For this exam, please do all of your work on the bottom of the page or on the back side. To pass, you must exhibit mastery of the concepts examined. You may use a calculator and the provided normal table. **Please return the normal table with your exam and please do not write on it so we may use them again.** You may also use the fact that the binomial probability formula is

$$\Pr\{Y = j\} = {}_nC_j p^j (1-p)^{n-j} \qquad \text{for } j = 0, 1, 2, \ldots, n, \text{ where } {}_nC_j = \frac{n!}{j!(n-j)!}.$$

In a large population of university students, six percent of the students have blood type B. In addition, heights are approximately normal with mean 68.3 inches and standard deviation 3.7 inches.

(a) In a random sample of ten students, what is the probability that exactly one student has blood type B?

Solution: Let X be the number of sampled students with blood type B. $\Pr\{X = 1\} = {}_{10}C_1 (0.06)^1 (0.94)^9 = 0.3438$

(b) In a random sample of ten students, what is the probability that one or fewer students have blood type B?

Solution: $\Pr\{X \leq 1\} = {}_{10}C_0 (0.06)^0 (0.94)^{10} + {}_{10}C_1 (0.06)^1 (0.94)^9 = 0.8824$

(c) In a random sample of ten students, what is the probability that one or more students have blood type B?

Solution: $\Pr\{X \geq 1\} = 1 - \Pr\{X = 0\} = 1 - {}_{10}C_0 (0.06)^0 (0.94)^{10} = 0.4614$

(d) What proportion of the students are taller than 72 inches?

Solution: Let Y be the height of a randomly selected student. $\Pr\{Y > 72\} = \Pr\{(Y - 68.3)/3.7 > (72 - 68.3)/3.7\} = \Pr\{Z > 1.00\} = 0.1587$

(e) What proportion of the students have heights between 60 and 70 inches?

Solution: $\Pr\{60 < Y < 70\} = \Pr\{(60-68.3)/3.7 < (Y-68.3)/3.7 < (70-68.3)/3.7\} = \Pr\{-2.24 < Z < 0.46\} = 0.6647$

(f) The middle 90% of the students will have heights between which two values? *[Hint: Which percentiles cut off the middle 90%?]*

Solution: The middle 90% of the distribution is between the 5th and 95th percentiles. Using the normal table, these are $z = \pm 1.645$. So, the values are $68.3 \pm 1.645(3.7)$ or 62.2 and 74.4.