directly. μ is called the absorption coefficient, and it depends on the material of the absorber.



THE EXPERIMENT

Using a Cs^{137} source, set up the scaling equipment and plateau of the Geiger-Müller tube as usual.

I. ABSORPTION MEASURMENTS IN STEEL

Mount the Geiger-Müller tube in a ring stand holder so that its center is about 2.5 inches above the table (See Fig. 4). With no source present measure the number of counts in a two-minute interval. This is a background measurement and m ust be subtracted from each subsequent measurement to get the true
rate.

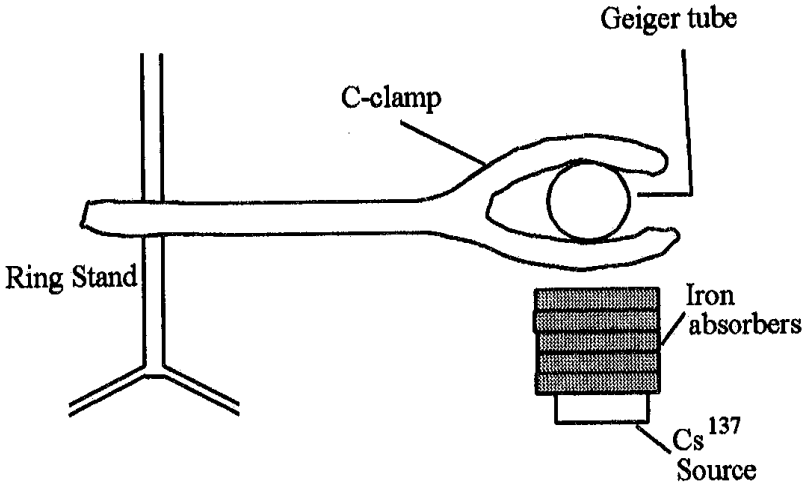


Figure 4. Experimental set-up for attenuation measurements

Next put the Cs^{137} source in place with one of the thin steel absorbers to stop the betas. In this condition we should have almost a pure photon beam coming towards the Geiger-Müller tube. Considering this to be zero thickness measure the rate for thickness 0, 1, 2 … absorbers up to a total thickness of about ~ 5 cm. A time interval of at least 30 seconds is suggested. However, for the sake of precision make sure that your number of counts for 0 thickness is 200 or above, and adjust your time interval accordingly. Make a table for recording the data with columns for the time interval used, thickness, the results of Trials 1 and 2, the background correction, and the corrected counts for trials 1 and 2 along with their uncertainties. Use the procedure described in Experiment #5 and #6 to determine the error in the corrected number of counts.

II. DETERMINATION OF THE ABSOPRTION COEFFICIENT

For each absorber thickness, average the corrected results of trials 1 and 2 and combine the uncertainties to get the uncertainty of the average. Give a sample calculation and tabulate the results for the thickness used. Remember that with the first absorber the thickness is =0.

Plot on semi-$\log_{10}$ paper, N versus x. Include the error bars. Draw a straight line which best approximates the data taking into account the error bars and determine the absorption coefficient. Make an estimate of the error of your value of determining an upper and lower limit for μ from your graph.

From your value of μ, determine the thickness of iron required to stop half of the incoming photons. Is this value reasonable given the number of iron plates you used? Based on the value, can you justify our assumption that the 2mm thick iron plate only slightly degraded the photon beam?

PRELAB QUESTIONS

Q.1 What is gamma decay?

Q.2 Which of the three possible processes will the photons from Cs^{137} decay undergo when they interact with the iron?

Q.3 Why is the interaction of gamma rays with matter fundamentally different from that of alpha rays and beta rays?

Q.4 How is this difference seen in practice?

THE INTERACTION OF ELECTRONS WITH MATTER

Aurora Borealis: Interaction of electrons with Oxygen and molecular Nitrogen

The purpose of this experiment is to measure the attenuation of electrons in aluminum. The technique is to record the number of electrons passing through the aluminum as a function of its thickness. These results should be contrasted with those obtained for the interaction of gamma rays with matter. The "range" of the electrons will be measured for two different incident energies.

DISCUSSION

I. THE RANGE OF ELECTRONS MATTER

A charged particle moving through matter loses energy by electromagnetic interactions which raise individual electrons in the matter to excited energy states. In some cases an electron may receive enough energy to separate it from the parent atom and the atom becomes ionized. As the charged particle proceeds through the material it makes many such collisions and steadily loses energy. The magnitude of the force between the charged particle having charge q and the electron in the atom having charge e is proportional to $\mathbf{qe/r^2}$. The distance between the charges, **r**, changes as a charged particle flies by. The energy transferred to the electron is the net work done on the electron by that changing force. A calculation of that work (not done here) shows that the amount of energy transferred per unit distance is proportional to $\mathbf{q^2e^2}$ and inversely proportional to $\mathbf{MV^2}$ where **M** is the mass, charge **q** and **V** is its velocity. The effect of this is that an alpha particle (**q** = 2) will lose four times the energy in a given material than will a proton of the same kinetic energy. It does not matter whether **q** is positive or negative, the energy transferred to the electrons is about the same for both. At very low energies the transfer of energy per collision is greater than it is at higher energies because it spends more

time near the electron. Obviously, the denser the material the more energy will be lost per unit thickness, because there are more electrons with which to collide.

In the case of larger charged particles, such as protons and alpha particles (Helium nuclei), this mechanism is predominant. Particles of the same energy pass through the material in an essentially straight line, losing energy in small increments, until all the particles are stopped at approximately the same thickness.

The process by which electrons are stopped as they travel through matter is relatively more complicated than that undergone either by alpha radiation or gamma radiation. The coulomb interactions that the electrons undergo as they pass through the matter means that they are continually losing energy over the path that they travel, as is the case for alpha particles. However, the electrons have much smaller mass (compared to alpha particles or protons) and so they are easily scattered through large angles by collisions with other electrons and the atomic nuclei in the matter. The ease with which the electrons are scattered and change direction means that the path they take is not likely to be straight. Therefore, like the gamma radiation in Experiment #7, electrons of a single initial energy will be removed from the beam as a function of the thickness of the material, since chances of a "catastrophic" scattering event increase with thickness. The thickness of a given material at which all of the electrons will get removed from the beam is called the "range". The range corresponds to the maximum straight line distance that an average electron can travel in the material, and therefore can be estimated by considering only the coulomb interactions. (See question 2 in the Prelab Questions)

II. SOURCE OF ELECTRONS

It is possible to get electrons with energies of about one MeV from the beta decay of radioactive nuclei. The electrons from beta decays are not monoenergetic, but have a spectrum of momenta ranging from zero up to some maximum value p_{max}. Experiment #6 dealt with this momentum spectrum for the betas emitted from a Tl^{204} source. We will use a magnet to bend the betas and some heavy absorbers to collimate them, thereby selecting a small range of momenta with which to make our measurements.

The source we will use is Tl^{204} which has electrons with a maximum kinetic energy equal to 0.764 MeV. Within the magnet the electrons bend in a circle of radius R given by

$$R = \frac{P}{qB} \qquad (1)$$

Where P is the momentum, and B is the magnetic field which is assumed to be uniform. It turns out that Eqn. (1) also holds for relativistic particles.

THE EXPERIMENT

Run a curve of the number of counts per unit time as a function of the detector voltage using the Tl^{204} source placed about 5~10cm from the Geiger tube (at the location where you obtain ~200 counts in 20 secs at ~900 volts) Set the voltage to be in the middle of the plateau region. Measure the field B in between the pole pieces of the beta-ray spectrometer. It is essential that the magnetic field strength be no larger than 0.14 Tesla, and no less than 0.095Tesla. The experiment is sensitive to the background levels. We will discuss how to properly account for them below.

I. MEASUREMENT OF THE ATTENUATION OF ELECTRONS IN AlUMINIUM(Al)

Fig. 1, is a sketch of the experimental set-up. Note that the source is placed face down and that the electrons bend in a semi-circle and exit through the slot in the aluminum collimating plate and enter the Geiger tube above.

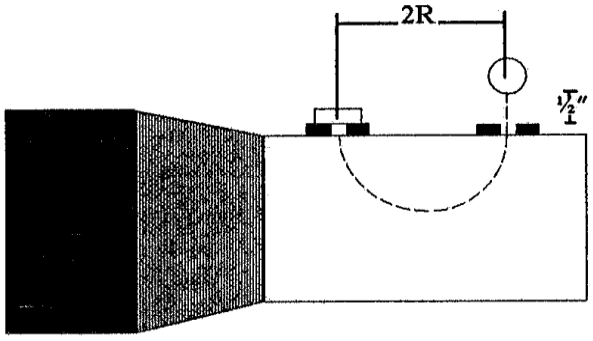


Figure 1. Experimental Set-up

The aluminum foil which will serve as our target material will be placed over the hole in the collimator bracket below the Geiger tube. The beta-ray spectrometer allows one to select the momentum, and therefore the energy, of the betas which will be directed into the aluminum target. You will measure the number of counts at the same measurement position for various thicknesses of aluminum foil in order to determine the range of the betas at that energy.

In Experiment #6, we determined that the betas with the highest energy of 0.764 MeV will perform a semi-circle in the spectrometer with the largest radius, which we called R_{max}. At that radius, however, the number of counts obtained would be just slightly above the background count. Since the relative accuracy of our measurements varies with the inverse square root of the number of counts, we should take the measurements at values of R which select betas with energy near the peak of the spectrum (See Exp. 6, Fig.). By using $R= 1/2R_{max}$ and $R=3/4R_{max}$ you will be selecting betas with energies just below and just above the energy of the peak. Your

initial number of counts should be approximately equal at these two positions, but the difference in energy of betas is large enough to see significantly different behavior in terms of their "range".

To locate these two positions, you must first determine the value of R_{max} for your experimental set-up. Measure the magnetic field in between the pole pieces of your magnet using the gaussmeter. Calculate the value of R for a magnetic field of this strength, assuming that the betas have an energy equal to the maximum energy of 0 .764 MeV. This is your expected value of R_{max}. Place your Tl^{204} source at a distance greater than twice this value and move it in slowly until you begin to obtain counts above your background rate. Use the measured distance as your value for $D_{max} = 2R_{max.}$

Since your Geiger tube must be elevated above the collimator bracket, there is a chance that stray electrons may enter the tube without having passed through the magnetic field. To account for this possibility, you should measure the background rate at each of the measurement positions, separately. After you have set-up the experiment at the R = ¾ R_{max} and ½ R_{max}positions, respectively, and before you begin to take measurements, place a thick slab of aluminum over the hole in the collimator bracket and record the number of counts over a 2 minute period. This background rate should account for any additional counts due to the closeness of the source to the tube. All counts obtained for the number of betas passing through the Al foil should be corrected for the background rate at that measurement position.

The absorber to be used is household Al foil about 0.02 mm thick. By folding the foil you can make up absorbers of various thickness. The thickness you will use depends a lot on the experimental set-up, but you might start working with sheets folded 4 times. At each of the measurements locations you have chosen, measure the rate in 60 seconds as a function of absorber thickness from zero up to the thickness at which the electrons are completely stopped. Record the measurement in tabular form. Make sure that you take into account the error of the background in determining the error in the corrected number of counts. Be sure that your beginning reading is the direct reading with no absorber.

For the two sets of data plot the counting rate versus Al thickness, remembering to include error bars. Estimate the apparent stopping thickness (range) and the uncertainty in that quantity from you graph. The accepted method for obtaining the range is to ignore the long tail on your graph and extrapolate the curve from the initial and middle sections of the graph until it intersects the zero line for corrected counts. Calculate the momentum and the kinetic energy (in MeV) for each of the values of R used, using your measured value of the B-field strength, and indicated the energy on each curve.

Q.1 If the electrons all followed the same straight vertical path through the absorber, what curve of counts vs. thickness would be expected? (Illustrate)

Q.2 Why do not all the electrons stop at exactly the same thickness as alphas do? What contributes to the spread?

Q.3 Compare your results for the two different momenta used. Does the difference in range seem reasonable? Explain.

PRELAB QUESTIONS

Q.1 Calculate the quantity pc for an electron with kinetic energy of 0.764 MeV.

Q.2 Assuming that the incoming electrons have low enough momentum so that ½ MV^2 is a reasonable estimate of the initial kinetic energy, determine the stopping distance for electrons as a function of the initial energy. (Hint: proceed as in the discussion for Experiment #7 on gamma rays. Instead of considering the incremental loss in the number of particles per thickness dx, set up an equation for the incremental loss of energy, dE, per dx. The integral is not difficult.)

APPENDIX A

EXPERIMENTAL ERROR

Every measurement is subject to errors. In the simple case of measuring the distance between two points by means of a meter rod, a number of measurements usually give different results, especially if the distance is several meters long and the measurements are made to fractions of a millimeter. The errors here arise from inaccuracy in setting, inaccuracy in estimating the fraction of a division, parallax in reading, faults in the meter rod, expansion due to change in temperature, and so on. Blunders in putting down a wrong reading or in not adding correctly are not classed as errors. Errors may be grouped into two general types:

(a) Random experimental errors are always present in any series of measurements, because no measurement can be carried out in exactly the same way twice. Repetition of any measurement gives a number of different values. These values tend to cluster about the mean value of the group, with small deviations from the mean occurring more frequently than large deviations.

(b) Systematic errors are often called constant errors to indicate their tendency to be of one value. They commonly arise from incorrectly calibrated measuring equipment (e.g., a meter stick that shrank 1%), from consistently improper use of equipment (reading the scale of an oscilloscope as "ms" instead of "μs"), from misidentifying the measured quantity (measuring a diameter and failing to convert it to a radius called for by the theoretical expression), etc.

In general, all measurements will include errors of both the random and systematic types, with the values tending to cluster about a mean value displaced somewhat from the so-called "true" value. In practical measurements the "true" or "correct" value is not known, although there may exist an "accepted" value by general agreement among experts. Further, in using a given piece of equipment for which the systematic errors may be unknown, the values obtained may deviate consistently from the accepted value. Therefore, without performing an error analysis, we can make no claims regarding the accuracy of our results.

SIGNIFICANT FIGURES

The presence of error means that we cannot know a measured quantity exactly. Rather, we are able to measure a quantity only approximately. Therefore, not all figures in a measured quantity are significant. When recording measurements, the student must be sure to record **only** significant figures, and **all** significant figures. For instance, if a distance has been measured to hundredths of a centimeter and found to be 50.00 cm, it is not correct to put the distance down as 50 cm, for the statement that the distance is 50 cm, implies that the distance has been only roughly measured and is found to be more nearly equal to 50 cm, than it is to 49 or 51 cm, whereas the statement that the distance is 50.00 cm implies that the distance lies between 49.99 and 50.01 cm.

On the other hand, if the precision of an instrument is very high, but the ability of the experimenter to use it is restricted, one should not overestimate the number of significant figures. For instance, if a digital ammeter reads current to 0.001μA, but the reading on the meter is fluctuating between 0.515μA and 0.522μA, then the number of significant figures is only two, and the quantity should be recorded as 0.52μA.

The number of significant figures in a quantity is irrespective of the decimal point; thus there is one significant figure in 0.01, two in 0.000026, and three in 2.10. A convenient method of writing a number so as to show its precision is to place the point after the first significant figure and multiply by an appropriate positive of negative power of ten. If a mass, for instance, is about 25,000 gm and the third figure is the last in which any confidence can be placed, this fact is indicated by saying that the mass is 2.50×10^4 gm.

UNCERTAINTIES

The last significant figure in a measured quantity has some uncertainty. The absolute uncertainty in a measurement is, in general, given by the smallest division which may be read directly on an instrument. We express the uncertainty in the same units as used in the measurement. Thus, the absolute uncertainty in measuring with an ordinary meter stick is 0.1 cm or 1 mm. However, this uncertainty may be much larger in fact, if it is not possible to make a reading to the precision of the instrument, as in the example given in the previous section. The fluctuating current in that example had an uncertainty of 0.007μA, although the smallest division which may be read is 0.001μA. A stopwatch may be able to read to 0.01s, but human reaction time is about 0.1s, therefore, the uncertainty of a measured time is better estimated to be 0.1s.

The relative uncertainty is obtained by dividing the absolute uncertainty by the actual value of the measurement, in the same units. The percent uncertainty is 100 times the relative uncertainty. In most cases the relative uncertainty is more important than the absolute uncertainty. If we measure the length of a rod with a meter stick and find it to be 241 mm, we usually regard an error of 1 mm as less important than we do if we measure the diameter of the rod to be 4 mm with an error of 1 mm. In the first case the error is 0.4%; in the second case it is 25%. Relative or percent errors are important also because they may be used to find the error in an answer determined by multiplying or dividing several measurements. Quantities to be added or subtracted should be measured to the same decimal value regardless of size, and their absolute uncertainties added. Any measured quantity must be written with the proper number of significant figures and with an estimate of the uncertainty of the measurement. For example:

Absolute error: $m = 1.3 \pm 0.1$ kg; $d = 245 \pm 4$ mm, etc., or

Percent error: m = 1.3 $\pm$ 7.7%; d = 245 $\pm$ 1.6%.

CALCULATION OF THE STANDARD DEVIATION (σ)

In many cases, the error in a measured quantity can be reduced by measuring the same quantity several times and taking an average of the results. Assuming that the errors in the measurements are random, the average or mean value will be a better estimate of the "true" value. The standard deviation is an estimate of the error in the average or mean value. The procedure is as follows:

1. Take the average of all the values obtained from a series of measurements. This is x (the "best value" for the quantity)
2. Subtract each measurement from x. These are $x - x_i$.
3. Square each of the items found in (2).
4. Add all the items in (3). (There will be n of them)
5. Divide (4) by the total number of measurements n minus one, i.e., n – 1
6. Take the square root of (5). This is σ, the standard deviation.
7. Write the result in the form x $\pm\sigma$

   In short, $\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x-x_i)^2}{n-1}}$ and $x = x \pm \sigma$

   The standard deviation is an absolute error. The result is interpreted as follows:
   There is a 68% probability that the next measured value of x will fall in the range from x + σ to x –σ and a 32% probability that it will fall outside this range.

PROPAGATION OF ERRORS

Error in the measured quantities means that there is an uncertainty in any quantity which is calculated using the measured values. There is more than one approach to determining how error propagates in computations. However, the following rules are standard in Physics experiments:

Addition and Subtraction

Add the uncertainties. e.g.,
(12.24 $\pm$ 0.02) gm + (5.07 $\pm$ 0.03) gm = (17.31 $\pm$0.05) gm, and
(11.37 $\pm$ 0.03) cm – (0.00 $\pm$ 0.02) cm = (11.37 $\pm$ 0.05) cm

Multiplication

Find the percent uncertainty of each factor and take the square root of the sum of the squares of the percentages. e.g.,

(4.20 $\pm$0.03) cm x (7.35 $\pm$ 0.04) cm

$\frac{.03}{4.20} = 0.0071$; $\frac{.04}{7.35} = 0.0054$.

$\sqrt{(0.71\%)^2 + (0.54\%)^2}$ = 0.89% uncertainty in answer

4.20 cm x 7.35 cm = 30.87 cm^2

0.89% of 30.87 cm^2 = 0.27 cm^2

Final answer: (30.87 $\pm$0.27) cm^2

Division

Find the percent uncertainty of dividend and of divisor and take the square root of the sum of the square of the percentages. e.g.,

(20.4 $\pm$ 0.3) ft / (4.6 $\pm$ 0.1) sec

$\frac{.3}{20.4}$ = 0.015; $\frac{0.1}{4.6}$ = 0.022

$\sqrt{(1.5)^2 + (2.2)^2}$ = 2.7% uncertainty in answer

20.4 ft/ 4.6 sec = 4.43 ft/sec

2.7% of 4.43 ft/sec = 0.12 ft/sec

Final answer: (4.43 $\pm$ 0.12) ft/sec

Powers of Single Values

Find the percent uncertainty. e.g.,

$$(2.21 \pm 0.02)^3$$

$$\frac{.02}{2.21} \times 100 = 0.9\%$$

Multiply the percent uncertainty by the exponent of 3

0.9% x 3 = 2.7% uncertainty in answer

2.7% of $(2.21)^3$ = $\frac{10.79\ x\ 2.7}{100}$ = 0.29

Final answer: 10.79 ± 0.29

Effect of Multiplying a Physical Quantity by an Exact Number

Since the uncertainty of the number zero, the percent uncertainty of the physical quantity is applied to the product. e.g.,

20 x (40.0 $\pm$0.2) lbs.

$$\frac{.2}{40.0} \times 100 = 0.5\%$$

20 x 40.0 lbs = 800 lbs

0.5% of 800 lbs = 4 lbs

Final Answer: (800 $\pm$4) lbs.

Trigonometric Functions

Look up the value at one end of the angular range to determine the range of the function. e.g.,

$\theta = 33.2 \pm 0.4^{o}$

$\tan 33.6^{o} = 0.664$; $\tan 33.2^{o} = 0.654$

$\Delta\tan\theta = 0.010$

Final answer: $\tan (33.2 \pm 0.4^{o}) = 0.654 \pm 0.010$

Logarithmic Functions

The relative error of N is the absolute error of the function. e.g.,

$N = 100 \pm 5$

$\frac{5}{100} = 0.05$

Final answer: $\text{Log}_{10}(100 \pm 5) = 2 \pm 0.05$

APPENDIX B

HANDLING OF RADIOACTIVE SOURCES

Several laboratory experiments will involve the use of radioactive sources. Handled in accordance with instructions and with the use of common sense, they are not dangerous. However, careless and irresponsible behavior may result in serious consequences. In using radioactive materials, be sure to comply with the following guidelines:

- Always sign the checkout sheet when obtaining a radioactive source to be used in an experiment
- Be sure to replace the source and to cross off your name on the checkout sheet at the end of the laboratory period.
- Never remove a source from the laboratory
- Never eat, drink or smoke in the laboratory when radioactive source are in use.
- Never bring a source close to your eyes, since corneal damage may result.
- Take extreme care not to damage the thin aluminum foil covering the radioactive material in the source disk. Scratching, bumping or puncturing the foil, or placing tape directly on it, may result in the escape of radioactive material. Never touch the foil with your finger or any other object.
- The intensity of a radioactive source is generally characterized by the number of decays occurring per second. The conventional unit of intensity is the activity of 1 gram of radium, which is 3.7 x 10^{10} decays per second. The unit is called the curie, and is abbreviated Ci. The maximum activities of the sources employed in our experiments that may be used without special government license are 50μCi of $^{204}_{81}Tl$ and 10μCi of $^{137}_{55}Cs$. Hence, the maximum activities of 60μCi - - about 2.2 x 10^{6} disintegrations per second - - may be encountered in the laboratory.

In assessing the health and safety hazard of a specific level of radiation, the important consideration is the amount of ionization produced in body tissue, since it is ionization that results in tissue damage. Biological damage due to radiation is usually measured in units called rem (roentgen equivalent, man). One rem is defined as the amount of radiation which produces the same biological damage in man as does gamma radiation depositing 97 ergs of energy in

one gram of body tissue. The Nuclear Regulatory Commission has recommended an upper limit of 1.25 rem per calendar quarter.

Sensible handling of the radioactive sources in the laboratory in accordance with the guidelines described above should result in a radiation exposure which is, at most, only a small fraction of the limit recommended by the Commission. The most practical way to minimize exposure in the lab is by handling the source only when absolutely necessary, and for as short a time as practicable.

A table indicating the specific exposure (i.e., the dose per unit of radioactive material, per unit time) at different distances from a source follows.

DOSES FROM ISOTOPES COMMONLY USED IN ISOTOPE LABORATORIES IN PHYSICS

R-FACTORS: DOSES AT 1.0 cm

Tl^{204} : 2.4 R/H – mCi @ 1.0 cm

Cs^{137} : 3.2 R/H – mCi @ 1.0 cm

Co^{60} : 13.2 R/H – mCi @ 1.0 cm

TABLE OF DOSES AT VARIOUS DISTANCES

DOSE (R/H – mCi)			
Distance	Cs^{137}	Tl^{204}	Co^{60}
1 mm	320	240	1320
1 cm	3.2	2.4	13.2
25 cm	0.005	0.004	0.0050
50 cm	0.00125	0.0010	0.0050
1 m	3.0×10^{-4}	2.5×10^{-4}	1.25×10^{-3}
2 m	7.5×10^{-5}	6.25×10^{-5}	3.12×10^{-4}
4 m	1.88×10^{-5}	1.56×10^{-5}	7.8×10^{-5}

APPENDIX C

USE OF THE OPTICAL SPECTROMETER

The spectrometer that you will be using consists of three main parts (see figure 2):

- **The collimator**: The collimator has an adjustable opening at one end and a lens at the other. The adjustable opening is put near a light source (incandescent, mercury vapor, hydrogen vapor, etc.) and the collimator passes the light as a narrow beam through the vertical slit.
- **The center stand:** The center stand is to put the device, e.g., a diffraction grating, which is going to interact with the light. It has three screws on the bottom to adjust its height and to level it off. The light comes from the collimator and passes over the center stand.
- **The telescope:** The telescope is mounted on a rotating base and has a lens at one end and an eyepiece with crosshairs at the other end. The eyepiece can move in and out for focusing. The base of the telescope sits on a scale that measures angles in degrees. It has half-degree increments, and is equipped with a vernier-type scale to read the angle to 1/60 of a degree (minutes).

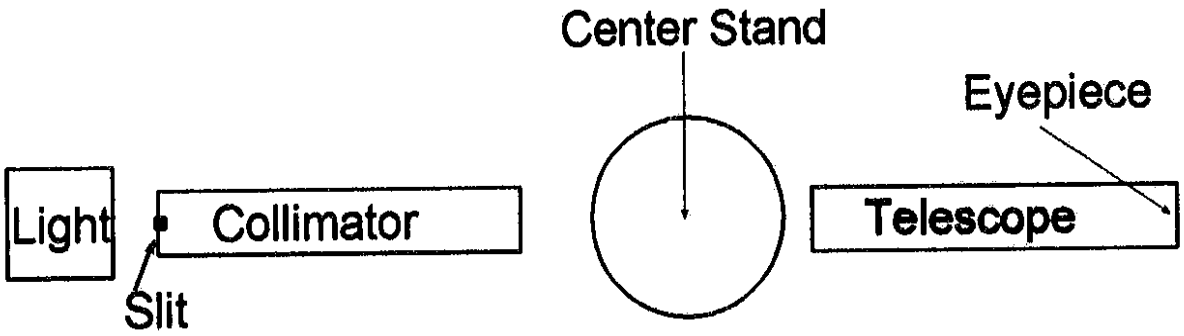


Figure 1. Spectrometer

The spectrometer should be set so that the collimator slit is in the center of whatever light source is being studied. With nothing on the center stand, look through the eyepiece at the light coming from the collimator. Make sure that the light source is bright and clearly visible and is in the center of the eyepiece. Check to see that the center stand is level and that the light will pass through the center of whatever object (diffraction grating, prism, etc.) is being studied,

Taking data with the spectrometer involves rotating the telescope until the light of interest is visible in the eyepiece and then placing the crosshairs in the center of that band or line and then reading that angle on the base. Outside light may interfere with the light being studied and make it impossible to read anything with accuracy. It is usually a good idea to turn off all

other lights, center the crosshairs in the desired spot, and then turn on a small light to read the angle. At all times, it is necessary to consider what effect the light coming from your experimental set up will have on other students' labs.

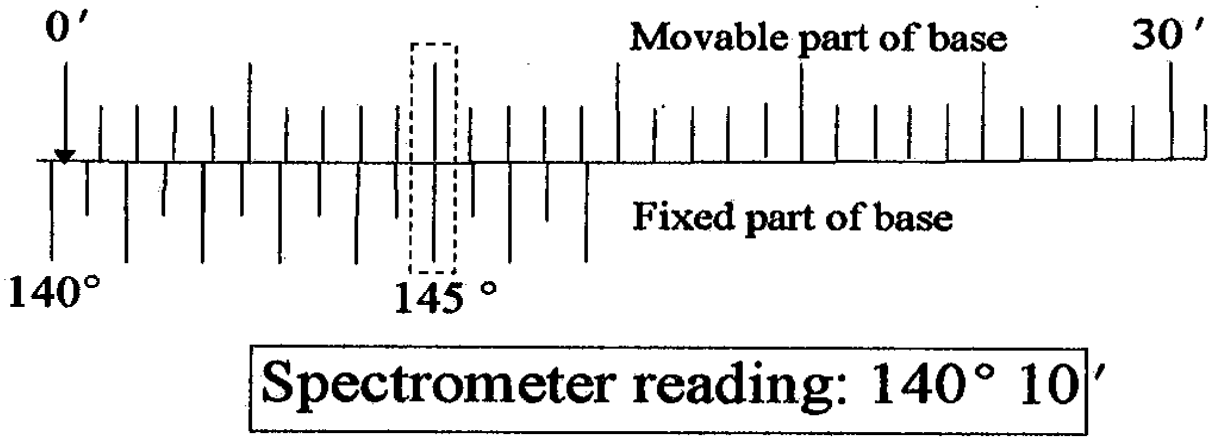


Figure 2. Vernier scale at the base of the spectrometer

Reading the angle on the spectrometer requires two steps. Looking at the base of the spectrometer, (see Figure 3) you will see that there is a scale going from 60 – 315 degrees, measured in half-degree increments. Attached to the bottom of the telescope is a smaller scale that ranges from 0 – 30. After you have centered the crosshairs in the desired location, note where the '0' mark on the smaller 0'-30' scale is. It will either be between a whole degree and a half-degree mark, in which case you record the angle as X degrees, or between the half0degree mark and the following whole angle, in which case you record the angle as X degrees + 30 minutes. Now, if you look closely at the two scales, you should see one line on the 0' – 30' scale matching up exactly with one line on the main, larger scale. The number on the smaller scale is the number of minutes you should add to the first measurement to find the angle accurate to degrees and minutes. Your final result should be in the form of X-degrees Y minutes, where Y is a number between 0 and 60: (0 or 30) + the number read off the smaller scale. For example, the angle in Fig. 2 would be 140^0 + 10'.

Note: You may have an older model spectrometer. The major difference is that the vernier scale is given in 10ths of a degree.

APPENDIX D

USE OF THE OSCILLOSCOPE

An oscilloscope (see Fig. 4) is a device designed to measure the dependence of a voltage on time. It does this using an electron beam and a phosphorescent screen, where the x-position of the beam is controlled by a time-scale, and the y-position is controlled by a voltage amplitude-scale. The screen on the oscilloscope is overlaid with a square grid, with markings along the center axis denoting every 0.2 square divisions. There is a set of dials and switches designed to control the position, size, and clarity of the beam for study. They are:

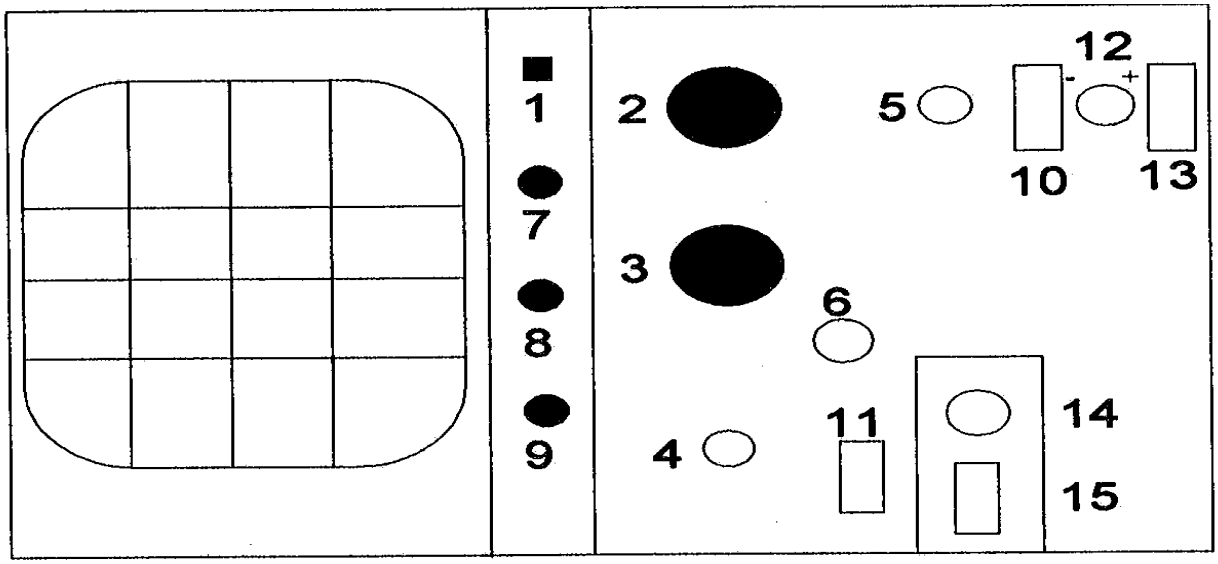


Figure 4. Oscilloscope

1. Power – On-Off
2. Sweep rate – This dial changes the time scale of the oscilloscope. It sets the scale at some unit of time/division, such that if it is set at 5 msec/div, every whole square division along the x-axis is 5 msec.
3. Voltage – Similar to the sweep rate dial, this sets the voltage amplitude scale to some voltage per division, such that if it set at 0.5 Volts/div, every whole square division along the y-axis is 0.5 V.
4. Input – This is where the coaxial cable carrying the input is connected.

5. This dial moves the entire signal left and right
6. This dial moves the entire signal up and down
7. Intensity – This dial changes the brightness of the beam. Do not set this very high – if left on for long periods of time it could damage the phosphorescent screen.
8. Focus – This dial focuses the beam.
9. Illumination – This dial turns on a light bulb to brighten the screen and scales.
10. Trigger – This switch should be set to 'Auto'.
11. This switch sets the input voltage to AC, DC, or the base-line Ground. It should usually be set to 'DC'.
12. Level – This dial sets the signal level at which the Oscilloscope begins its time sweep in order to display the input signal. If the signal seems to be moving around or if you are not seeing the very beginning of the signal, try raising or lowering the level.
13. Source – This switch should be set at 'Int'.
14. Mode – This dial should be set at 'Ch 1'.
15. Int. Trig – This switch should be set at 'Ch 1'.