1. Comparison of Paired Samples
   In a paired design, the observations \((Y_1, Y_2)\) occur in pairs (not independent). Instead of considering \(Y_1\) and \(Y_2\) independently, we consider the DIFFERENCE \(d\), defined as \(d = Y_1 - Y_2\).

   - Relationship between sample means and population means:
     \[
     \bar{d} = \bar{y}_1 - \bar{y}_2 \\
     \mu_d = \mu_1 - \mu_2
     \]

   - Standard error of \(\bar{d}\) is
     \[
     SE_{\bar{d}} = \frac{s_d}{\sqrt{n_d}}
     \]

   - \(t\) Test
     \[
     H_0: \mu_d = 0 \\
     t_s = \frac{\bar{d} - 0}{SE_{\bar{d}}}
     \]

   - \((1 - \alpha)\) Confidence Interval for \(\mu_d\) is
     \[
     \bar{d} \pm t_{\frac{\alpha}{2}} SE_{\bar{d}}
     \]

     e.g., If \(\alpha = 10\%\), then 90\% confidence interval of \(\mu_d\) is \(\bar{d} \pm t_{0.05} SE_{\bar{d}}\)

2. Analysis of Categorical Data
   - The chi-square goodness-of-fit test
     \[
     H_0: \Pr\{\text{categorical 1}\} = p_1, \Pr\{\text{categorical 2}\} = p_2, \ldots
     \]

     \(H_A\): At least one of the probabilities specified in \(H_0\) is incorrect

     Chi-square Statistics is
     \[
     \chi^2 = \sum \frac{(O - E)^2}{E}
     \]
where the summation is over all the categories. \( O \) represents the observed frequency of the category and \( E \) represents the expected frequency. For categorial \( i \), \( E = n \times p_i \), where \( n \) is the total number of observations: \( n = \sum O \).

Under \( H_0 \), if the sample size is large enough, the distribution of \( \chi^2 \) can be approximated by \( \chi^2 \) distribution with degree of freedom as
\[
df = (\text{number of categories}) - 1
\]

- The chi-square test for the 2x2 contingency table

\[
H_0 : p_1 = p_2
\]

The Chi-square statistics is
\[
\chi^2 = \sum \frac{(O - E)^2}{E}
\]

where the sum is taken over all four cells in the contingency table. \( O \) represents the observed frequency and \( E \) represents the correspondint expected frequency according to \( H_0 \), and
\[
E = \frac{(\text{Row total}) \times (\text{Column total})}{\text{Grand total}}
\]

The degree of freedom is
\[
df = 1
\]