# Math 340 — Problem Solving Seminar, Fall 2001, Problem Set #5

Please give each of these problems a try. We do not expect you to solve them all.

**Problem 1: Rotated digits.** Find a positive integer the first digit of which is 1, which has the property that, if this digit is transfered to the end of the number, the number is tripled.

**Problem 2: A sequence of odd integers.** The sequence $\{a_n\}$ of integers is defined by $a_1 = 2$, $a_2 = 7$, and

$$-\frac{1}{2} < a_{n+1} - \frac{a_n^2}{a_{n-1}} \leq \frac{1}{2}$$

Prove that $a_n$ is odd for all $n > 1$.

**Problem 3: Passing pennies.** Players 1, 2, 3, ..., $n$ are seated around a table. Each player begins with one penny. Player 1 begins by passing one penny to Player 2. Player 2 then passes two pennies on to Player 3. Player 3 passes one penny to Player 4, and then Player 4 passes two pennies on to Player 5. This pattern continues with each player passing alternately one or two pennies to the next player who still has pennies remaining. When a player runs out pennies, he is out of the game. (Note that both Players 1 and 2 exit the game after the first two passes.) Find an infinite set of odd numbers $n$ in which one player winds up with all of the pennies.

**Problem 4: An interesting integral.** Evaluate the integral

$$\int_0^\infty \left(x - \frac{x^3}{2} + \frac{x^5}{2 \cdot 4} - \frac{x^7}{2 \cdot 4 \cdot 6} + \cdots\right)\left(1 + \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} + \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \cdots\right) \mathrm{d}x$$